Chapter Two
(Turbulence Structure)

2.1 Turbulent and its Spectrum:

Turbulent is an intrinsic part of the atmospheric boundary layer that must be quantified in order to study it. The randomness of turbulence makes deterministic description difficult, we are forced to retreat to the use of statistics, where we are limited to average or expected measures of turbulence. in this chapter we review some basic statistical methods and show how measurements of turbulence can be put into statistical framework .statistical descriptors such as the variance or covariance are limited usefulness unless we can physically Interpret them . variances are shown to be measures of turbulence intensity or turbulence kinetic energy , and covariance are shown to be measures of flux or stress . On other hand the turbulent spectrum is analogous to the spectrum of colors that appears when you shine a light through a prism. White light consists of many colors (i.e., many wavelengths or frequencies) superimposed on another. The prism is a physical device that separates the colors. We could measure the intensity of each color to learn the magnitude of its contribution to the original light beam. We can perform a similar analysis on a turbulent signal using mathematical rather than physical devices to learn about the contribution of each different size eddy to the total turbulence kinetic energy. Figure 2.1 shows an example of the spectrum of wind speed measured near the ground. The ordinate is a measure of the portion of turbulence energy that is associated with a particular size eddy. The abscissa gives the eddy size in terms of the time period and frequency of the wind –speed variation. Small eddies have shorter time periods than large eddies (again, using Taylor's hypothesis).Peaks in the spectrum show which size eddies contribute the most to the turbulence kinetic energy. the first peak with a period of near 100hour corresponds to wind speed variations associated with the passage of fronts and weather systems , in other words , there is evidence of the Rossby waves cyclone in our wind speed record . The next peak, at 24 hour, shows the diurnal increase of wind speed during the day and decrease at night. The third peaks is the one we will study in this book, its indicates the microscale eddies that having durations of 10sec to 10min.

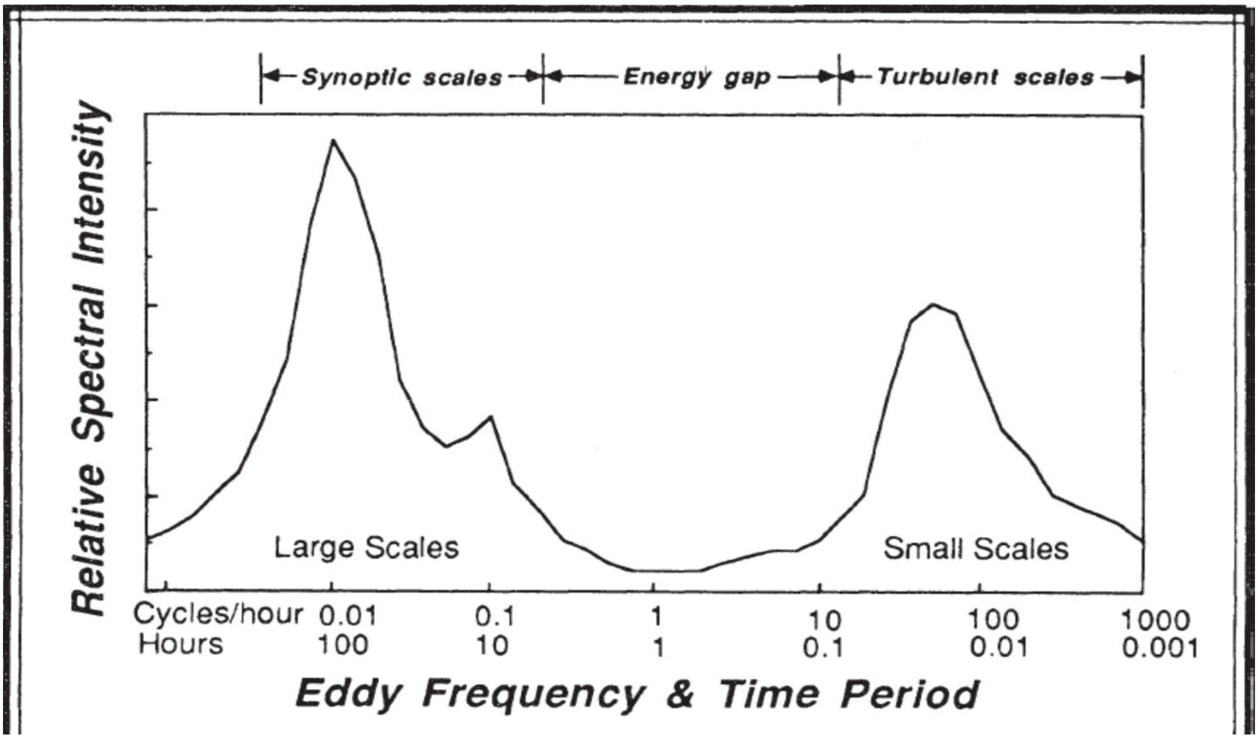

Figure (2.1) : Schematic spectrum of wind speed near ground .

In the first peak we see that the largest eddies are usually the most intense. the smaller high frequency eddies are very weak , as previously discussed , large – eddies motions can create eddy –size wind shear regions , which can generate smaller eddies , such a net transfer of turbulence energy from the larger to smaller eddies is known as the energy cascade , at the smallest size eddies , this cascade of energy is dissipate in to heat by molecular viscosity .

