THE INTEGRATION

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1. **Anti-derivative**

We have studied how to find the derivative of a function. However, many problems require that we recover a function from it known derivative. For instance, we many know the velocity function of an object falling from an initial height and need to know its height at any time. More generally, we want to find a function F from its derivative f. If such a function F exists, it is called an *antiderivative* of f.

Definition 1.1. A function F is an **antiderivative** of f on an interval I if $F^{'}(x) = f(x)$ for all x in I.

Example 1.1. x^3 is an antiderivative of $3x^2$, since $D_x(x^3) = 3x^2$. But $x^3 + 5$ is also an antiderivative of $3x^2$, since $D_x(5) = 0$.
In general, if $F(x)$ is an antiderivative of $f(x)$, then $F(x) + C$ is also an antiderivative of $f(x)$, where C ia any constant.

2. **Indefinite Integrals**

A special symbol is used to denoted the collection of all antiderivatives of a function f.

Definition 2.1. The collection of all antiderivative of f is called the **indefinite integral** of f which respect to x, and denoted by

$$\int f(x)dx.$$

The symbol $\int$ is an **integral sign**. The function f is the **integrand** of the integral, and x is the **variable of integration**.

2.1. **Low of Antiderivative (Indefinite Integral).**

(1) $\int dx = x + C.\quad C$ is constant of integral.
(2) $\int af(x)dx = a\int f(x)dx.\quad a$ is any constant.
(3) $\int x^m dx = \frac{x^{m+1}}{m+1} + C, \quad m \neq -1, m \in R$
(4) $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx.$
(5) $\int [f(x)]^m \cdot f(x)dx = \frac{[f(x)]^{m+1}}{m+1} + C, \quad m \neq -1.$

Example 2.2. *evaluate the following integrals:*

(1) $\int (x^2 + 3x + 5)dx.$

(2) $\int (x^2 + 2)^2 dx.$

(3) $\int 3\sqrt[3]{(3x+1)}dx.$

(4) $\int (\frac{x^3+3x^2-2}{x^2})dx.$

(5) $\int x^4 \sqrt[3]{3-5x^5}dx.$

Solution

(1) $= \int x^2dx + \int 3xdx + \int 5dx = \int x^2 + 3\int x + 5\int dx = \frac{x^3}{3} + 3\frac{x^2}{2} + 5x + C.$

(2) $= \int (x^4+4x^2+4)dx = \int x^4 dx + 4\int x^2 + 4\int dx = \frac{x^5}{5} + 4\frac{x^3}{3} + 4x + C.$

(3) $= \int 3(3x+1)^{\frac{1}{2}} dx = \frac{(3x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{(3x+1)^{\frac{3}{2}}}{\frac{3}{2}} + C.$

(4) $= \int (\frac{x^3}{x^2} + 3\frac{x^2}{x^2} - \frac{2}{x^2})dx = \int x dx + 3\int dx - 2\int x^{-2} dx = \frac{x^2}{2} + 3x + \frac{2}{x} + C.$

(5) $= \int x^4 (3-5x^5)^{\frac{1}{3}} dx = \frac{-1}{25} \int -25x^4 (3-5x^5)^{\frac{1}{3}} dx = \frac{-1}{25} \frac{(3-5x^5)^{\frac{4}{3}}}{\frac{4}{3}} + C.$