(تمارين محلولة) ①

اوجد التكاملات الاتية

1- $\int \left(\frac{7}{x^{\frac{5}{4}}} - \sqrt[3]{x} + 4\sqrt{x} \right) dx$

$$= 7\int x^{-\frac{5}{4}} dx - \int x^{\frac{1}{3}} dx + 4\int x^{\frac{1}{2}} dx$$

$$= 7\,\frac{x^{\frac{1}{4}}}{\frac{1}{4}} - \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + 4\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = 28x^{\frac{1}{4}} - \frac{3}{4}x^{\frac{4}{3}} + \frac{8}{3}x^{\frac{3}{2}} + c$$

2- $\int \frac{(\ln x^2)^2}{x} dx = \frac{1}{2}\int 2\,\frac{(\ln x^2)^2}{x} dx = \frac{1}{2}\,\frac{(\ln x^2)^3}{3} + C$ o.k

مشتقة $\ln x^2$ هي $\frac{1}{x^2}\cdot 2x = \frac{2}{x}$

3- $\int (2+x^2)^2 dx = \int (4+4x^2+x^4)\, dx$

$$= 4\int dx + 4\int x^2 dx + \int x^4 dx = 4x + \frac{4x^3}{3} + \frac{x^5}{5} + c$$

4. $\int e^{2x}\sqrt{1+e^{2x}}\, dx = \int e^{2x}\cdot(1+e^{2x})^{\frac{1}{2}} dx = \frac{1}{2}\int 2e^{2x}\cdot(1+e^{2x})^{\frac{1}{2}} dx$

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$$= \frac{1}{2}\cdot\frac{(1+e^{2x})^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{1}{3}(1+e^{2x})^{\frac{3}{2}} + c$$

5. $\int \frac{dx}{(\frac{1}{3}x-8)^5} dx = \int (\frac{1}{3}x-8)^{-5} dx = 3\int \frac{1}{3}(\frac{1}{3}x-8)^{-5} dx$

$$= 3\,\frac{(\frac{1}{3}x-8)^{-4}}{-4} + c = \frac{-3}{4(\frac{1}{3}x-8)^4} + c$$

6. $\int \frac{(\sqrt{x}+2)^8}{\sqrt{x}} dx = 2\int \frac{(\sqrt{x}+2)^8}{2\sqrt{x}} dx = 2\cdot\frac{(\sqrt{x}+2)^9}{9} + c$

7. $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx = 2\int \frac{\cos\sqrt{x}}{2\sqrt{x}} dx = 2\sin\sqrt{x} + c$

8. $\int \frac{\cos(\ln x)}{x} dx = \int \frac{1}{x}\cos(\ln x)\, dx = \sin\ln x + c$ o.k

9. $\int \frac{\sin(\frac{5}{x})}{x^2} dx = \frac{-1}{5}\int \frac{-5}{x^2}\sin(\frac{5}{x})\, dx = -\frac{1}{5}\left[-\cos(\frac{5}{x})\right] + c$

$$= \frac{1}{5}\cos(\frac{5}{x}) + C$$

10. $\int \sin(\sin x)\cos x\, dx = -\cos(\sin x) + C$

[illegible]

11. $\int \frac{\sin 2x}{\cos x}\,dx = \int \frac{2\sin x \cancel{\cos x}}{\cancel{\cos x}}\,dx = 2\int \sin x = -2\cos x + C$

12. $\int x^2 \sec^2 x^3\,dx = \frac{1}{3}\int 3x^2 \sec^2 x^3\,dx = \frac{1}{3}\tan x^3 + C$

13. $\int \frac{\sec x}{\cos x}\,dx = \int \sec x \cdot \frac{1}{\cos x}\,dx = \int \sec x \sec x\,dx = \int \sec^2 x\,dx$

$= \tan x + C$

14. $\int \sec^2(\sin 5x)\cos 5x\,dx = \frac{1}{5}\int \sec^2(\sin 5x)\cdot 5\cos 5x\,dx = \frac{1}{5}\tan(\sin 5x) + C$

16. $\int \left(\frac{\cos^3 x - 5}{\cos^2 x}\right) dx = \int \left(\cos x - \frac{5}{\cos^2 x}\right) dx = \int \cos x\,dx - 5\int \frac{1}{\cos^2 x}\,dx$

o.k

$= \sin x - 5\int \sec^2 x\,dx = \sin x - 5\tan x + C$

17. $\int \frac{dx}{\csc(2x)} = \int \sin(2x)\,dx = \frac{1}{2}\int 2\sin(2x)\,dx = -\frac{1}{2}\cos(2x) + C$

o.k

18. $\int e^{4x}\sec(e^{4x})\tan(e^{4x})\,dx = \frac{1}{4}\int 4e^{4x}\sec(e^{4x})\tan(e^{4x})\,dx = \frac{1}{4}\sec(e^{4x})$

19. $\int \sqrt{\sin \pi x}\,\cos \pi x\,dx = \frac{1}{\pi}\int (\sin \pi x)^{\frac{1}{2}}\cdot \pi\cos \pi x\,dx$

o.k

$= \frac{1}{\pi}\,\frac{(\sin \pi x)^{3/2}}{3/2} + C = \frac{2}{3\pi}(\sin \pi x)^{3/2} + C$