2.2 Mean and Turbulent parts:

There is a very easy way to isolate the large-scale variations from the turbulent once, by averaging our wind speed measurements over a period of 30 minutes to one hour, we can eliminate or "average out" the positive and negative deviations of the turbulent velocities about the mean-once we have the mean velocity $\bar{u}$, for any time period, we can subtract it from the actual instantaneous velocity, u, to give us just the turbulent part.

$$u' = u - \bar{u}$$

We can think of u as the gust that is superimposed on the mean wind. It represents the part of the flow that varies with periods shorter than about one hour. Micro scale turbulent is a three – dimensional phenomena. therefore we expect that gusts in the x-direction might be accompanied by the gust in the y- and z- directions . Turbulence by definition is the type of motion, yet motions frequently cause variations in the temperature, moisture, and pollutant fields if there is some mean gradient of that variable across the turbulent. Hence, can partition each of these variable in to mean and turbulent parts:

$$u' = u - \bar{u}$$

$$v' = v - \bar{v}$$

$$w' = w - \bar{w}$$

$$\theta' = \theta - \bar{\theta}$$

$$q' = q - \bar{q}$$

$$c' = c - \bar{c}$$

2.3 Some Basic Statistical Method:

Because one of the primary possibilities for studying turbulent flow is the stochastic approach. This section will survey some of the basic methods of statistics, including the mean, variance, standard deviation, covariance, and correlation coefficient.

2.3.1 The Mean:

An ensemble average consists of the sum over N identical experiments:

$$\overline{A(t,s)} = \frac{1}{N}\sum_{i=0}^{N-1} A(t,s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

We can summarize the rules of averaging:

$$\bar{C} = \mathrm{C} \qquad \text{where C= constant}$$

$$(\overline{\mathrm{C}A}) = C\bar{A}$$

$$\left(\bar{\bar{A}}\right) = \bar{A}$$

$$\left(\overline{\bar{A}\,B}\right) = \bar{A}\,.\bar{B}$$

$$(\overline{A+B}) = \bar{A} + \bar{B}$$

$$\left(\overline{\frac{dA}{dt}}\right) = \frac{d\bar{A}}{dt}$$

The averaging rules of the last section can now be applied to variables that are split in to mean and turbulent parts. Let a $A = \bar{A} + A'$, and $B = \bar{B} + B'$. starting with the instantaneous value , A, for example , we can find its mean using the fifth and third rules of the previous section .

$$(\bar{A}) = \overline{(A' + \overline{A})} = (\overline{\bar{A}}) + \bar{A'} = \bar{A} + \bar{A'} \quad , \qquad (\bar{A}) = (\bar{A})$$

$$where \quad (\overline{A'}) = 0$$

This result is not surprising if one remembers the definition of the mean value. By definition, the sum of the positive deviations from the mean must equal the sum of the negative deviations. Thus the deviations balance when summed, as implied in the above average. The nonlinear product $\overline{(A'B')}$ is not necessarily zero. The same conclusion holds for other variables such as:

$$\overline{A'^2} \quad , \qquad \overline{A'B'^2} \quad , \quad \overline{A'^2B'^2}$$

2.3.2 Variance, Standard deviation and Turbulent Intensity:

One statistical measure of the dispersion of data about the mean is the variance defined by:

$$\sigma_A{}^2 = \frac{1}{N}\sum_{i=0}^{N-1}(A - \bar{A})^2 = \overline{A'^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Recall that the turbulent part (or the perturbation or gust part) of turbulent variable is given by $A' = A + \bar{A}$, substituting this in to the variance gives:

$$\sigma_A{}^2 = \frac{1}{N}\sum_{i=0}^{N-1} A'^2 = \overline{A'^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\sigma_A = \left(\overline{A'^2}\right)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Thus, whenever we encounter the average of the square of a turbulent part of a variable, such as $\overline{u'^2}, \overline{v'^2}, \overline{w'^2}, \overline{\theta'^2}, \overline{r'^2}$ or $\overline{q'^2}$ we can interpret these as the variance. The standard deviation always has the same dimensions as the original variable. Figure 2.6 show the relationship between a turbulent trace of wind speed and the corresponding standard deviation. It can be interpreted as a measure of the magnitude of the spread or dispersion of the original data from its mean, for this reason, its used as a measure of the intensity of the turbulent.

