

④ Right hand

$$(a|b) * [c|d] * (e|f) =$$

$$(a|b) * (c+e, ~~d+f~~ d+f+2df) =$$

$$(a+c+e, b+d+f+2df+2bd+2bf+4dbf) \dots \textcircled{2}$$

$$\therefore \textcircled{1} = \textcircled{2}$$

* is assoc.

① $a * b = b \quad \forall a, b \in (\mathbb{Z}, *)$

sol. : let $a, b, c \in \mathbb{Z}$

$$(a * b) * c \stackrel{?}{=} a * (b * c)$$

$$b * c \stackrel{?}{=} a * c$$

$$c = c$$

$\therefore *$ is asso.