20. $\int \tan^3(5x)\sec^2(5x)\,dx = \frac{1}{5}\int \tan^3(5x)\cdot 5\sec^2(5x)\,dx = \frac{1}{5}\,\frac{(\tan 5x)^4}{4} + C$

o.k

21. $\int \sqrt{\tan x}\,\sec^2 x\,dx = \frac{(\tan x)^{\frac{3}{2}}}{\frac{3}{2}} + C$

o.k

22. $\int \frac{\sin(2x)\,dx}{(5+\cos 2x)^3} = \int \sin(2x)\,(5+\cos(2x))^{-3}\,dx$

$= -\frac{1}{2}\int -2\sin(2x)\,(5+\cos 2x)^{-3}\,dx = -\frac{1}{2}\,\frac{(5+\cos 2x)^{-2}}{-2} + C$

$= \frac{1}{4(5+\cos 2x)^2} + C$

③

بالتوزيع

ex.6 $\int \frac{dx}{x\ln x} = \int \frac{\frac{dx}{x}}{\frac{x\ln x}{x}} = \int \frac{\frac{dx}{x}}{\ln x} = \int \frac{\frac{1}{x}dx}{\ln x} = \ln|\ln x| + C$

ex.7 $\int (3x+1)\cot(3x^2+2x)\,dx = \int (3x+1)\frac{\cos(3x^2+2x)}{\sin(3x^2+2x)}\,dx$ D (H.W)

$= \frac{1}{2}\int (6x+2)\frac{\cos(3x^2+2x)}{\sin(3x^2+2x)}\,dx = \frac{1}{2}\ln|\sin(3x^2+2x)| + C$

ex.8 $\int \frac{\sin 3x}{1+\cos 3x}\,dx = -\frac{1}{3}\int \frac{-3\sin 3x}{1+\cos 3x}\,dx = -\frac{1}{3}\ln|1+\cos 3x| + C$ B

ex.9 $\int \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\,dx = \ln|e^{x}-e^{-x}| + C$ B+C+D (H.W) KWZ

نضرب البسط والمقام ب e^{-x}

ex.10 $\int \frac{dx}{1+e^{x}} = \int \frac{e^{-x}\,dx}{(1+e^{x})e^{-x}} = \int \frac{e^{-x}\,dx}{e^{-x}+1} = -\int \frac{-e^{-x}\,dx}{e^{-x}+1} = -\ln|e^{-x}+1| + C$ B+C+D

نضرب

ex.11 $\int \frac{\ln x}{x}\,dx = \int \frac{1}{x}\ln x\,dx = \frac{(\ln x)^2}{2} + C$ B+C+D

ex.12 $\int \frac{2e^{2x}+1}{e^{2x}+x}\,dx = \ln|e^{2x}+x| + C$ C

ex.13 $\int \frac{\sec^2 x}{3\tan x+5}\,dx = \frac{1}{3}\int \frac{3\sec^2 x}{3\tan x+5}\,dx = \frac{1}{3}\ln|3\tan x+5| + C$ D (H.W)

ex.14 $\int \frac{\sec(5x)\tan(5x)}{2\sec(5x)-1}\,dx = \frac{1}{10}\int \frac{10\sec(5x)\tan(5x)}{2\sec(5x)-1}\,dx$ D+B+C

$= \frac{1}{10}\ln|2\sec(5x)-1|$

ex.15 $\int e^{\tan 10x}\sec^2 10x\,dx = \frac{1}{10}\int e^{\tan 10x}(10\sec^2 10x)\,dx = \frac{1}{10}e^{\tan 10x} + C$

(4)

" تمارين محلولة "

1. $\int e^{\tan 10x} \sec^2 10x \, dx$

$= \frac{1}{10} \int e^{\tan 10x} (10 \sec^2 10x) \, dx = \frac{1}{10} e^{\tan 10x} + c$

2. $\int \pi^{\sin 2x} \cos 2x \, dx = \frac{1}{2} \int \pi^{\sin 2x} \cdot 2 \cos 2x \, dx$

$= \frac{1}{2} \frac{\pi^{\sin 2x}}{\ln \pi} + c$

3. $\int 5^{x^2+6x} (x+3) \, dx = \frac{1}{2} \int 5^{x^2+6x} (2x+6) \, dx = \frac{1}{2} \cdot \frac{5^{x^2+6x}}{\ln 5} + c$

4. $\int \frac{dx}{\sqrt{x}\, e^{\sqrt{x}}} = \int \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx = -2 \int -\frac{1}{2\sqrt{x}} e^{-\sqrt{x}} dx = -2 e^{-\sqrt{x}} + c$

5. $\int (3x+1) \cot(3x^2+2x) \, dx = \int (3x+1) \frac{\cos(3x^2+1)}{\sin(3x^2+1)} dx$

$= \frac{1}{2} \int (6x+2) \frac{\cos(3x^2+1)}{\sin(3x^2+1)} dx = \frac{1}{2} \ln |\sin(3x^2+1)| + c$

6. $\int$