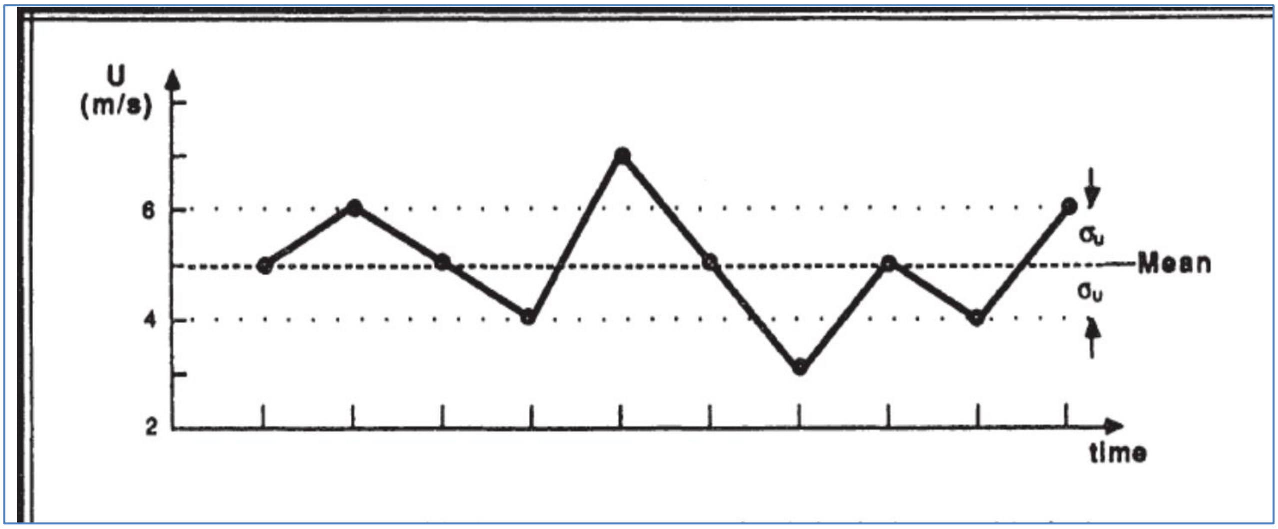

Figure (2.2): relationship between the standard deviation and turbulence variation. Solid line connects the data points, heavy dashed line is the average , dotted lines are drawn one standard deviation above and below the mean.

Near the ground, the turbulence intensity might be expected to increase as the mean wind speed, M, increase. For this reason a dimensionless measure of turbulence intensity, I, is often defined as:

$$I = \frac{\sigma_M}{\overline{M}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5)$$

For mechanically generated turbulence, one might expect σ_M to be simple function.

2.2.3 Covariance and Correlation:

In statistics, the covariance between variances is defined as:

$$covar\,(A,B) = \frac{1}{N}\sum_{i=0}^{N-1}(A-\bar{A})\,(B-\bar{B})$$

Using averaging methods, we can show that:

$$covar\,(A,B) = \frac{1}{N}\sum_{i=0}^{N-1} A'\,B' \; = \overline{A'B'} \dots\dots\dots\dots\dots\dots\dots\dots (6)$$

The covariance indicates the degree of common relationship between the two variables $A\,,B$. for example , let A represent air temperature , T , and let B be the vertical velocity , W , on a hot summer day over land , we might expect the warmer than average air to rise (positive T' and positive W') and cooler than average air to sink (negative T' and negative W') . Thus, the product $W'T'$ will be positive on average indicating that W and T very together. The covariance $\overline{W'T'}$ is indeed found to be positive throughout the bottom 80% of convective mixed layer . Sometimes, one is interested in a normalized covariance. Such a relationship is defined as the linear correlation coefficient

$$r_{AB} = \frac{\overline{A'B'}}{\sigma_A\;\sigma_B} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

This variable ranges between -1 and +1 by definition. Two variables that are perfectly correlated (i.e, very together) yield r =1 two variables that are perfectly negatively correctly yield r = -1 variables with no net variation together yield r = 0.

Example : suppose we erect a short mast instrumented with anemometer to measure the u and w components , we record the instantaneous wind speed every 6 second for minute ,resulting in the following 10 pairs of wind observations .

U m/s	5	6	5	4	7	5	3	5	4	6
V m/s	0	-1	1	0	-2	1	2	-1	1	-1

2.3 Flux and Kinematic Flux :

Flux is the transfer of a quantity per unit area per unit time. In BL meteorology we are often concerned with mass, heat, moisture, momentum and pollutants fluxes. The dimensions of these fluxes are summarized below, using SI units show table 2.1:

Table 2.1 : Show units of flux and kinematic flux .

Flux	Units of flux	Units of kinematic flux
Mass	$\frac{kg_{air}}{m^2 \cdot s}$	$\frac{m}{s}$
Heat	$\frac{j}{m^2 \cdot s}$	$K.\frac{m}{s}$
Moisture	$\frac{kg_{water}}{m^2 \cdot s}$	$\frac{kg_{water}}{kg_{air}} \cdot \frac{m}{s}$
momentum	$\frac{kg \cdot (\frac{m}{s})}{m^2 \cdot s}$	$\frac{m^2}{s^2}$
pollutant	$\frac{kg_{pollutant}}{m^2 \cdot s}$	$\frac{kg_{pollutant}}{kg_{air}} \frac{m}{s}$

We rarely measure quantities such as heat or momentum directly instead we measure things like temperature or wind speed. Therefore, for convenience the above flux can be redefined in kinematic form by dividing by the density of the moist air, ρ_{air} in case of sensible heat flux, we also divide by the specific heat of air C_p.

These kinematic fluxes are now expressed in units that we can measure directly: wind speed for mass and momentum fluxes, temperature and wind speed for heat flux, and specific humidity q and wind speed for moisture flux. The pollutant flux is frequently expressed in either form: concentration and wind speed, or mass ratio (like parts per million, ppm) and wind speed. Each of these fluxes can be split in to three components, for example, there might be a vertical component of heat flux, and two horizontal components of heat flux, similar fluxes could be expected for mass, moisture, and pollutants. Hence, we can picture these fluxes as vectors.

2.3.1 Eddy Turbulent fluxes

Covariance can be interpreted as fluxes using the following conceptions. Consider a portion of the atmosphere with a constant gradient of potential temperature, as sketched in Fig. 2.8. Consider an idealized eddy circulation consisting of an updraft portion that moves an air parcel from the bottom to the top of the layer, and a compensating downdraft that moves a different air parcel downward. Air mass is conserved (i.e., mass up = mass down). However, the air parcels carry with them small portions of the air from their starting points, and these portions preserve their potential temperatures as they move, resulting in a flux as will now be shown. In Fig. 2.8a, the thick line represents a statically unstable mean environment $[\theta(z)]$. For this case when the rising air parcel reaches its destination, its potential temperature is warmer than the surrounding environment at that altitude. Namely, its deviation from its new environment is θ' = (+). This air parcel had to move upward to get to its destination, so w' = (+). The contribution of this rising parcel to the total covariance is w' times θ' = $\theta' w'$ = (+) · (+) = (+). Similarly, for the downward-moving [w' = (-)] portion of this eddy, the cold air from aloft finds itself colder [θ' = (-)] than its new surrounding environment at its final low altitude. Thus, its contribution to the covariance is $\theta' w'$ = (-) · (-) = (+).

