

$$\textcircled{8} \quad \bar{4} +_6 \bar{5} \stackrel{?}{=} \bar{5} +_6 \bar{4}$$

$$\bar{3} = \bar{3}$$

$$\therefore a * b = b * a \quad \forall a, b \in G$$

$$\text{i.e. } \bar{a} +_6 \bar{b} = \bar{b} +_6 \bar{a} \quad \forall a, b \in \mathbb{Z}_6$$

$\therefore (\mathbb{Z}_6, +_6)$ is comm. gp.

Q/ Show whether the math. sys. $(\mathbb{Z}\sqrt{3}, +)$ is gp. or not?

Sol.:

$$\textcircled{1} \text{ Let } a + b\sqrt{3}, c + d\sqrt{3} \in \mathbb{Z}\sqrt{3}$$

$$(a + b\sqrt{3}) + (c + d\sqrt{3}) = (a + c) + (b + d)\sqrt{3} \in \mathbb{Z}\sqrt{3}$$

\therefore closure is satisfy

$$\textcircled{2} \text{ Let } \left. \begin{array}{l} a_1 + b_1\sqrt{3} \\ a_2 + b_2\sqrt{3} \\ a_3 + b_3\sqrt{3} \end{array} \right\} \in \mathbb{Z}\sqrt{3}$$

$$\left[(a_1 + b_1\sqrt{3}) + (a_2 + b_2\sqrt{3}) \right] + (a_3 + b_3\sqrt{3}) \stackrel{?}{=} (a_1 + b_1\sqrt{3}) + \left[(a_2 + b_2\sqrt{3}) + (a_3 + b_3\sqrt{3}) \right]$$

L.H.

$$\left[(a_1 + a_2) + (b_1 + b_2)\sqrt{3} \right] + (a_3 + b_3\sqrt{3}) =$$

$$(a_1 + a_2 + a_3) + (b_1 + b_2 + b_3)\sqrt{3} \quad \dots \textcircled{1}$$

R.H.

$$(a_1 + b_1\sqrt{3}) + \left[(a_2 + a_3) + (b_2 + b_3)\sqrt{3} \right] =$$

$$(a_1 + a_2 + a_3) + (b_1 + b_2 + b_3)\sqrt{3} \quad \dots \textcircled{2}$$

$$\therefore \textcircled{1} = \textcircled{2}$$