Electrical Resistivity of Materials

As mentioned previously, most metals are extremely good conductors of electricity because of the large numbers of free electrons. Thus *n* has a large value in the conductivity expression, Eq. (6) in lecture – 1.

At this point it is convenient to discuss conduction in metals in terms of the resistivity, the reciprocal of conductivity.

Since crystalline defects serve as scattering centers for conduction electrons in metals, increasing their number raises the resistivity (or lowers the conductivity). The concentration of these imperfections depends on ***temperature***, ***composition***, and the ***degree of cold work*** of a metal specimen. In fact, it has been observed experimentally that the ***total resistivity*** of a **metal** ***is the sum of the contributions from thermal vibrations, impurities, and plastic deformation***. This may be represented in mathematical form as follows:

$$\rho_{total} = \rho_{th} + \rho_{imp} + \rho_{def}$$

in which ρth, ρimp and ρdef represent the individual thermal, impurity, and deformation resistivity contributions, respectively. Eq. (7) is sometimes known as **Matthiessen's rule**.

- ***Influence of Temperature***

When the temperature of a metal increases, thermal energy causes the atoms to vibrate, Figure (1). At any instant, the atom may not be in its equilibrium position, and it therefore interacts with and scatters electrons. The mean free path decreases, the mobility of electrons is reduced, and the resistivity increases.

The change in resistivity of pure metal as a function of temperature, thus,

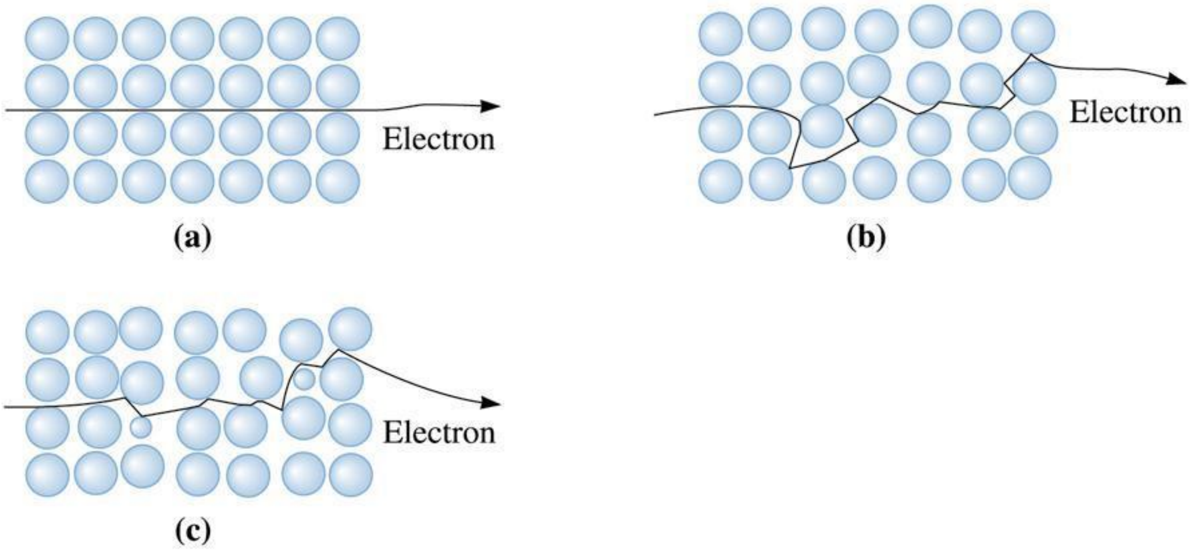


Figure (1): Movement of an electron through (a) a perfect crystal, (b) a crystal heated to a high temperature, and (c) a crystal containing atomic level defects. Scattering of the electrons reduces the mobility and conductivity.

$$\rho_{th} = \rho_{RT}(1 + \alpha\Delta T)$$

where ρ_{th} the resistivity at any temperature T, ρ_{RT} the resistivity at room temperature (i.e., 25°C), $\Delta T = (T - T_{RT})$ is ***the difference between the temperature of interest and room temperature***, and α is the temperature resistivity coefficient. This dependence of the thermal resistivity component on temperature is due to the increase with temperature in thermal vibrations and other lattice irregularities (e.g., vacancies), which serve as electron – scattering centers. So, the relationship between resistivity and temperature is linear over a wide temperature range, Figure (2). Table (1), given some examples of the temperature resistivity coefficient.

