CHAPTER FOUR
(ATMOSPHERIC STABILITY)

4-1 Atmospheric Stability :

Know that most clouds form as air rises , expands, and cools . but why does the air rise on some occasions and not on the other ? and why does the size and shape of clouds vary so much when the air does rise ? to answer these questions , lets focus on the concept of ***atmosphere stability*** .

We call that a balloon like drop air parcel . figure 4.1 . when an air parcel rises , it moves in to a region where the air pressure surrounding it is lower . this situation allows the air molecules inside to push outward on the parcel walls , expanding it , as the air parcel expands, the air inside cools . if the same parcel is brought back to the surface , the increasing pressure around the parcel squeezes (compresses) it back to its original volume , and the air inside warms. Hence , a rising parcel of air expands and cools , while a sinking parcel is compressed and warms .

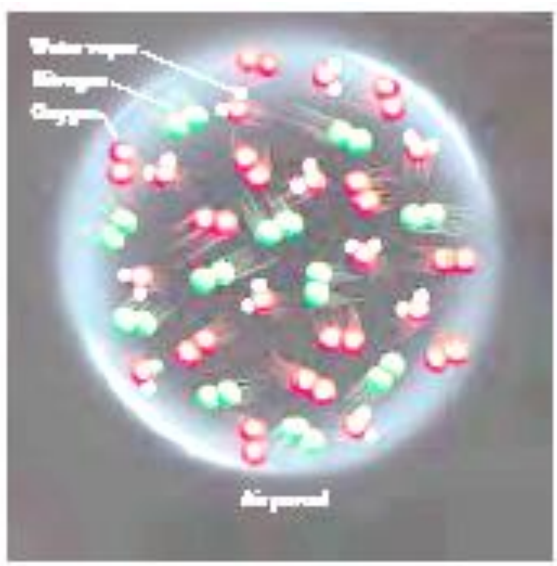


Figure (4.1) : the this air parcel be expanded and compress in to a number of ways

if a parcel of air expands and cools , or compresses and warms, and with no interchange of heat with its outside surroundings , this situation is called an ***adiabatic process*** . as long as the air in the parcel is unstatureated (the relative humidity is less than 100 percent) , the rate of adiabatic cooling or warming remains constant and is about 10^0C for every 1000 meters of change in elevation . since this rate of cooling or warming applies only to unsaturated air , it is called the ***dry adiabatic rate*** , see figure 4.2 .

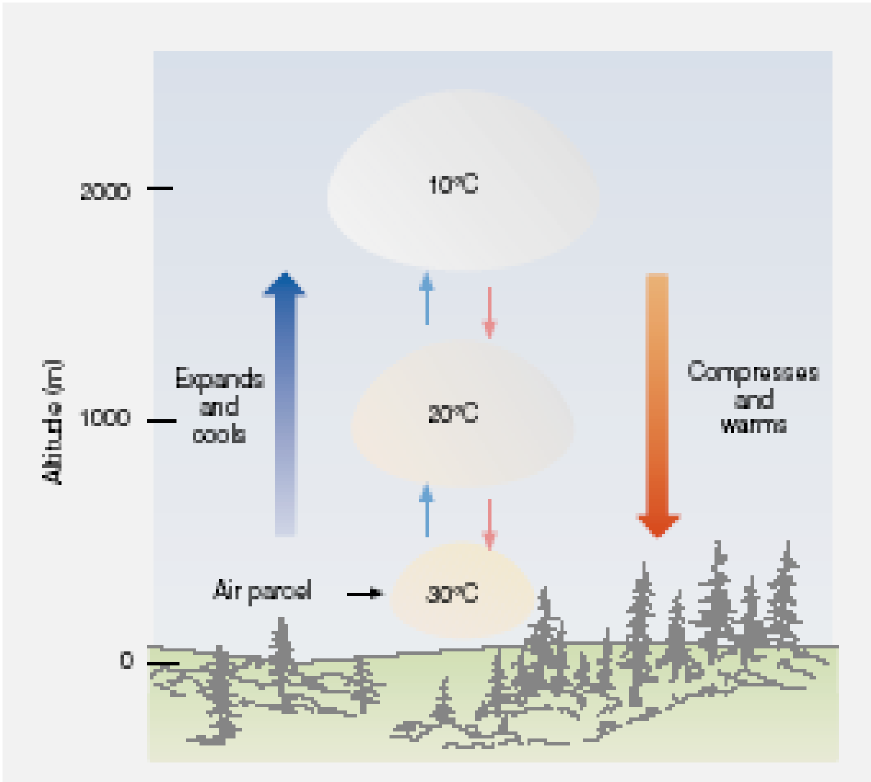


Figure 4.2 :The dry adiabatic rate. As long as the air parcel remains unsaturated, it expands and cools by 10°C per 1000 m; the sinking parcel compresses and warms by 10°C per 1000 m.

when the rising air cools , its relative humidity increases as the air temperature approaches the dew-point temperature. further lifting results in condensation , a cloud forms , and latent heat is released in to the rising air . because the heat added during condensation offsets some of cooling due to expansion , the air no longer cools at the dry adiabatic rate but at a lesser rate called the ***moist adiabatic rate***.(because latent heat is added to the rising saturated air , process is not really adiabatic) if a saturated parcel containing water droplets were to sink, it would compress and warm at the moist adiabatic rate because evaporation of the liquid droplets would offset the rate of compressional warming . unlike the dry adiabatic rate , the moist adiabatic rate is not constant , but varies greatly with temperature and, hence , with moisture content – as warm saturated air produces more liquid water than cold saturated air. the added condensation in warm , saturated air liberates more latent heat. consequently , the moist adiabatic rate is much less than the dry adiabatic rate when the rising air is quit warm , however the two rate are nearly the same when the rising air is very cold . although the moist adiabatic rate does vary , we will use an average of 6^0C per 1000m in most of our examples and calculation .

Stable air :

Suppose we release a balloon –borne instrument – a radiosonde , and measure the air temperature data as shown in figure (4.3) . we measure the air temperature in the vertical and find that it decreases by 4^0C for every 1000m ,we refer to as the ***environmental lapse rate*** . notice in figure 4.3a that (with environmental lapse rate of 4^0C per 1000m) a rising parcel of unsaturated dry air is colder and heavier than the air surrounded it at all levels . even if the parcel is initially saturated (figure 4.3b) , as it rises too, would be colder than its environment at all levels. in both cases, the atmosphere is ***absoulutely stable*** because the lifted parcel of air is colder and heavier than the air surrounding it .the atmosphere is stable when the evironmental lapse rate is small ; that is , when there is a relatively small difference in temperature between the surface air and the air aloft . Consequently, the atmosphere tends to become more stable—it ***stabilizes***—as the air aloft warms or the surface air cools. The ***cooling*** of the *surface air* may be due to:

1. Nighttime radiational cooling of the surface.
2. air moving over a cold surface.

It should be apparent that, on any given day, the air is generally most stable in the early morning around sunrise, when the lowest surface air temperature is recorded. Recall that sinking (subsiding) air warms as it is compressed. The warming may produce an inversion, where the air aloft is actually warmer than the air at the surface. An inversion that forms by slow, sinking air is termed a ***subsidence inversion*** . Because inversions represent a very stable atmosphere, they act as a lid on vertical air motion. When an inversion exists near the ground, stratus, fog, haze, and pollutants are all kept close to the surface . in Fig. 4.4 is shown A typical profile of a plume during a subsidence inversion and in the case of radiation inversion in Fig. 4.5.