The average of these two air parcels represents the covariance, and since each contribution is positive, the average (indicated by the overbar) is also positive: $\overline{w'\theta'} = 0$ Thus, positive $w'\theta'$ covariance is associated with warm air moving up and/or cold air moving down; namely, a positive heat flux $F_H(= w'\theta')$. This form of flux is called a kinematic heat flux, and has units of (K m s^{-1}). It is related to the traditional heat flux Q_H (W m^{-2}) by

$$Q_H = c_P\, F_H = cp\, \overline{w'\theta'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (2.8)$$

Where ρ the mean air density and C_p is the specific heat of air at constant pressure. Figure 2.8b shows the contrasting behavior observed in a statically stable environment. In this case, both the upward and downward moving parcels contribute negatively to the covariance. Thus, a downward heat flux is associated with cold air moving up or warm air moving down. Hence, the covariance $FH = w'\theta'$ is negative.

The net result of this turbulence is that warmer and colder layers are mixed to yield an intermediate potential temperature.

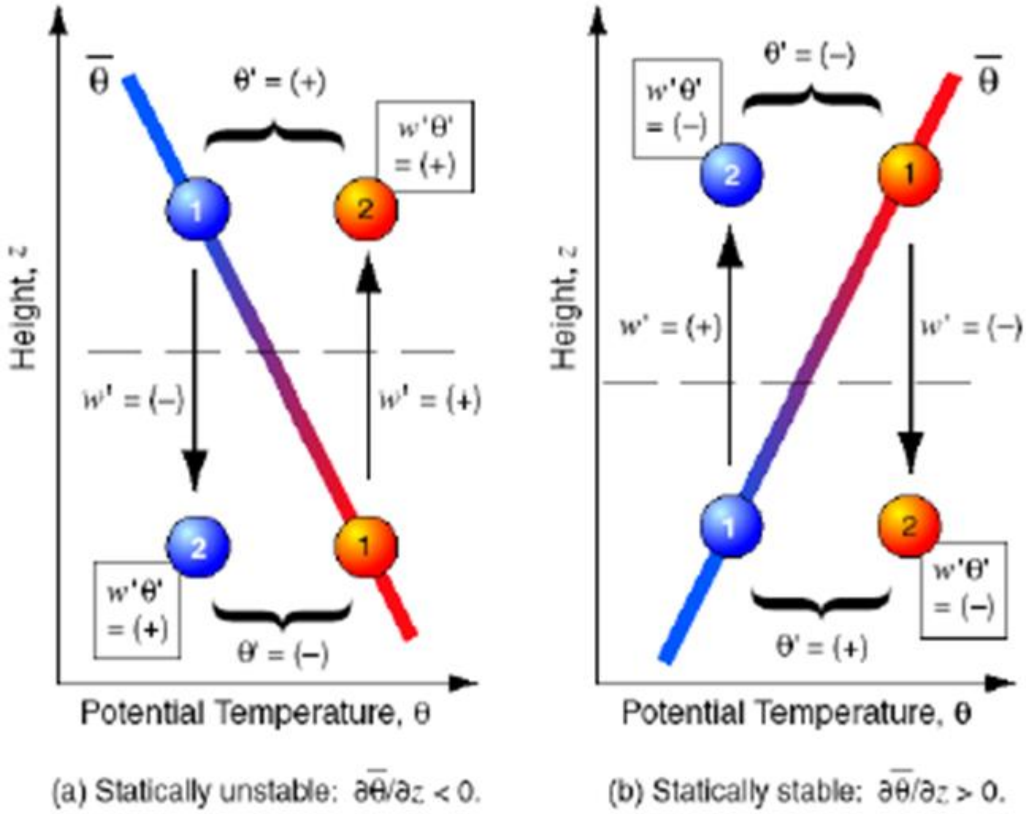

Figure 2.8. Illustration of how to anticipate the sign of turbulent heat fluxes, for small-eddy (local) vertical mixing across a region with a linear gradient in the mean potential temperature (thick colored line). Assuming an adiabatic process (no mixing), air parcels (sketched as spheres) preserve their potential temperature (as indicated by their color) of the ambient environment at their starting points (1), even as they arrive at their destinations (2). (a) Statically unstable lapse rate. (b) Statically stable lapse rate.

In a similar manner, one can conceive of turbulent mixing of moisture, pollutants, and even momentum. In each case, turbulence tends to homogenize a fluid. Turbulence is an extremely efficient mixer. For example, when milk is added to coffee or tea, most people prefer not to wait hours for molecular diffusion to homogenize their drink. Instead, they stir the fluid to generate turbulence, which homogenizes their drink within a few seconds. Atmospheric turbulence is equally efficient at causing mixing-so much so that molecular diffusion and molecular viscosity can be neglected for all motions except the tiniest eddies. In fact, during the daytime over land, convective turbulence is so effective at mixing that the boundary layer is also known as the mixed layer, because pollutants are so quickly distributed in the vertical.

