③ Q/ Are the following binary operations asso. or not ?

(a) $a * b = \frac{1}{2}(a+b) \quad \forall a, b \in (\mathbb{Q}, *)$

Sol.: let $a, b, c \in \mathbb{Q}$

$(a*b)*c \stackrel{?}{=} a*(b*c)$

$\left[\frac{1}{2}(a+b)\right]*c \stackrel{?}{=} a*\left[\frac{1}{2}(b+c)\right]$

$\frac{1}{2}\left[\frac{1}{2}(a+b)\right]+\frac{1}{2}c \stackrel{?}{=} \frac{1}{2}a+\frac{1}{2}\left[\frac{1}{2}(b+c)\right]$

$\frac{1}{4}a+\frac{1}{4}b+\frac{1}{2}c \neq \frac{1}{2}a+\frac{1}{4}b+\frac{1}{4}c$

$\therefore$ $*$ is not asso.

(b) $a * b = a+b-1 \quad \forall a, b \in (\mathbb{Q}, *)$

Sol.: let $a, b, c \in \mathbb{Q}$

$(a*b)*c \stackrel{?}{=} a*(b*c)$

$(a+b-1)*c \stackrel{?}{=} a*(b+c-1)$

$a+b+c-2 = a+b+c-2$

$\therefore$ $*$ is asso.

(c) $(a,b) * (c,d) = (a+c, b+d+2bd) \quad \forall (a,b),(c,d) \in (\mathbb{R}\times\mathbb{R}, *)$

Sol.: let $(a,b),(c,d),(e,f) \in \mathbb{R}\times\mathbb{R}$

$[(a,b)*(c,d)]*(e,f) \stackrel{?}{=} (a,b)*[(c,d)*(e,f)]$

$(a+c, b+d+2bd)*(e,f)$

$(a+c+e, b+d+f+4bdf+2bd+2bf+2df)$ --- (1)