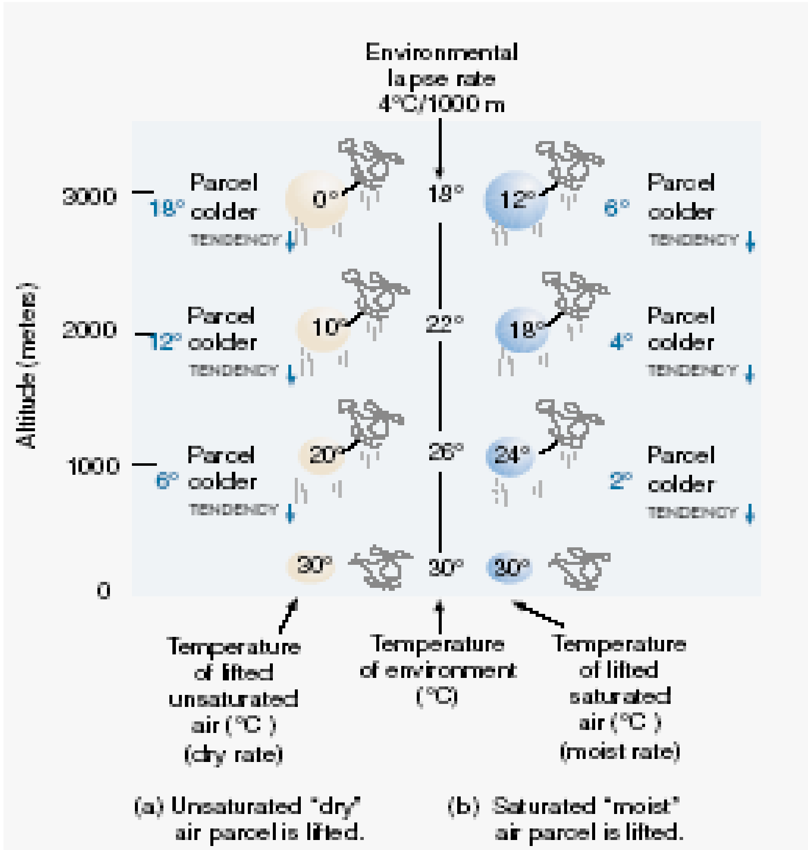


FIGURE 4.3 :A stable atmosphere. An *absolutely stable atmosphere* exists when a rising air parcel is colder and heavier (i.e., more dense) than the air surrounding it. If given the chance (i.e., released), the air parcel in both situations would return to its original position, the surface.

<u>Unstable air</u> : The atmosphere is unstable when the air temperature decreases rapidly as we move up into the atmosphere. For example, in Fig. 4.6 , notice that the measured air temperature decreases by 11°C for every 1000-meter rise in elevation, which means that the **environmental lapse rate** is 11°C per 1000 meters. Also notice that a lifted parcel of unsaturated “dry” air in Fig.4.6a, as well as a lifted parcel of saturated “moist” air in Fig. 4.6b, will, at each level above the surface, be warmer than the air surrounding them. Since, in both cases, the rising air is warmer and less dense than the air around them, once the parcels start upward, they will continue to rise, away from the surface. Thus, we have an ***absolutely unstable atmosphere.*** The atmosphere becomes more unstable as the environmental lapse rate steepens; that is, as the temperature of the air string rapidly with increasing height. This circumstance may be brought on by either the air aloft .becoming colder or the surface air becoming warmer (see Fig. 4.7).

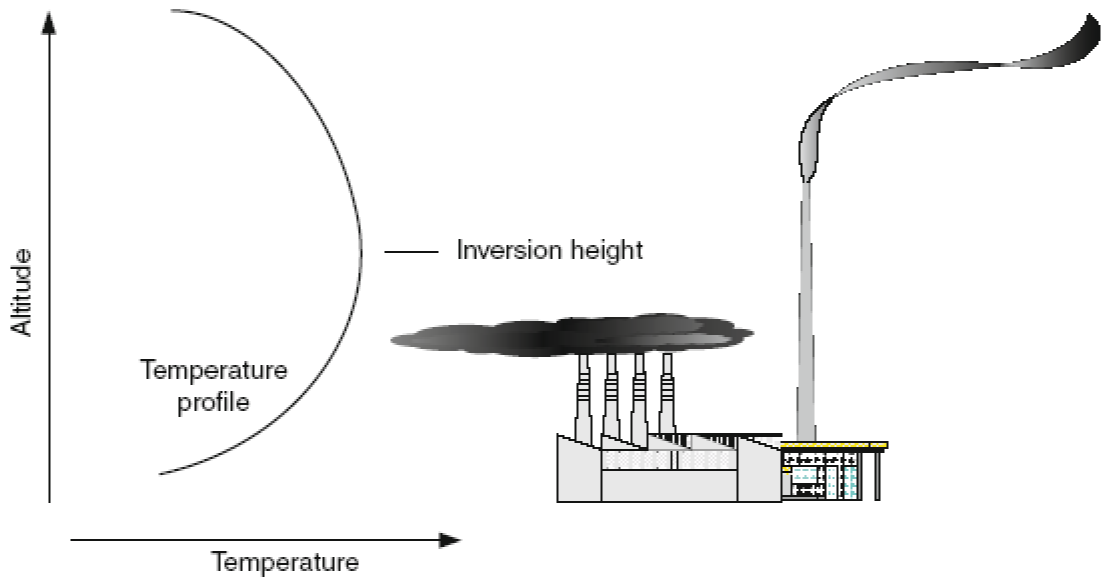


Fig. 4.4 The plume from the lower chimneys is trapped inside the inversion, whereas, the plume from the tall chimney which is located above the inversion is rising, mixing and transported

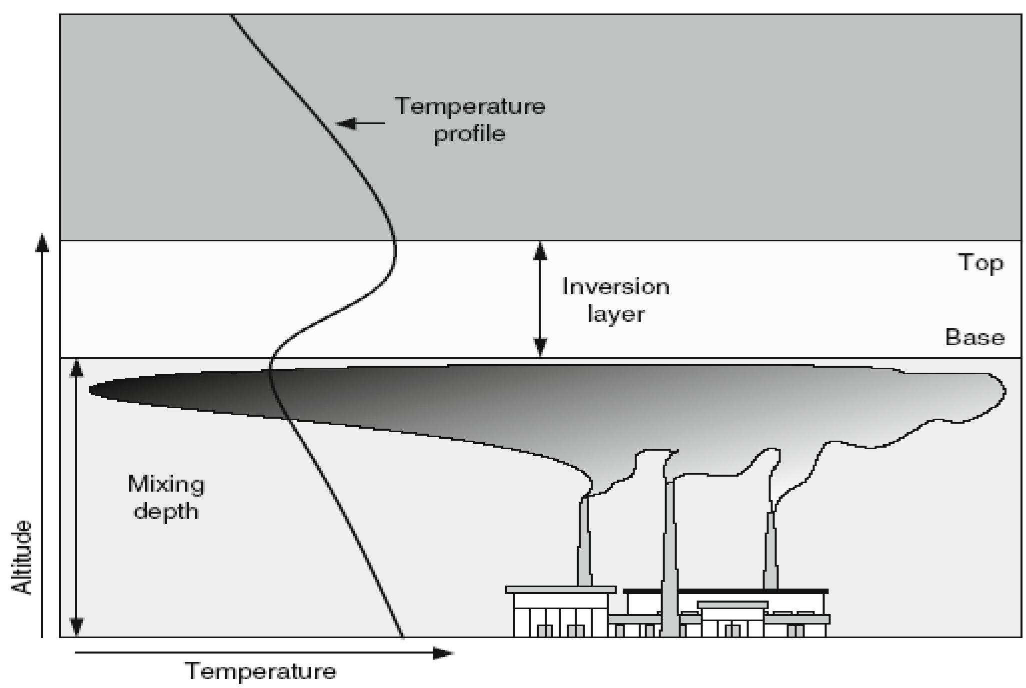


Fig. 4.5 The inversion height is not allowing the escape of gaseous pollutants under it. If the inversion height is becoming lower, then the mixing height is reduced and the pollutants are trapped in a smaller air volume

The *warming* of the *surface air* may be due to:

1. daytime solar heating of the surface.
2. influx of warm air brought by the wind.
3. air moving over a warm surface.

Generally, as the surface air warms during the day, the atmosphere becomes more unstable—it *destabilizes*. The air aloft may cool as winds bring in colder air or as the air (or clouds) emit infrared radiation to space (radiation cooling). Just as sinking air produces warming and a more stable atmosphere, rising air, especially an entire layer where the top is dry and the bottom is humid, produces cooling and a more unstable atmosphere. The lifted layer becomes more unstable as it rises and stretches out vertically in the less dense air aloft .