2.3.2 Shear Turbulent Flux:

Momentum flux is analogous to a stress .where stress is define as the force tending to produce deformation in the body. It is measured as a force per unit area. And there is three types of stress appear frequency in studies of the atmosphere, such as pressure, Reynolds stress, and viscous shear stress .pressure is type of stress that can act on fluid at rest. For an infinitesimally small fluid element, such as idealized as cube sketched in figure 2.9a pressure acts equally in all directions. Isotropic is the name given to characteristics that are the same in all directions. If we consider just one face of this cube, as in fig 2.9b, we see that the isotropic nature of pressure tends to counteract itself in all directions except in direction normal to the surface of the cube. Forces acting normal to all faces of the cube tend to compress or expand the cube, thereby deforming it, figure 2.9c.

Reynolds stress exists only when the fluid is in turbulent motion. A turbulent eddy can mix air of different wind speeds in to our cube of interest, figure 2.9d. When this different-speed air is incorporated in to one face of the cube and not the opposite face, the cube deforms because of the velocity differences between those two faces, figure 2.9e. The rate that air of different speeds is transported across any face of the cube is just the momentum flux. The effect of this flux on the cube is identical to what we would observed if we applied a force on the face of the cube, namely the cube would deform. Thus, turbulent momentum flux acts like a stress, and is called the Reynolds stress. On other hand viscous stress exists when there are shearing motions in the fluid. The motion can be laminar or turbulent. When one portion of a fluid moves, the intermolecular forces tend to drag adjacent fluid molecules in the same direction, figure 2.9j. The strength of these intermolecular forces depend on the nature of fluid: molasses has stronger forces than water, which in turn has stronger forces than air. A measure of these forces is the viscosity. The result of this stress is a deformation of the fluid, figure 2.9k. These viscous forces can act in any of the three Cartesian directions on any of the three faces of our conceptual cube, figure 2.9f. Thus the viscous stress is also a tensor with nine components . Like the Reynolds stress, the viscous –stress tensor is symmetric leaving six independent components.

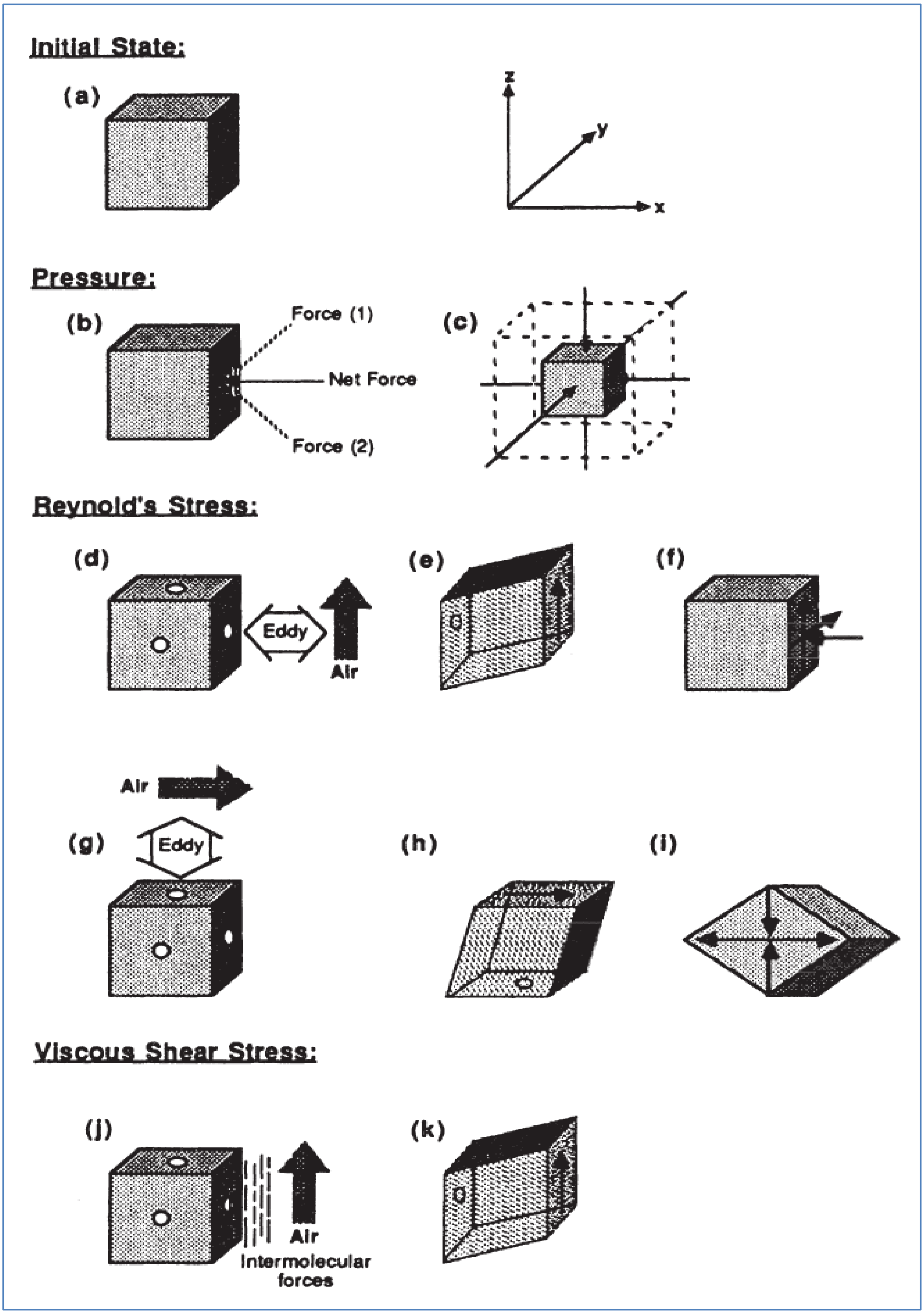

Figure 2.9: show the effect of stress on a conceptual cube of fluid.

