

⑦ Q / Show whether the math. sys. $(Z_6, +_6)$ is comm. gp. or not?

Sol. : $Z_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$

$+_6$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$
$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{0}$
$\bar{2}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{0}$	$\bar{1}$
$\bar{3}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{0}$	$\bar{1}$	$\bar{2}$
$\bar{4}$	$\bar{4}$	$\bar{5}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{5}$	$\bar{5}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$

① Closure is satisfy from the table

② Asso., let $\bar{2}, \bar{3}, \bar{5} \in Z_6$

$$(\bar{2} +_6 \bar{3}) +_6 \bar{5} \stackrel{?}{=} \bar{2} +_6 (\bar{3} +_6 \bar{5})$$

$$\bar{5} +_6 \bar{5} \stackrel{?}{=} \bar{2} +_6 \bar{2}$$

$$\bar{4} = \bar{4}$$

\therefore Asso. is satisfy

③ Identity element is $\bar{0}$ as we show from the table

④ Inverse

$$\bar{0}^{-1} = \bar{0}$$

$$\bar{1}^{-1} = \bar{5}$$

$$\bar{2}^{-1} = \bar{4}$$

$$\bar{3}^{-1} = \bar{3}$$

$$\bar{4}^{-1} = \bar{2}$$

$$\bar{5}^{-1} = \bar{1}$$

$\therefore (Z_6, +_6)$ is gp.