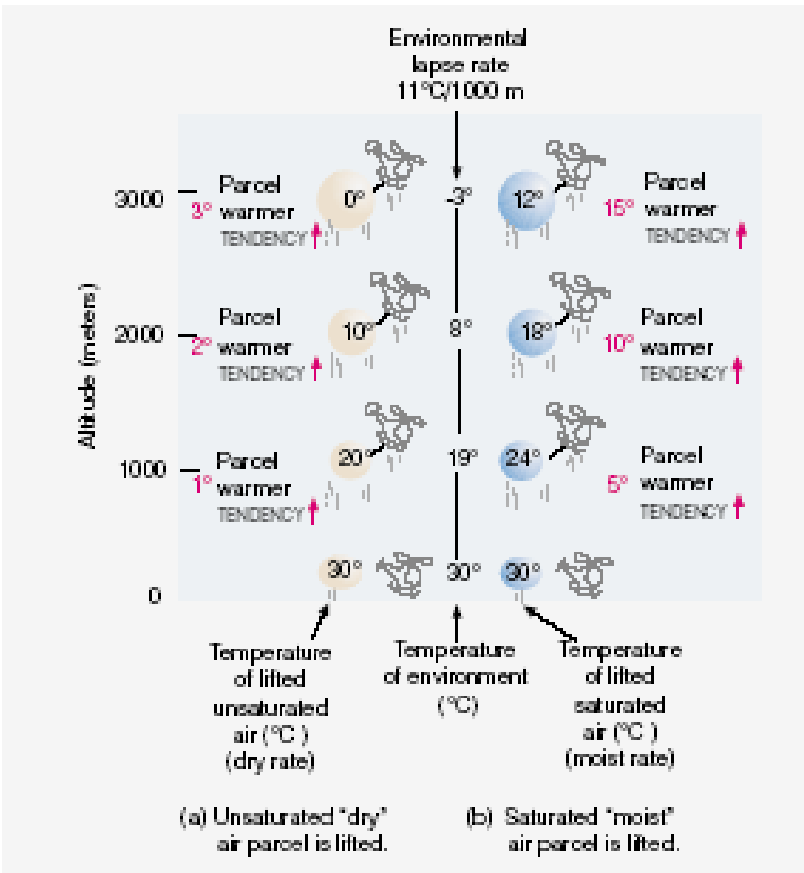


Figure 4.6:An unstable atmosphere. An *absolutely unstable atmosphere* exists when a rising air parcel is warmer and lighter (i.e., less dense) than the air surrounding it. If given the chance (i.e., released), the lifted parcel in both (a) and (b) would continue to move away (accelerate) from its original position.

This extending effect steepens the environmental lapse rate as the top of the layer cools more than the bottom. Instability brought on by the lifting of air is often associated with the development of severe weather, such as thunderstorms and tornadoes, It should be noted, however, that deep layers in the atmosphere are seldom, if ever, absolutely unstable. Absolute instability is usually limited to a very shallow layer near the ground on hot, sunny days. Here, the environmental lapse rate can exceed the dry adiabatic rate, and the lapse rate is called ***superadiabatic***.

Conditionally Unstable Air:

Suppose an unsaturated (but humid) air parcel is somehow forced to rise from the surface, as shown in Fig. 4.7. As the parcel rises, it expands, and cools at the *dry adiabatic rate* until its air temperature cools to its dew point. At this level, the air is saturated, the relative humidity is 100 percent, and further lifting results in condensation and the formation of a cloud. The elevation above the surface where the cloud first forms (in this example, 1000 meters) is called the **condensation level.** In Fig. 4.7, notice that above the condensation level, the rising saturated air cools at the ***moist adiabatic rate***. Notice also that from the surface up to a level near 2000 meters, the rising, lifted air is colder than the air surrounding it. The atmosphere up to this level is ***stable***.

However, due to the release of latent heat, the rising air near 2000 meters has actually become warmer than the air around it. Since the lifted air can rise on its own agreement, the atmosphere is now ***unstable***. The level in the atmosphere where the air parcel, after being lifted, becomes warmer than the air surrounding it, is called the ***level of free convection***.The atmospheric layer from the surface up to 4000 meters in Fig. 4.7 has gone from stable to unstable because the rising air was humid enough to become saturated, form a cloud, and release latent heat, which warms the air. Had the cloud not formed, the rising air would have remained colder at each level than the air surrounding it. From the surface to 4000 meters, we have what is said to be a **conditionally unstable atmosphere**— the condition for instability being whether or not the rising air becomes saturated. Therefore, *conditional instability means that, if unsaturated stable air is somehow lifted to a level where it becomes saturated, instability may result*. In Fig. 4.7, we can see that the environmental lapse rate is 9°C per 1000 meters. This value is between the dry adiabatic rate (10°C/1000 m) and the moist adiabatic rate (6°C/1000 m). Consequently, ***conditional instability*** *exists whenever the environmental lapse rate is between the dry and moist adiabatic rates*. when the boundary layer is unstable.

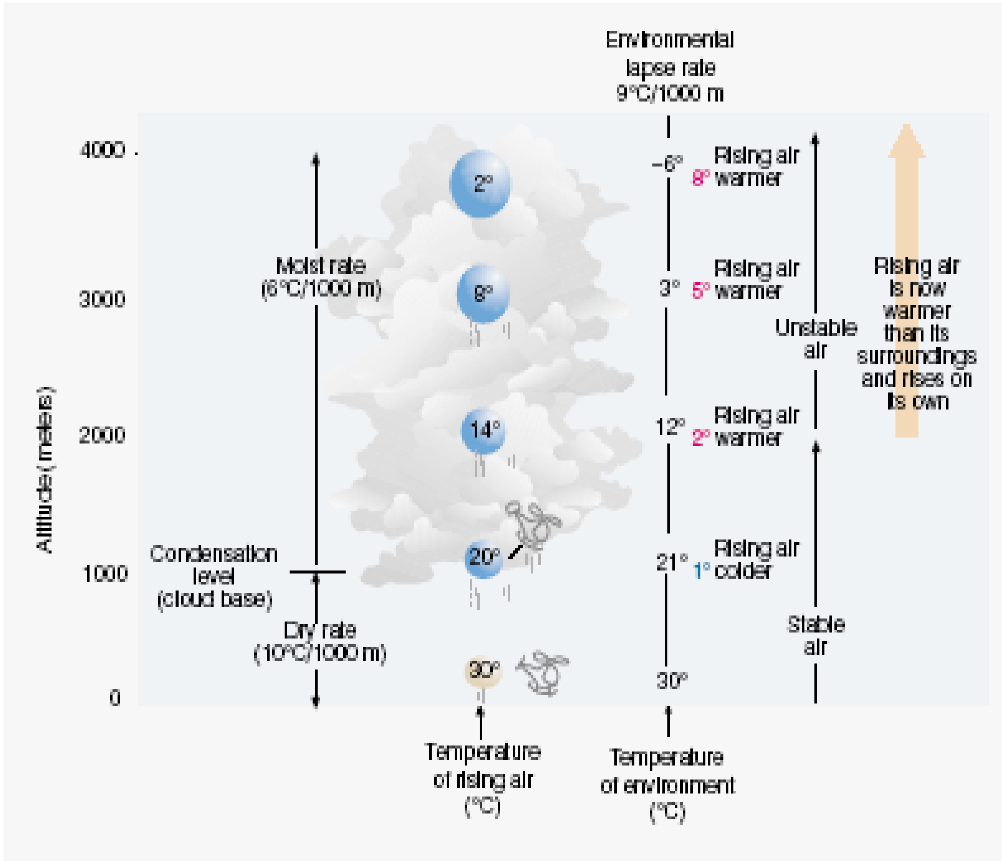


Figure 4.7 : Conditionally unstable air. The atmosphere is conditionally unstable when unsaturated, stable air is lifted to a level where it becomes saturated and warmer than the air surrounding it. If the atmosphere remains unstable, vertical developing cumulus clouds can build to great heights.

4.2 Static Stability :

the word “static” means “ having no motion “ this type of stability does not depend on wind . air is statically unstable when less– dense air (warmer and or moister) underlies more dense air. the flow responds to this instability by supporting convective circulations such as thermals that allow buoyant air to rise to the top of the unstable layer , thereby stabilizing the the fluid . thermals also need some trigger mechanism to get them started . in the real boundary layer , there are so many triggers (hills , buildinings , trees , dark fields, or other perturbations to the mean flow) .

Local Definitions : *the static stability is determined by the local lapse rate* . the local definition frequently fails in convective MLs , because the rise of thermals from near surface or their decent from cloud top depends on their excess buoyancy and not on the ambient lapse rate .

As an example , in the middle 50% of the convective ML the lapse rate is nearly adiabatic , causing an incorrect classification of neutral stability if the traditional local definition is used . we must make a clear distinction between the phrases " adiabatic lapse rate "and neutral stability ". **an adiabatic lapse rate** (in the virtual potential temperature sense) my statically stable , neutral, or unstable , depending on convection and the buoyancy flux . neutral stability implies a very specific situation: adiabatic lapse rate AND no convection .We conclude that measurement of the local lapse rate alone is insufficient to determine the static stability. either knowledge of the whole $\overline{\theta_v}$ profile is needed ,or measurement of the turbulent buoyancy flux must be made .