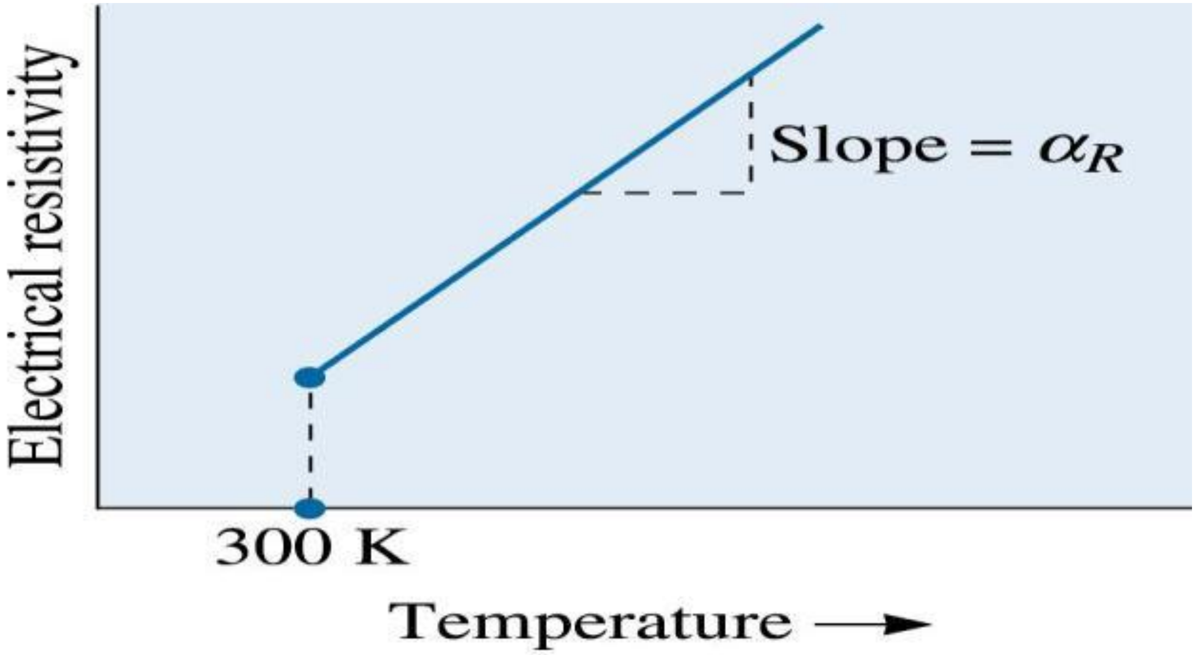


Figure (2): The effect of temperature on the electrical resistivity of a metal with a perfect crystal structure. The slope of the curve is the temperature resistivity coefficient.

Table (1): The temperature resistivity coefficient *α* for selected metals.

TABLE 18-3 ■ ***The temperature resistivity coefficient α_R for selected metals[1]***

Metal	Room Temperature Resistivity (ohm · cm)	Temperature Resistivity Coefficient (α_R) (ohm/ohm · °C)
Be	4.0×10^{-6}	0.0250
Mg	4.45×10^{-6}	0.0037
Ca	3.91×10^{-6}	0.0042
Al	2.65×10^{-6}	0.0043
Cr	12.90×10^{-6} (0°C)	0.0030
Fe	9.71×10^{-6}	0.0065
Co	6.24×10^{-6}	0.0053
Ni	6.84×10^{-6}	0.0069
Cu	1.67×10^{-6}	0.0043
Ag	1.59×10^{-6}	0.0041
Au	2.35×10^{-6}	0.0035
Pd	10.8×10^{-6}	0.0037
W	5.3×10^{-6} (27°C)	0.0045
Pt	9.85×10^{-6}	0.0039

(Source: Reprinted by permission from Handbook of Electronic Materials, *by P.S. Neelakanta, pp. 215–216, Table 9-1. Copyright © 1995 CRC Press, Boca Raton, Florida.)*

Example – 1: Resistivity of Pure Copper

Calculate the electrical conductivity of pure copper at (a) 400°C and (b) –100°C

Solutions

Since the conductivity of pure copper is 5.98* 10^5 $\Omega^{-1}.cm^{-1}$. the resistivity of copper at room temperature is 1.67 * 10^{-6} ohm.cm. The temperature resistivity coefficient is 0.0068 $(°C)^{-1}$.