2.4 Friction Velocity :

During situations where turbulence is generated or modulated by wind shear near the ground, the magnitude of the surface Reynolds s' stress proves to be an important scaling variable. The total vertical flux of horizontal momentum measured near the surface is

$$\tau_{xz} = -\bar{\rho}\overline{u'w'} \quad and \qquad \tau_{yz} = -\bar{\rho}\overline{v'w'} \ldots\ldots\ldots\ldots\ldots\ldots..(2.9)$$

$$\left|\tau_{reynolds}\right| = \left[\tau_{xz}^2 + \tau_{yz}^2\right]^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2.10)$$

Based on this relationship, a velocity scale called the friction velocity, u_*, is defined as :

$$u_*^2 = \left[\overline{u'w'}_s^2 + \overline{v'w'}_s^2\right]^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.11)$$

$$u_*^2 = \frac{\left|\tau_{reynolds}\right|}{\bar{\rho}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2.12)$$

For the special case where the coordinate system is aligned so the x-axis points in the direction of the surface stress , we can rewrite the lasted equation as :

$$u_*^2 = \frac{\left|\tau_{reynolds}\right|}{\bar{\rho}} = \left|\overline{u'w'}_s\right| \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.13)$$

Example Problem 1

The following mean flow and turbulence measurements were made at the 22.6 m height level of the micrometeorological tower during an hour-long run of the 1968 Kansas Field Program:

Mean wind speed = 4.89 m s^{-1}.
Velocity variances: $\overline{u^2} = 0.69$, $\overline{v^2} = 1.04$, $\overline{w^2} = 0.42\ m^2\ s^{-2}$.
Turbulent fluxes: $\overline{uw} = -0.081\ m^2\ s^{-2}$, $\overline{\theta w} = 0.185$ K m s^{-1},
$\overline{\theta u} = -0.064$ K m s^{-1}.

Calculate the following:
(a) standard deviations of velocity fluctuations;
(b) turbulence intensities;
(c) correlation coefficients;
(d) turbulence kinetic energy (TKE) and the ratio TKE/$\overline{uw}$.

Solution

(a) From the definition, standard deviations are

$$\sigma_u = (\overline{u^2})^{1/2} = 0.83\ \text{m s}^{-1}$$
$$\sigma_v = (\overline{v^2})^{1/2} = 1.02\ \text{m s}^{-1}$$
$$\sigma_w = (\overline{w^2})^{1/2} = 0.64\ \text{m s}^{-1}$$

(b) The corresponding turbulence intensities can be computed as

$$i_u = \sigma_u/|\mathbf{V}| = 0.17$$
$$i_v = \sigma_v/|\mathbf{V}| = 0.21$$
$$i_w = \sigma_w/|\mathbf{V}| = 0.13$$

(c) The correlation coefficients can be calculated as

$$r_{uw} = \overline{uw}/\sigma_u\sigma_w = -0.15$$
$$r_{w\theta} = \overline{\theta w}/\sigma_\theta\sigma_w = 0.59$$
$$r_{u\theta} = \overline{\theta u}/\sigma_\theta\sigma_u = -0.16$$

which indicate that θ and w are positively well correlated, while θ and u are only weakly correlated with a negative correlation coefficient of -0.16 in the convective surface layer.

(d) Turbulence kinetic energy can be calculated as

$$\text{TKE} = \tfrac{1}{2}(\overline{u^2} + \overline{v^2} + \overline{w^2}) = 1.075\ \text{m}^2\ \text{s}^{-2}$$
$$\text{TKE}/\overline{uw} = -13.27$$

Problems and Exercises

1. Discuss the various types of local or small-scale instability mechanisms that might be operating in the atmosphere and their possible effects on atmospheric motions.

2.
If you have available only the upper air velocity and temperature soundings, how would you use this information to determine which layers of the atmosphere might be turbulent?

3.
(a) What are the possible mechanisms of generation and maintenance of turbulence in the atmosphere?
(b) What criterion may be used to determine if turbulence in a stably stratified layer could be maintained or if it would decay?

4. What are the special characteristics of turbulence that distinguish it from a random wave motion?

5. Compare and contrast the time and ensemble averages. Under what conditions might the two types of averages be equivalent?

6. What is the possible range of turbulence intensities in the atmospheric PBL and under what conditions might the extreme values occur?

