**Exercise: (College algebra 4th by Raymond D. Barnett Michal R. Ziegler)**

**Q1. Using algebraic long division to find the following:**

$i) \frac{4m^{2}-1}{2m-1}$ ii) $\frac{6-6x+8x^{2}}{2x+1}$ iii) $\frac{x^{3}-1}{x-1}$ iv) $\frac{3y-y^{2}+2 y^{3}-1}{y+2}$ v)$ \frac{x^{3}-3x^{2}+x-3}{x-i}$ vi) $\frac{16x-5x^{3}-8+6x^{4}-8x^{2}}{2x-4+3x^{2}}$.

**Q2. Using synthetic division to write the polynomial in the form** $f\left(x\right)=\left(x-r\right)Q\left(x\right)+R.$

i) $\frac{3x^{4}-x-4}{x+1} $ ii)$ \frac{3x^{4}+2x^{3}-4x-1}{x+3}$ iii)$ \frac{x^{5}+1}{x+1}$ iv)$ \frac{2x^{6}-13x^{5}+75x^{3}+2x^{2}-50}{x-5}$ v) $\frac{4x^{4}+10x-9}{x+3}.$

**Q3. Using synthetic division and remainder theorem to find the following:**

i) $p\left(x\right)=3x^{2}-x-10, $ find $ p(-2)$.

ii) $p\left(x\right)=2x^{3}-5x^{2}+7x-7$, find$ p(2)$.

iii) $p\left(x\right)=x^{4}-10x^{2}+25x-2$, find$ p(-4)$.

**Q4. Write** $p(x)$ **as a product of first degree factors**

i) $p\left(x\right)=x^{3}+9x^{2}+24x+16;$ -1 is a zero.

ii) $p\left(x\right)=x^{4}-1;$ 1 and -1 are zero.

ii) $p\left(x\right)=2x^{3}-17x^{2}+90x-41;$ $\frac{1}{2}$ is a zero.

iv) $p\left(x\right)=x^{3}-4x^{2}-3x+18;$ 3 is a double zero.

**Q5. Find all other zeros of** $f(x)$**, given the indicated zero.**

i$) p\left(x\right)=x^{3}-5x^{2}+4x+10; $ $3-i$ is one zero.

ii) $p\left(x\right)=x^{3}-3x^{2}+25x-75;$ $-5i$ is one zero.

iii) $p\left(x\right)=x^{4}-4x^{3}+3x^{2}+8x-10;$ $2+i$ is one zero.

**Q6. Find the upper and lower bounds, for the real zero of the following polynomials.**

$i) p\left(x\right)=x ^{2}-2x+3$.

ii) $p\left(x\right)=x^{3}-3x+5.$

iii) $p\left(x\right)=x^{4}-x^{2}+3x+2.$

**Q7. Find the following foe each polynomial**

1) what is the degree of each polynomial.

2) write the zero of each polynomial.

3) indicate the multiplicity of each zero.

i) $p\left(x\right)=\left(x+8\right)^{3}(x-6)^{2}. $

ii) $p\left(x\right)=3\left(x+4\right)^{3}(x-3)^{2}(x+1).$

**Q8. What are the possible combinations of real and imaginary zero?**

i$) p\left(x\right)=2x^{3}-3x^{2}+x-5.$

ii) $p\left(x\right)=3x^{6}-5x^{5}+3x^{2}-4.$

iii) $p\left(x\right)=x^{5}-2x^{4}+\sqrt{5 }x^{2}-7.$

iv) $p\left(x\right)=5x^{4}-2x^{2}+x-8.$

**Q9. Construct a table showing the possible combinations of positive, negative and imaginary zeros using Descartes rule of signs.**

i$) p\left(x\right)=2x^{2}+x-4.$

ii) $p\left(x\right)=7x^{2}+2x+4.$

iii) $p\left(x\right)=2x^{3}-4x^{2}+x-3.$

**Q10. Find the smallest positive integer and the largest negative integer using upper and lower bound theorem.**

i$) p\left(x\right)=x^{2}-2x+3.$

ii) $p\left(x\right)=x^{3}-3x+5.$

iii) $p\left(x\right)=x^{4}-x^{2}+3x+2.$

**Q11. Show that, for each given polynomial, there is at least one real zero between the given values of** $a$ **and** $b$**.**

i$) p\left(x\right)=x^{2}-3x-2, a=3, b=4.$

ii$) p\left(x\right)=x^{3}-3x+5, a=-3, b=-2.$

iii$) p\left(x\right)=x^{3}-3x^{2}-3x+9, a=1, b=2.$

**Q12. Find all roots (rational, irrational and imaginary) for each polynomial equation.**

i$) p\left(x\right)=2x^{3}-5x^{2}+1.$

ii)$ p\left(x\right)=x^{4}+4x^{3}-x^{2}-20x-20.$

iii$) p\left(x\right)=x^{4}-2x^{3}-5x^{2}+8x+4.$

**Q13. For each polynomial,**

 **(a) list all possible rational zeros (b) find all rational roots.**

i) $p\left(x\right)=x^{3}-2x^{2}-5x+6.$

ii) $p\left(x\right)=3x^{3}-11x^{2}+8x+4.$

iii) $p\left(x\right)=12x^{3}-16x^{2}-5x+3.$