1. At 400°C:

$$\rho = \rho_{RT}(1 + \alpha_R \Delta T) = (1.67 \times 10^{-6})[1 + 0.0068(400 - 25)]$$

$$\rho = 5.929 \times 10^{-6} \text{ ohm} \cdot \text{cm}$$

$$\sigma = 1/\rho = 1.69 \times 10^5 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$$

2. At –100°C:

$$\rho = (1.67 \times 10^{-6})[1 + 0.0068(-100 - 25)] = 0.251 \times 10^{-6} \text{ ohm} \cdot \text{cm}$$

$$\sigma = 39.8 \times 10^5 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$$

Example 2: Calculate the electrical conductivity of nickel at - 50°C and at +500°C. Hence the resistivity at room temperature 6.84×10^{-6} Ω.cm and α = 0.0069 1/°C.

Solution

$$\rho_{th} = \rho_{RT}(1 + \alpha\Delta T)$$

$$\rho_{500} = (6.84\times10^{-6}\ \Omega.cm)[1 + (0.0069)(500 - 25)] = 29.26\times10^{-6}\ \Omega.cm$$

$$\sigma_{500} = 1/\rho_{500} = 1/29.26\times10^{-6}\ \Omega.cm = 0.34\times10^{5}\ (\Omega.cm)^{-1}$$

$$\rho_{-50} = (6.84\times10^{-6}\ \Omega.cm)[1 + (0.0069)(-50 - 25)] = 3.3003\times10^{-6}\ \Omega.cm$$

$$\sigma_{-50} = 1/\rho_{-50} = 1/3.3003\times10^{-6}\ \Omega.cm = 3.03\times105\ (\Omega.cm)^{-1}$$

Example 3: The electrical resistivity of pure chromium is found to be $18*10^{-6}$ Ω.cm. estimate the temperature at which the resistivity measurement was made, where the resistivity at room temperature $12.9*10^{-6}$ Ω.cm and α = 0.0030 Ω.cm/°C.

Solution

$$\rho_{th} = \rho_{RT}(1 + \alpha\Delta T)$$

$$18\times10^{-6} = (12.9\times10^{-6}\ \Omega.cm)[1 + (0.0030)(T - 25)]$$

$$1.395 - 1 = (0.003)(T - 25)$$

$$T = 156.8°C$$

- ***Influence of Impurities***

For additions of a single impurity that forms a solid solution, the impurity resistivity ρ_{imp} is related to the impurity concentration c_{imp} in terms of the atom fraction as follows:

$$\rho_{imp} = Ac_{imp}(1 - c_{imp})$$

where A is the impurity resistivity coefficient. The influence of nickel impurity additions on the room temperature resistivity of copper is demonstrated in Figure (3), up to 50 wt% Ni; over this composition range nickel is completely soluble in copper Figure (4). Again, nickel atoms in copper act as scattering centers, and increasing the concentration of nickel in copper; results in an enhancement of resistivity.

