32* H.W. 3

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(3)

س/ جد التكامل الآتي:

ex/ $\int e^{2x} \cos 3x \, dx$

sol/ let $u = e^{2x} \Rightarrow du = 2e^{2x} dx$

& let $dv = \cos 3x \, dx \Rightarrow \int dv = \int \cos 3x \, dx = \frac{1}{3}\int 3\cos 3x \, dx = \boxed{\frac{1}{3}\sin 3x}$

$\therefore \int e^{2x} \cos 3x \, dx = \int u \, dv = u.v - \int v \, du$

$= e^{2x} . \frac{\sin 3x}{3} - \int \frac{\sin 3x}{3} . 2e^{2x} dx$

$\therefore \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3}\int \sin 3x \, e^{2x} dx \quad \text{------}(1)$

الان نجد $\int e^{2x}\sin 3x dx$ باستخدام ايضا طريقة التجزئة

$\int e^{2x} \sin 3x \, dx$

let $u = e^{2x} \Rightarrow du = 2e^{2x} dx$

& let $dv = \sin 3x \, dx \Rightarrow v = \frac{-\cos 3x}{3}$

$\therefore \int e^{2x} \sin 3x \, dx = u.v - \int v \, du$

$= e^{2x} . \left(-\frac{\cos 3x}{3}\right) - \int -\frac{\cos 3x}{3} . 2e^{2x} dx$

$= -\frac{1}{3}\cos 3x . e^{2x} + \frac{2}{3}\int e^{2x} \cos 3x \, dx$

وبالتعويض في (1) ينتج:

$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3}\int \sin 3x \, e^{2x} dx$

$= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3}\left[-\frac{1}{3} e^{2x} \cos 3x + \frac{2}{3}\int e^{2x} \cos 3x \, dx\right]$

$\Rightarrow \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9}\int e^{2x} \cos 3x \, dx$

let $\int e^{2x} \cos 3x \, dx = I$

$\therefore I = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9} I$

$\Rightarrow I + \frac{4}{9} I = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x$

$I\left(1 + \frac{4}{9}\right) = I\left(\frac{13}{9}\right) = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x$

$\therefore \frac{13}{9} I = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x$

وبقسمة الطرفين على $\frac{13}{9}$ ينتج ان:

$I = \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x$

نرجع قيمة I ينتج:

$\int e^{2x} \cos 3x \, dx = \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + c$

H-W 4 — 3

تمارين محلولة

①

1. $\int \ln(2x+3)\,dx$

Sol. let $u=\ln(2x+3) \Rightarrow du=\frac{2}{2x+3}\,dx$

& let $dv=dx \Rightarrow v=x$

$\therefore \int \ln(2x+3)\,dx = \int u\,dv = u.v-\int v\,du$

$= x\ln(2x+3)-\int x\cdot\frac{2}{2x+3}\,dx$

$= x\ln(2x+3)-\boxed{\int\frac{2x}{2x+3}\,dx} \longrightarrow$ ممكن حلها امتحان هذا الاختيار لوحده

$= x\ln(2x+3)-\int\frac{2x+3-3}{2x+3}\,dx$ بإضافة 3 وطرح 3 إلى البسط

$= x\ln(2x+3)-\int\frac{\cancel{2x+3}}{\cancel{2x+3}}\,dx+\int\frac{-3}{2x+3}\,dx$

$= x\ln(2x+3)-\int dx+3\int\frac{1}{2x+3}\,dx$

$= x\ln(2x+3)-x\mp\frac{3}{2}\int\frac{2\,dx}{2x+3}$ حيث هنا نوفر المشتقة للمقام نضرب × 2 ونقسم على 2

$= x\ln(2x+3)-x\mp\frac{3}{2}\ln|2x+3|+c$

2. $\int \tan^{-1}(3x)\,dx$

let $u=\tan^{-1}(3x) \Rightarrow du=\frac{3\,dx}{1+(3x)^2}$

& let $dv=dx \Rightarrow v=x$

$\therefore \int \tan^{-1}(3x)\,dx=\int u\,dv=uv-\int v\,du$

$= \tan^{-1}(3x)\cdot x-\int x\cdot\frac{3\,dx}{1+9x^2}$

$= x\tan^{-1}(3x)-\int\frac{3x\,dx}{1+9x^2} = x\tan^{-1}(3x)-\frac{1}{6}\int\frac{18x\,dx}{1+9x^2}$

$= x\tan^{-1}(3x)-\frac{1}{6}\ln|1+9x^2|+c$

3. $\int \sin(\ln x)\, dx$

Solv. let $u = \sin(\ln x) \Rightarrow du = \cos(\ln x) \cdot \frac{1}{x}\, dx$

$*$ let $dv = dx \Rightarrow v = x$

$\therefore \int \sin(\ln x)\, dx = \int u\, dv = u \cdot v - \int v\, du$

$$= \sin(\ln x) \cdot x - \int \cancel{x} \cos(\ln x) \cdot \frac{1}{\cancel{x}}\, dx$$

$\therefore \int \sin(\ln x)\, dx = x \sin(\ln x) - \int \cos(\ln x)\, dx$ ---------- (1)

الآن نجد التكامل $\int \cos(\ln x)dx$ أيضاً باستخدام طريقة التجزئة

$\therefore \int \cos(\ln x)\, dx = uv - \int v\, du$

let $u = \cos(\ln x) \Rightarrow du = -\sin(\ln x) \cdot \frac{dx}{x}$

$*$ let $dv = dx \Rightarrow v = x$

$\therefore \int \cos(\ln x)\, dx = x \cos(\ln x) - \int \cancel{x} \cdot -\sin(\ln x) \frac{dx}{\cancel{x}}$

$$= x \cos(\ln x) + \int \sin(\ln x)\, dx$$

وبالتعويض في معادلة (1) ينتج :

$\int \sin(\ln x)\, dx = x \sin(\ln x) - \left[x \cos(\ln x) + \int \sin(\ln x)\, dx \right]$

$\therefore \int \sin(\ln x)\, dx = x \sin(\ln x) - x \cos(\ln x) - \int \sin(\ln x)\, dx$

$\Rightarrow 2 \int \sin(\ln x)\, dx = x \sin(\ln x) - x \cos(\ln x) + C$

بالقسمة ÷ 2

$\therefore \int \sin(\ln x)\, dx = \frac{1}{2} x \sin(\ln x) - \frac{1}{2} x \cos(\ln x) + C$