Non Local Definitions : it is better to examine the stability of the whole , and make a layer determination of stability , for example , if $\overline{w'\theta'}$ at the earth surface is positive , or if displaced air parcels will rise from the ground or sink from cloud top as thermals traveling across a BL , then the whole BL is said to be unstable or convective . if $\overline{w'\theta'}$ is negative at the surface ,or if displaced air parcels return to their starting point, then BL is said to be stable . If ,when integrated over the depth of the boundary layer , the mechanical production term in the TKE equation is much larger than the buoyancy term, or if the buoyancy term is near zero , then the boundary layer is said to be neutral . the boundary layer of this latter case is also sometimes referred to as an **Ekman boundary layer** . during fair weather conditions over land , the BL touching the ground is rarely neutral . neutral conditions are frequently found in the RL aloft . in overcast conditions with strong winds but little temperature difference between the air and the surface , the BL is often close to neutral stability .

In the absence of knowledge of convection or measurements of buoyancy flux , an alternate determination of static stability is possible if the $\overline{\theta_v}$ profile over the whole BL is known , as sketched in figure 5.8 . as is indicated in the figure , if only portions of the profile are known , then the stability might be indeterminate .

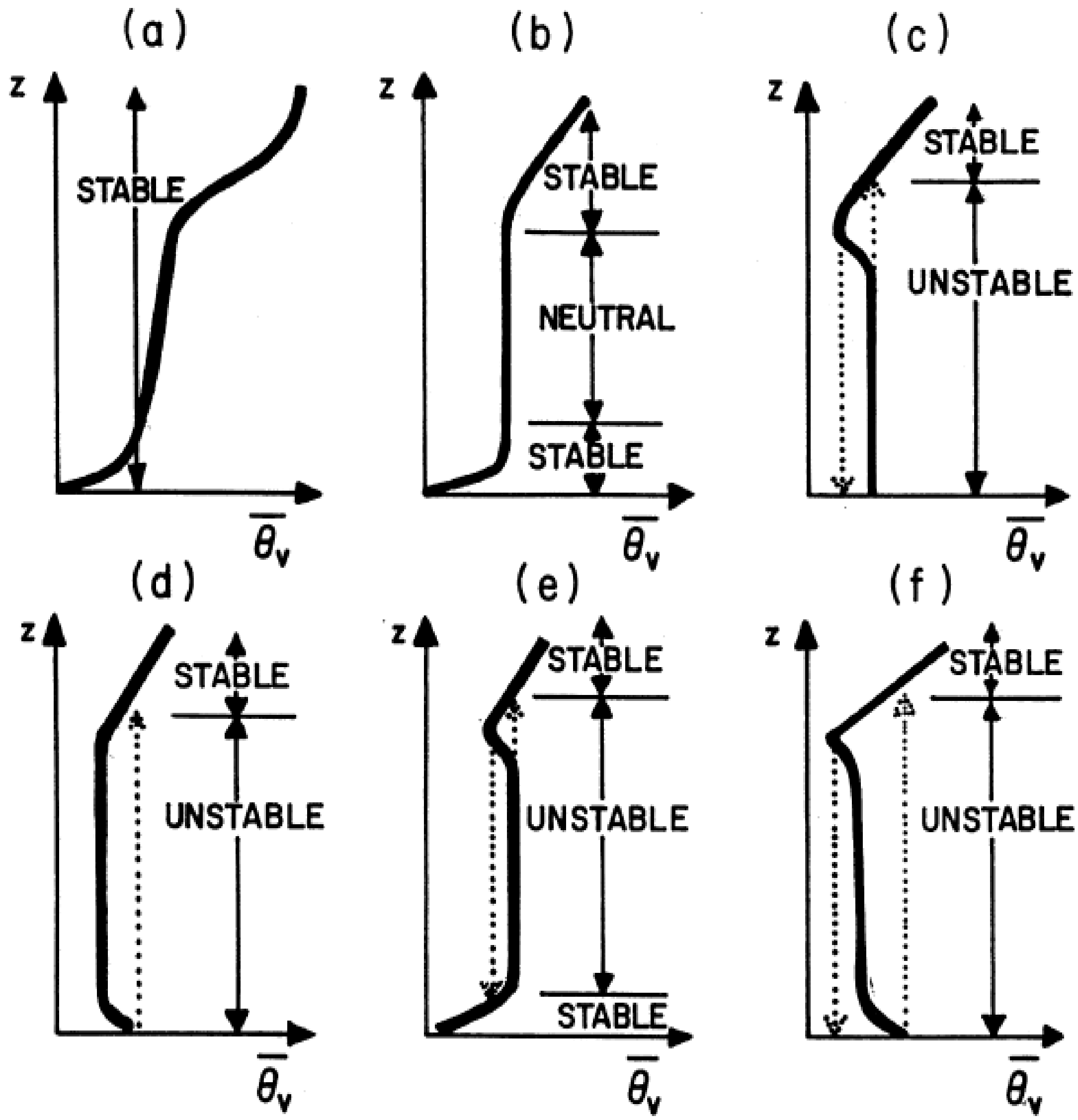


Figure (4.8): Nonlocal Stability Characteristic

4.3 Dynamic Stability :

The word “ dynamic “ refers to motion , hence , dynamic stability depends in part on the winds . even if the air is statically stable , wind shears may be able to generate turbulence dynamically .

Some laboratory experiments have been performed using denser fluids underlying less – dense fluids with a velocity shear between the layers to simulate the stable stratification and shears of the atmosphere . figure 5.9 is a sketch of the resulting flow behavior . the typical sequence of events is :

1. A shear exists across a density interface . initially , the flow is laminar .
2. If a critical value of shear is reached , then the flow becomes dynamically unstable , and gentle waves begin to form on the interface . the crests of these waves are normal to the shear direction .
3. These waves continue to grow in amplitude , eventually reaching a point where each wave begins to “roll up “ or “ break “ . this “ breaking “ wave is called **a Kelvin – Helmholtz (KH) wave** , and is based on different physics than surface waves than “ break “ on an ocean beach .
4. Within each wave , there exists some lighter fluid that has been rolled under denser fluid , resulting in patches of static instability .
5. The static instability , combined with the continued dynamic instability , causes each wave to become turbulent .
6. The turbulence then spreads throughout the layer , causing a diffusion or mixing of the different fluids . during this diffusion process , some momentum is transferred between the fluids , reducing the shear between the layer . what was formerly a sharp , well -defined , interface becomes a broader , more diffuse shear with weaker shear and static stability .
7. This mixing can reduce the shear below a critical value and eliminate the dynamic instability .
8. In the absence of continued forcing to restore the shears , turbulence decays in the interface region , and the flow becomes laminar again .

This sequence of evens is suspected to occur during the onset of **clear air turbulence** (**CAT**) . these often occur above and below strong wind jet , such as the nocturnal jet and the planetary – scale jet stream . in these situations , however, continued dynamic forcings can allow turbulence to continue for hours to days . these regions of **CAT** have large horizontal extent (hundreds of kilometers in some cases) , but usually limited vertical extent (tens to hundreds of meters).

Although KH waves are probably a frequent occurrence within statically stable shear layers , they are only rarely with the naked eye . occasionally , there is sufficient moisture in the atmosphere to allow cloud droplets to act as visible