7. Write down all the Reynolds stress components and indicate which of these have the same magnitudes.

8. The following mean flow and turbulence measurements were made at a height of 22.6 m during the 1968 Kansas Field Program under different stability conditions:

Run no.	19	20	43	54	18	17	23
Ri	−1.89	−1.15	−0.54	−0.13	0.08	0.12	0.22
U (m s^{-1})	4.89	6.20	8.06	9.66	7.49	5.50	5.02
σ_u (m s^{-1})	0.83	1.04	1.27	1.02	0.73	0.45	0.20
σ_v (m s^{-1})	1.02	1.07	1.26	0.91	0.56	0.36	0.17
σ_w (m s^{-1})	0.65	0.73	0.73	0.64	0.48	0.28	0.10
σ_θ (K)	0.48	0.61	0.56	0.25	0.14	0.15	0.18
$\overline{uw}$ (m^2 s^{-2})	−0.081	−0.111	−0.244	−0.214	−0.118	−0.043	−0.005
$\overline{\theta w}$ (m s^{-1} K)	0.185	0.273	0.188	0.072	−0.029	−0.016	−0.005
$\overline{\theta u}$ (m s^{-1} K)	−0.064	−0.081	−0.175	−0.129	0.068	0.039	0.023

(a) Calculate and plot turbulence intensities as functions of Ri.
(b) Calculate and plot the magnitudes of correlation coefficients r_{uw}, $r_{w\theta}$, and $r_{u\theta}$ as functions of Ri. Explain their different signs under unstable and stable conditions.
(c) Calculate the turbulent kinetic energy (TKE) for each run and plot the ratio $-\mathrm{TKE}/\overline{uw}$ as a function of Ri.
(d) What conclusion can you draw about the variation of the above turbulence quantities with stability?

9.

(a) Calculate Kolmogorov's microscales of length and velocity for the following values of the rate of energy dissipation observed in the atmospheric boundary layer under different stability conditions and at different heights in the PBL:

$$\varepsilon = 10^{-4},\ 10^{-3},\ 10^{-2},\ \text{and}\ 10^{-1}\ \mathrm{m^2\,s^{-3}}$$

(b) Comment on the possible variation of η with height in the surface layer, considering that ε generally decreases with height (it is inversely proportional to z near the surface).

2.7 Basic equations for turbulent flow :

We can put the equations of continuity ,motion , and thermo-dynamic energy as mathematical expressions of the conservation of mass momentum , and heat in an elementary volume of fluid . these equation can applicable to laminar as well as turbulent flows .in the turbulent flow, all variables and their temporal and spatial derivative are highly irregular and rapidly varying function of time and space .this property of turbulence makes all the terms in the conservation equations significant ,and no further simplification of them are feasible . we can put the instantaneous equation of motion as :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (\,continuity\,)$$

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial z} = fv - \frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu\nabla^2 u \\
&\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \\
&\qquad = -fu - \frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu\nabla^2 v
\end{aligned}\right\} \quad (\,motion\,)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \frac{g}{T_0}T - \frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu\nabla^2 w$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = \propto_h \nabla^2\theta \qquad (\,thermodynamic\,)$$

$$P = \rho_{air}\ \Re\ T_v \qquad (\,state\,)$$

$$\left.\begin{aligned}
&\frac{\partial q_T}{\partial t} + u\frac{\partial q_T}{\partial x} + v\frac{\partial q_T}{\partial y} + w\frac{\partial q_T}{\partial z} = \nu_q\frac{\partial^2 q}{\partial x^2} + s_{qT}/\rho_{air} \\
&\frac{\partial q_T}{\partial t} + u\frac{\partial q_T}{\partial x} + v\frac{\partial q_T}{\partial y} + w\frac{\partial q_T}{\partial z} = \nu_q\frac{\partial^2 q}{\partial xy} + \frac{s_{qT}}{\rho_{air}} \\
&\frac{\partial q_T}{\partial t} + u\frac{\partial q_T}{\partial x} + v\frac{\partial q_T}{\partial y} + w\frac{\partial q_T}{\partial z} = \nu_q\frac{\partial^2 q}{\partial xz} + s_{qT}/\rho_{air}
\end{aligned}\right\} \quad (\,mositure\,)$$

$$\left.\begin{aligned}
&\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \nu_c\frac{\partial^2 C}{\partial x^2} + S_C \\
&\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \nu_c\frac{\partial^2 C}{\partial xy} + S_C \\
&\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \nu_c\frac{\partial^2 C}{\partial xz} + S_C
\end{aligned}\right\} \quad scalar$$

Where

u, v, w : is the instantaneous wind speed in the x , y , z direction

ƒ : carioles parameter $f=2\omega\sin\Phi = 1.45 * 10^{-4} S^{-1} * \sin\Phi$

ν : kinematic viscosity $\nu=\mu/\rho$, μ : viscosity parameter .

∇^2 : Laplacian operator .

θ : instantaneous potential temperature .

T_0 , T_1= T_0 و ، درجة حرارة الهواء عند حالة معينة T_1 الانحراف لدرجة الحرارة من درجة الحرارة المعينة .

α_h : thermal diffusivity. or molecular diffusivity of heat .

$\Re$: dry gas constant of air $\Re$= 287 J. K^{-1} Kg^{-1}

T_V : virtual Absolut temperature .

ν_q: molecular diffusivity of water vapor in the air .

S_{qT}: net sources of moisture (units total mass of water per unit time .

q_T : total moisture ($q_T = q + q_L$) , q : moisture content vapor , q_L: moisture without vapor .

C : concentration (mass per volume).

ν_c : molecular diffusivity .

We can calculate the turbulent flow by split any institutions variable to mean and turbulent part . these condition can used in deriving the equations for mean variables from these of the instantaneous variables . the time and space averages often used in practice can satisfy the rules only under certain idealized conditions (stationary and homogeneity of the flow) .

The usual procedure for deriving turbulent equation is to substituted in equation above $u = u' + \bar{u}$, $v = v' + \bar{v}$ and taken average for the new variables , first we taken equation of continuity equation :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Which after averaging gives :

$$\frac{\partial(\bar{u}+u')}{\partial x}+\frac{\partial(\bar{v}+v')}{\partial y}+\frac{\partial(\bar{w}+w')}{\partial z} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}+\frac{\partial\bar{w}}{\partial z} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Subtracting equation 3 from 4 one obtains the continuity equation for the fluctuation motion :

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Thus , the form of continuity equation remains the same for instantaneous ,mean ,and fluctuating motions . this is not the case , however , for the equations of conservation of momentum and heat , because of the presence of nonlinear advection terms in those equations .