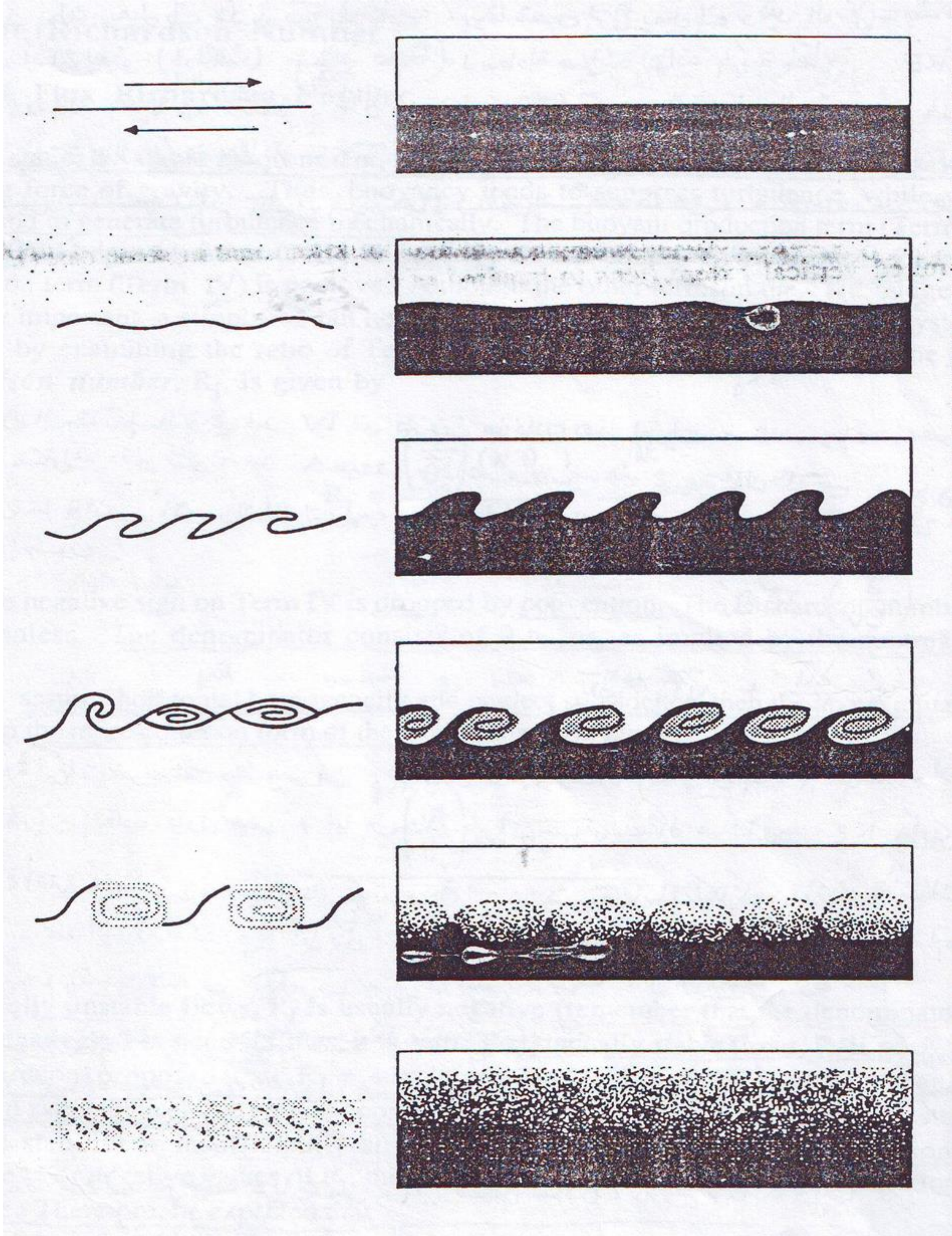


Fig. 4.9 Schematic diagram of Kelvin-Helmholtz instability in a laboratory experiment where shear flow has been generated. The upper layer, water, flows to the right, and the lower more dense fluid, dyed brine, flows to the left. The figures are about half a second apart. After Thorpe (1969,1973) and Woods (1969).

tracers . clouds that form in the rising portions of the waves often form parallel bands called **billow clouds** .

For both static and dynamic instabilities , and many other instabilities for that matter , it is interesting to note that the fluid reacts in a manner to undo the cause of the instability . this process is strikingly similar to **Lechatelier "s Principle** of chemistry , which states that " if some stress is brought to bear upon a system in equilibrium , a change occurs such that the equilibrium is displaced in a direction which tends to undo the effect of the stress " thus , turbulence is a mechanism whereby fluid flows tend to undo the cause of the instability . in the case of static instability , convection occurs that tends to move more buoyant fluid upward , thereby stabilizing the system . for dynamic instability , turbulence tends to reduce the wind shears , also stabilizing the system .

After the unstable system has been stabilized , turbulence acts to eliminate itself . given observations of turbulence occurring for long periods of time within the boundary layer , it is logical to conclusion that there must be external forcings tending to destabilize the BL over long time periods . in case of static instability , the solar heating of the ground by the sun is that external forcing . in case of dynamic instability , pressure gradients imposed by synoptic –scale features drive the winds against the dissipative effect of turbulence .

By comparing the relative magnitudes of the shear production and buoyant comsumption terms of the TKE equation , we can hope to estimate when the flow might become dynamically unstable . the Richadson number , described soon , can used as just such an indicator .

Quasions:

1- What is an adiabatic process?
2- How would one normally obtain the environmental lapse rate ?
3- Why are the moist and dry adiabatic rates of cooling different?
4- How can the atmosphere be made more stable ? more unstable ?
5- If the atmosphere is conditionally unstable , what does this mean ? what condition is necessary to bring on instability ?
6- Explain why an inversion represents an extremely stable atmosphere >
7- Why are cumulus clouds more frequently observed during the afternoon ?
8- What type of cloud would you most likely expect to see in a stable atmosphere ? in an unstable atmosphere .
9- There are usually large spaces of blue sky between cumulus clouds . explain why this is so .
10- Where would you expect the moist adiabatic rate to be greater in the tropics or near the north pole ? explain why?
11- What changes in weather conditions near the earth's surface are needed to transform an absolutely stable atmosphere in to absolutely unstable atmosphere ?

4-4 Method to Measure Stability

4-4-1 Flux Richadson Number

in a statically stable environment , turbulent vertical motions are acting a gainst the restoring force of gravity . thus, buoyancy tends to suppress turbulence , while wind shears tend to generate turbulent mechanically. the buoyant production term of the TKE budget equation is negative in this situation , while the mechanical production term is positive . the other terms in the TKE budget are certainty important, a simplified but nevertheless usefull approximation to the physics is possible by examining the ratio of term buoyant to term mechanical . this ratio, called the **flux Richardson number** . $\boldsymbol{R_f}$ is given by :

$$R_f = \frac{\left(\frac{g}{\overline{\theta_v}}\right)\left(\overline{w'\theta'_v}\right)}{\left(\overline{u'w'}\right)\frac{\partial\bar{u}}{\partial z} + \left(\overline{v'w'}\right)\frac{\partial\bar{v}}{\partial z}} \quad \ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

This equation is applied at horizontal homogeneity and neglect subsidence .for statically unstable flow R_f is usually negative (remember that the denominator is usually negative) . for neutral flows , it is zero . for Statically stable flows , R_f is positive . Richardson proposed that $R_f = +1$ is a critical value , because the mechanical production rate balances the buoyant consumption of TKE . at any value of R_f less than +1 , static stability is insufficiently strong to prevent the mechanical generation of turbulence . for negative values of R_f , the numerator even contributes to the generation of turbulence . therefore , he expected that :

Flow is turbulent (dynamically unstable) *when* $R_f < +1$

Flow becomes laminar (dynamically stable) *when* $R_f > +1$

We recognize that statically unstable flow is , by definition , always dynamically unstable .

4-4-2 Grident Richadson Number

A peculiar problem arises in the use of R_f , namely , we can calculate its value only for turbulent flow because it contains factors involving turbulent correlations like $\overline{w'\theta'_v}$. in other words , we can use it to determine whether turbulent flow will become laminar, but not whether laminar flow will become turbulent .

It is logical to suggest that the value of the turbulent correlation $-\overline{w'\theta'_v}$ might be proportional to the lapse rate $\frac{\partial\bar{\theta}_v}{\partial z}$. similarly we my suggest that $\overline{-u'w'}$ is proportion to $\frac{\partial\bar{u}}{\partial z}$ and that $\overline{-v'w'}$ is proportional to $\frac{\partial\bar{v}}{\partial z}$. these arguments from the basis of a theory known *as K-Theory or eddy diffusivity theory* . we will just assumed that the propertioalities are possible, and substitute those in above equation to given a new ratio called the **gradient Richadson number , R_i.**

$$R_i = \frac{\frac{g}{\theta_v}\left(\frac{\partial\bar{\theta}_v}{\partial z}\right)}{\left[\left(\frac{\partial\bar{u}}{\partial z}\right)^2 + \left(\frac{\partial\bar{v}}{\partial z}\right)^2\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (2)$$

When investigators refer to a " Richardson number " without specifying which one , they usually mean the gradient Richadson number .

Theoretical and laboratory research suggest that laminar flow becomes unstable to KH-wave formation and the ONSET of turbulence when R_i is smaller than the **critical Richardson number R_c** . another value R_T , indicats the termination of turbulence the dynamic stability criteria can be stated as follows :

Laminar flow becomes turbulent when $R_i < R_c$

Turbulent flow becomes laminar when $R_i > R_T$

Although there is still some debate on the correct values of R_c and R_T it is appears that R_c = 0.21 to 0.25 and R_T = 1.0 work fairly well . thus , there appears to be a hysteresis effect because R_T is greater than R_c .

4-4-3 Bulk Richadson Number

The theoretical work yielding $R_c = 0.25$ is based on local measurement of the wind shear and temperature gradient . Meterorologists rarely know the actual local gradients , but can approximate the gradient using observations made at a series of discrete height intervals . if we approximate $\frac{\partial \bar{\theta}_v}{\partial z}$ by $\overline{\Delta\theta_v}/\Delta z$, and approximate $\frac{\partial \bar{u}}{\partial z}$ and $\frac{\partial \bar{v}}{\partial z}$ by $\frac{\Delta \bar{u}}{\Delta z}$ *and* $\Delta\bar{v}/\Delta z$ respectively , then we can define a new ratio known as the **Bulk Richadson number** .

$$R_b = \frac{g\,\overline{\overline{(\Delta\theta)_v}}\,\Delta z}{\overline{\theta_v}\ \left[(\Delta U)^2 + (\Delta V)^2\right]} \dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

this form of the Richason number is used most frequently in Meterology , because Rawinsonde data and numerical weather forecsts supply wind and temperatue measurements at discrete points in space . the finite differences defined for example by ,

$$\Delta\bar{U} = \bar{U}\left(z_{top}\right) - \bar{U}(z_{bottom})$$

Unfortunately , the critical value of 0.25 applies only for local gradients , not for finite differences across thick layers . in fact , the thicker the layer is the more likely we are to average out large gradients that occur within small subregions of the layer of interest . the thinner the layer , the closer the critical Richadson number will likely be to 0.25 .

<u>Example</u> : A rawinsonde sounding through the lower troposphere gives the following profile information . which layers of air are turbulent , and why ?

Z (km)	T (^{0}C)	U (m/s)
13	-58	30
11	-58	60
8	-30	25
5	-19	20
3	-3	18
2.5	1	9
2	2	8
1.6	0	5
0.2	13	5
0	18	0
Assume v=0, q=0 , T_v=T		

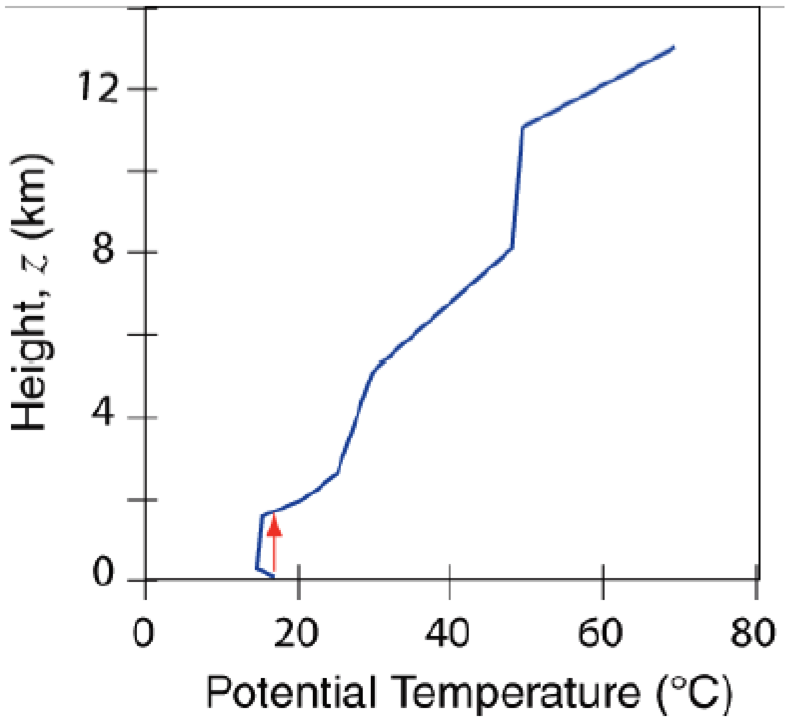
Solution : To determine the regions in which turbulent is occurring we need to examine both dynamic stability and non- local static stability . for dynamic stability , we use bulk Richadson number with finite differences :

$$R_b = \frac{\frac{g}{\overline{\theta_v}}\overline{\overline{(\Delta\theta)_v}}\,\Delta z}{[(\overline{\Delta u})^2 + (\overline{\Delta v})^2]}$$

Where the $\overline{\theta_v}$ represent the average across the whole layer . also used $\theta = T + \Gamma z$, where $\Gamma = 9.8\, m/s^2$ is the dry adiabatic lapse rate the results are show in table below , for static stability , plot the profile , and lift parcels from every relative maximum and lower from every relative minimum to identify statically unstable regions .

Z (km)	T (^{0}C)	U (m/s)	θ ^{0}C	T av.(k)	Δz (m	Δu ($\frac{m}{s}$	$\Delta\theta$ (k)					
13	-58	30	69.4									
11	-58	60	49.8	215.15	2000	-30	19.6					
8	-30	25	48.8	229.15	3000	35	1.4					
5	-19	20	30	248.65	3000	5	18.4					
3	-3	18	26.4	262.15	2000	2	3.6					
2.5	1	9	25.5	272.15	500	9	0.9					
2	2	8	21.6	274.65	500	1	3.9					
1.6	0	5	15.6	274.15	400	3	5.92					
0.2	13	5	14.9	279.65	1400	0	0.72					
0	18	0	18	288.65	200	5	-3.04					

For example the layer in figure profile below z=0 to 1.8 is statically unstable , now we consider the dynamic stability , using the bulk richardson number critersion .



Summary: The bottom turbulent region 0–1.8 km is the boundary layer. *Clear air turbulence* (CAT) exists near the jet stream, from 8 to 11 km. The other turbulent region is 2.5 to 3 km.

Layer (km)	R_B	Dynamically	Statically	Turbulent
11 to 13	1.98	Stable	Stable	no
8 to 11	0.15	Unstable	Stable	yes
5 to 8	87.02	Stable	Stable	no
3 to 5	67.29	Stable	Stable	no
2.5 to 3	0.20	Unstable	Stable	yes
2 to 2.5	69.58	Stable	Stable	no
1.6 to 2	9.41	Stable	Unstable to 1.8 km	yes to 1.8 km
0.2 to 1.6	$+\infty$	(undefined)	Unstable	yes
0 to 0.2	-0.83	Unstable	Unstable	yes

4-4-4 OBukhov Lenghth

The Obukhov length (L) is a scaling parameter that is useful in the surface layer to show this parameter is related to the TKE equation , first recall that one definition of the surface layer is that region where turbulent fluxes vary by less that 10% of their magnitude with height . by making the constant flux (with height) approximation , one can use surface values of heat momentum flux to define turbulent scales and nondimensionalize the TKE equation .

Start with the equation The TKE budget **:**

$$\frac{\partial \bar{e}}{\partial t} = \frac{g}{\theta}\,\overline{(w'\theta')} - \overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \frac{\partial\left(\overline{w'e}\right)}{\partial z} - \frac{1}{\rho}\frac{\partial\left(\overline{w'p'}\right)}{\partial z} - \varepsilon \quad \dots\dots (4)$$

and multiply the whole equation by $\frac{-kz}{u_*^3}$, assume all turbulent fluxes equal their respective surface values , and focus on just terms ($-\overline{u'w'}\frac{\partial \bar{u}}{\partial z}$) and ($\frac{g}{\theta}\,\overline{(w'\theta')}$ and (ε) we obtain :

$$= -kzg\,\frac{(w'\theta'_v)_s}{\overline{\theta_v}u_*^3} + kz\,\frac{(u'w')_s}{u*^3}\,\frac{\partial u}{\partial z} + \cdots \dots \frac{-kz\varepsilon_s}{u_*^3} \dots\dots\dots\dots\dots\dots\dots\dots (5)$$

Each of these terms is now dimensionless . the last , a dimensionless disspation rate , will not be pursued further here .

The von karmen constant , k , is a dimensionless number included by tradition . its importance in the log wind profile in the surface layer is discussed in next section .

Term buoyancy is usually assigned the symbol ζ , and is further defined as $\zeta = \frac{z}{L}$ where L is the obukhov length , thus

$$\zeta = \frac{z}{L} = \frac{-k\ z\ g\ \left(\overline{w'\theta'}\right)_s}{\overline{\theta_v}u_*^3} \dots\dots\dots\dots\dots\dots\dots\dots\dots.. (6)$$

The Obukhov length is given by :

$$L = \frac{-u_*^3}{k\,\frac{g}{\theta_v}\;\overline{(\overline{w'\theta'})_s}} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

One physical interpretation of the Obukhov length is that it is *proportional to height above the surface at which buoyant factor dominate over mechanical (shear) production of turbulence* . for convective situations , buoyant and shear production terms are approximately equal at z= -0.5 L .

The parameter ζ turns out to be very important for scaling and similarity arguments of the surface layer . its sometimes called a stability parameter , although it magnitude is not directly related to static nor dynamic stability . only its sign relates to static stability ; negative implies unstable positive implies statically stable . a better description of ζ is " **a surface-layer scaling parameter** " .