Let us consider , for example , the advection terms in the instantaneous equation of the conservation of heat .

$$A_\theta = u\left(\frac{\partial\theta}{\partial x}\right)+ v\left(\frac{\partial\theta}{\partial y}\right)+ w\left(\frac{\partial\theta}{\partial z}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.a)$$

$$A_\theta = \left(\frac{\partial}{\partial x}\right)(u\theta)+\left(\frac{\partial}{\partial y}\right)(v\theta)+\left(\frac{\partial}{\partial z}\right)(w\theta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.b)$$

Expressing variables as sums of their mean and fluctuating parts in equation 6.b and averaging one obtains the averaged advection terms.

$$A_\theta = \frac{\partial(\overline{u\theta})}{\partial x}+\frac{\partial(\overline{v\theta})}{\partial y}+\frac{\partial(\overline{w\theta})}{\partial z}+\frac{\partial}{\partial x}(\overline{u'\theta'})+\frac{\partial}{\partial y}(\overline{v'\theta'}) + \frac{\partial}{\partial z}(\overline{w'\theta'}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.a)$$

$$A_\theta = \bar{u}\frac{\partial\theta}{\partial x}+\bar{v}\frac{\partial\theta}{\partial y}+\bar{w}\frac{\partial\theta}{\partial z}+\frac{\partial\theta}{\partial x}(\overline{u'\theta'})+\frac{\partial}{\partial y}(\overline{v'\theta'}) + \frac{\partial}{\partial z}(\overline{w'\theta'}) \ldots\ldots\ldots\ldots (7.b)$$

thus , upon averaging , the nonlinear advection terms yield not only the terms which may be interpreted as advection or transport by mean flow , but also several additional terms involving covariance or turbulent fluxes . these latter terms are spatial gradients (divergence) of turbulent transports .

following the above procedure with each component of equation 1 , one obtain the average equations for the conservation of mass , momentum, and heat , as follow :

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}=0$$

$$\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}+\bar{w}\frac{\partial \bar{u}}{\partial z}=f\bar{v}-\frac{1}{\rho_0}\frac{\partial P}{\partial x}+v'\,\nabla^2\bar{u}-(\frac{\partial \overline{u'^2}}{\partial x}+\frac{\partial \overline{u'v'}}{\partial y}+\frac{\partial \overline{u'w'}}{\partial z})$$

$$\frac{\partial \bar{v}}{\partial t}+\bar{u}\frac{\partial \bar{v}}{\partial x}+\bar{v}\frac{\partial \bar{v}}{\partial y}+\bar{w}\frac{\partial \bar{v}}{\partial z}$$
$$=-f\bar{u}-\frac{1}{\rho_0}\frac{\partial P}{\partial x}+v'\nabla^2\bar{v}-\left(\frac{\partial \overline{u'v'}}{\partial x}+\frac{\partial \overline{v'^2}}{\partial y}+\frac{\partial \overline{v'w'}}{\partial z}\right)\dots\dots\dots\dots..(8)$$

$$\frac{\partial \bar{w}}{\partial t}+\bar{u}\frac{\partial \bar{w}}{\partial x}+\bar{v}\frac{\partial \bar{w}}{\partial y}+\bar{w}\frac{\partial \bar{w}}{\partial z}$$
$$=\frac{g}{T_0}T-\frac{1}{\rho_0}\frac{\partial P}{\partial x}+v'\,\nabla^2\bar{w}-(\frac{\partial \overline{w'u'}}{\partial x}+\frac{\partial \overline{w'v'}}{\partial y}+\frac{\partial \overline{w'^2}}{\partial z})$$

$$\frac{\partial \bar{\theta}}{\partial t}+\bar{u}\frac{\partial \bar{\theta}}{\partial x}+\bar{v}\frac{\partial \bar{\theta}}{\partial y}+\bar{w}\frac{\partial \bar{\theta}}{\partial z}=\propto_h\,\nabla^2\bar{\theta}-(\frac{\partial \overline{u'\theta'}}{\partial x}+\frac{\partial \overline{v'\theta'}}{\partial y}+\frac{\partial \overline{w'\theta'}}{\partial z})$$

When these equations are compared with the corresponding instantaneous equation, one finds that of the terms (except for the turbulent transport terms) are similar and can be interpreted in the same way. However , there are several fundamental difference between these two sets of equations , which forces us to treat them in entirely different ways . while the first group deals with instantaneous variables varying rapidly and irregularly in time and space, second group deals with mean variables which are comparatively well behaved and vary only slowly and smoothly . while all the terms in the former equation set may be significant and cannot be ignored a priori , the mean flow equations can be greatly simplified by neglecting the molecular diffusion terms outside of possible viscous sub layers and also other terms on the basis of certain boundary layer approximation and considerations of stationarity and horizontal homogeneity , whenever applicable . for example , its easy to show that for a horizontally homogeneous and stationary PBL the equations of mean flow reduce to :

$$-f(\bar{v}-\bar{v}_g) = -\frac{\partial \overline{u'w'}}{\partial z} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. (9)$$

$$f(\bar{u} - \bar{u}_g) = -\partial\overline{v'w'}/\partial z$$