We can write an alternative form for ζ by employing the definition of w_* .

$$\zeta = \frac{z}{L} = -\frac{k\,z\,w_*^3}{z_i\,u_*^3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

figure (4.10) shows the variation of R_i with ζ from slightly unstable to slightly stable conditions . for unstable situations , $R_i = \zeta$. one must keep in mind that ζ can be calculated only for turbulent flow , thus this figure shows only the subset of all data that was turbulent . nonturbulent flow can occur in stable situations , but it does not appear in this figure .

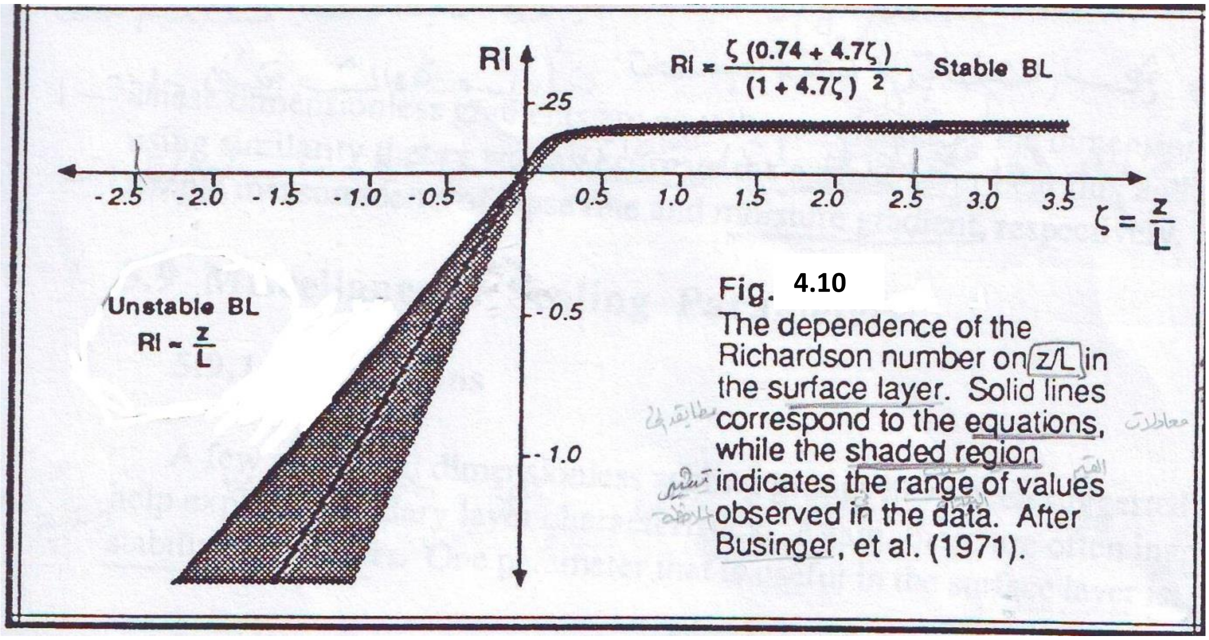


Fig. 4.10 The dependence of the Richardson number on z/L in the surface layer. Solid lines correspond to the equations, while the shaded region indicates the range of values observed in the data. After Businger, et al. (1971).

Example :

At a height of z = 300m in a 1000m thick mixed layer the following condition were Observed : $\partial\bar{u}/\partial z = 0.01s^{-1}$, $\overline{\theta_v} = 25^0C$, $\overline{W'\theta'} = 0.15\, k\frac{m}{s}$, and $\overline{u'w'} = -0.03\, m^2s^{-2}$, also , the surface virtual heat flux is 0.24 k. (m/s) . if the pressure and turbulent transports are neglected then :

(a) What dissipation rate is required to maintain a locally steady state at z= 300m .
(b) What are the values of the normalized TKE terms ?
(c) Calculate the flux Richadson number and comment on dynamic stability .
(d) Given a fictitious , SBL where $(g/\overline{\theta_v}) = 0.033\, m\, s^{-2}k^{-1}$, $\frac{\partial u}{\partial z} = \left[\frac{u_*}{0.4.z}\right]s^{-1}$, $u_* = 0.4m/s$, and where the lapse rate , c_1 , is constant with height such that there is 6^0C, $\overline{\theta_v}$ increase with each 200m of altitude gained . how deep is the turbulent ?

The answer :

a- Since no information was given about the v-component of velocity or stress , let 's assume that the x-axis has been chosen to be aligned with the mean wind . and we know that term local storage must be zero for steady state , and term disspation and turbulent transports are zero as specified in the statement of the problem . thus , the remaining terms can be manipulated to solve for ε .

$$\varepsilon = \frac{g}{\bar{\theta}_v}\ \overline{w'\theta'} - \overline{u'w'}\frac{\partial\bar{u}}{\partial z}$$

Plugging in the values given above yields :

$$\varepsilon = \left\{\left(\frac{9.8\, ms^{-2}}{[(273.15+25]}\right)\right\}.(\, 0.15\; k\; ms^{-1}\,) - (-0.03\; m^2 s^{-2}\,).(\, 0.01\; s^{-1}\,)$$

$$\varepsilon = 4.93x\; 10^{-3} + 3x\; 10^{-4} \;\left(\frac{m^2}{s^3}\right)$$

$$\varepsilon = 5.23\; x\; 10^{-3}\;\;(\, m^2\; s^{-3}\,)$$

b- To normalize the equation we first use $\frac{w_*^3}{z_i} = \left(\frac{g}{\theta_v}\right)\overline{w'\theta_v}'$, which for our case equals 7.89x 10^{-3} (m^2s^{-3}) . dividing our terms by this value yield :

0= 0.625 + 0.038 - 0- 0 - 0.663

Discussion :

This buoyant production term is about an order of magnitude larger than the mechanical production term , meaning that the turbulence is in a state of free convection . in regions of strong turbulent production , the transport term usually removes some of the TKE and deposits it where there is a net loss of TKE , such as in the entrainment zone . thus , we might expect that the local dissipation rate at z= 300m is smaller than the value calculated above .

c- Since the flux Richardson number is defined as the ratio of the buoyancy term to the negative of the shear term .

$$R_i = \frac{buoyancy}{-\,shear\;\; term} = \frac{0.00493}{-0.0003} = -16.4$$

Discussion : a negative Richardson number is without question less than +1 , and thus indicates dynamic instability and turbulence . this is a trivial conclusion , because any flow that is statically unstable is also dynamically unstable by definition .

d- We can use the gradient Richardson number as an indicator of dynamic stability and turbulence . using the prescribed gradients , we find that :

$$R_i = \frac{\frac{g}{\theta}\frac{\partial\bar{\theta}}{\partial z}}{\left(\frac{\partial\bar{u}}{\partial z}\right)^2} = \frac{\frac{g}{\theta_v}c_1}{\left(\frac{u_*}{0.4\,z}\right)^2} = \frac{(0.033)\,.(\,0.03)\;\; z^2}{\left(\frac{0.4}{0.4}\right)^2} = (\,0.00099\; m^{-2}\,)z^2$$

If we use R_i = 0.25 , then we can use this critical value in place of Ri above and solve for z at the critical height above which there is no turbulence .

$$z = \sqrt{(1010\; m^2)R_C} \;=\; \sqrt{252.5\;\; m^2} \;=\; 15.9\; m$$

Discussion : if we has used a critical termination value of R_T= 1.0 , then we would have found a critical height of 31.8m . thus , below 15.9 m we expect turbulence , while above 31.8m we expect laminar flow . between these heights the turbulent state depends on the past history of the flow at that height . if previously turbulent , it is turbulent now .

PROPLEMS:

Q1) Given surface measurements : $u_* = 0.2 m.s^{-1}$, $\frac{g}{\theta} = 0.0333\, m.s^{-1}\, k^{-1}$, *and* $\overline{w'\theta'} = -0.05\, k.ms^{-1}$, *.find scaling*

Parameters L , ζ at z=10m.

Q2) given the following wind speeds measured at various heights in the boundary layer ?

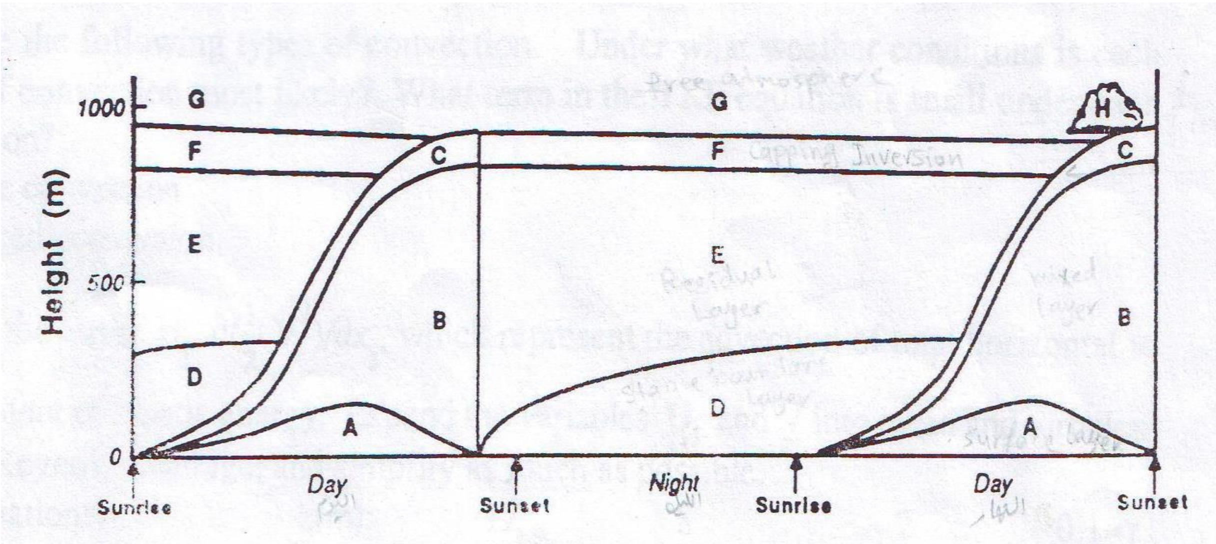
Z m	2000	1000	500	300	100	50	20	10	4	1
U m/s	10	10	9.5	9	8	7.4	6.5	5.8	5	3.7

Assume that the potential temperature increases with height at height at the constant rate of 6 k/km . calculate the Bulk Richadson Number for each layer and indicate the static and dynamic stability of each layer . also , show what part of the atmosphere is expected to be turbulent in these conditions .

Q3) fill in the table based on the regions A-H labeled on the attached diagram .

Property Choices	Lapse rate	Heat flux	Static stability	Turbulent ?	name
	Subadiab.	up	Stable	yes	Noct. inversion
	Adiabatic	zero	neutral	unknown	Cloud layer
	Superad.	down	unstable	no	Mixed laye
				sporadic	Entrainment zone
					Capping inversion
					Free atmosphere

Region					
A					Surface
B					
C	Subadiab.				
D					
E					Residual
F		zero			
G				unknown	
H			stable		



Q4) what is the Reynolds stress ? why is it called a stress ? how does it relate to u_* ?

Q5) observations:

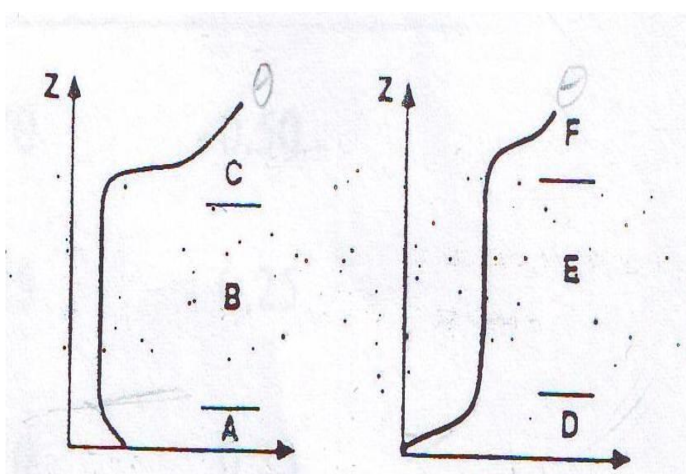
Z(m)	12	8	2	0.1 = z0
θ (k)	300	301	303	308
U (m/s)	5.4	5.0	3.4	0

Situation : daytime boundary layer over land .

a- Find R_b at 2 , 4 , and 10m

b- Comment on the static and dynamic stability of the air . is the flow turbulent ?

Q6) what is the static stability of each of the layers in the diagram at night ?



Q7) this problem is best saved until after the log – Wind profile has been introduced . given the following data :

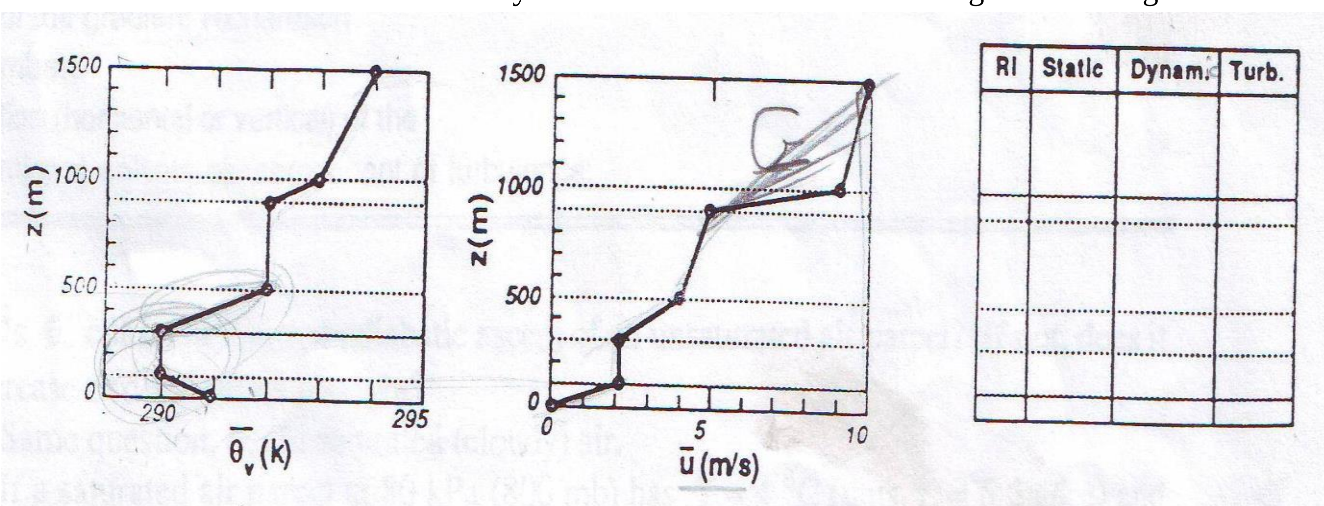
$\overline{w'\theta'} = 0.2\,k.\frac{m}{s}$, $u_* = 0.2\frac{m}{s}$, $zi = 500m$, $k = 0.4$, $\frac{g}{\theta} =$ 0.0 33 m s^{-2} k^{-1} z=6m

z_0= 0.01m = roughness , no moisture

Find :

a- L
b- z/l
c- w*
d- θ_*
e- *static stability*
f- Rf *at* $6\,m$ (*make assumpation to find this*)
g- Ri *at* $6m$ (*make assumpations to find this*)
h- *dynamic stability*
i- *flow state* (*turbulent or not*)

Q8) given the following sounding in the moring boundary layer . determine whether each layer is stable or unstable (in both the static and dynamic sense) , and state if the flow is turbulent . indicate your results in the table to the right of the figures .



Q9) what boundary layer flow phenomena or characteristics have scale sizes on the order of : a- 1mm b- 10m c- 1 km

Q10)

a) Is θ_v conserved during adiabatic ascent of an unsaturated air parcel ? if not , does it increase or decrease with height ?
b) Same equation , but in saturated (cloudy) air .

c) If a saturated air parcel at 80 kPa (800 mb) has T= $4^{O}C$ (thus , $r_s = 6.5$ g/kg) and has a total water mixing ratio of $r_T =$ 8 g/kg , then calculate the virtual potential temperature at that altitude .

Q11) given $w'\theta' = 0.3\ k\ .\frac{m}{s}\ , u'w' = -0.25\ m^2\ s^{-2}\ , and\ zi = 1\ km\ ,\ find$:

a- U*
b- W*
c- R_f (assumed $\frac{\partial u}{\partial z} = 0.1\ s^{-1}$
d- Obukhov length (L)

Q12) given the following sounding , indicate for each layer the :

a- Static stability
b- Dynamic stability
c- Existence of turbulence (assuming a laminar past history)

Z m	80	70	60	50	40	30	20	10	0
$\theta\ k$	305	305	301	300	298	294	292	292	293
U m/s	18	17	15	14	10	8	7	7	2

Q13) which Richardson number (flux , gradient , bulk) would you use for the following application ? (given the one best answer for each question) .

a- Diagnostic the possible existences of clear air turbulence using rawinsonde data .
b- Determine whether turbulent flow will become laminar .
c- Determine whether laminar flow will become turbulent in the boundary layer .