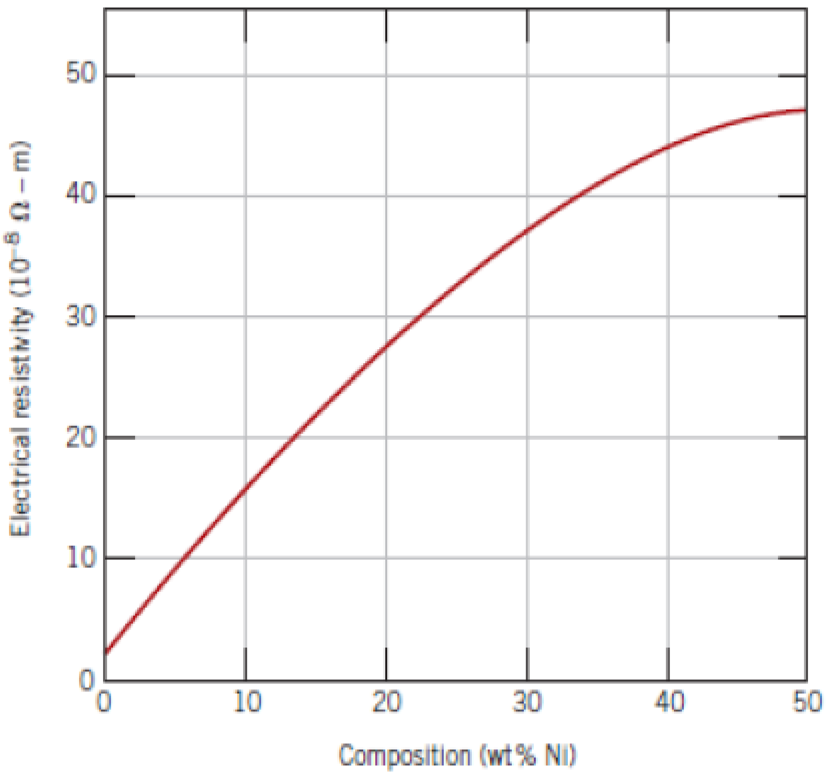


Figure (3): Room temperature electrical resistivity versus composition for copper– nickel alloys.

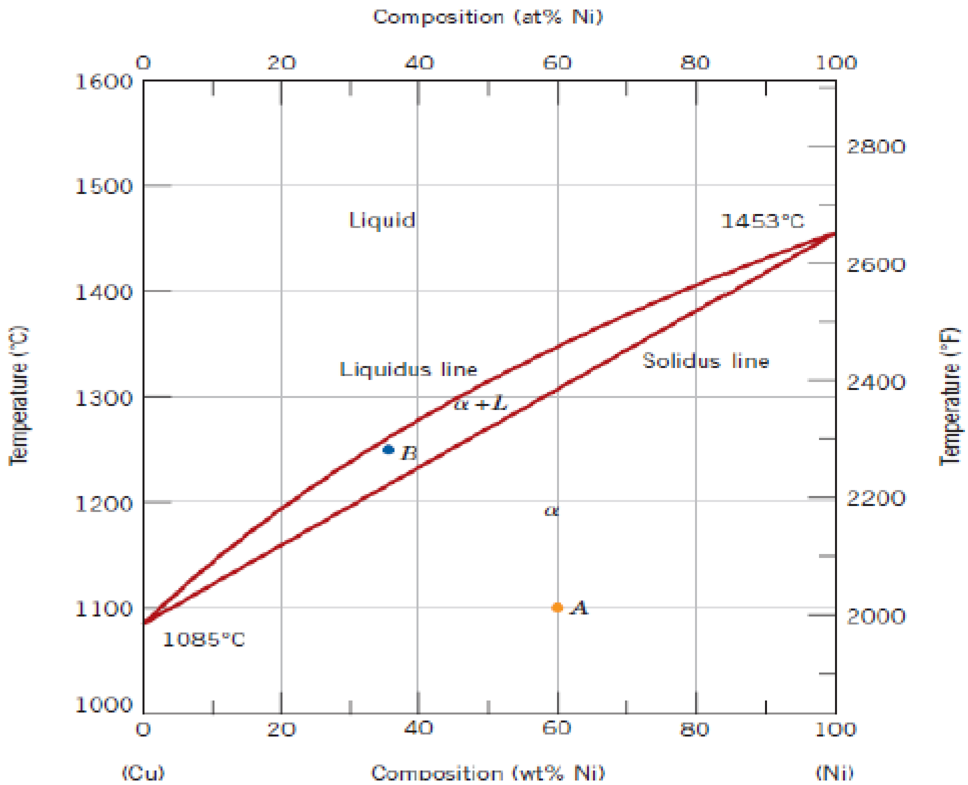
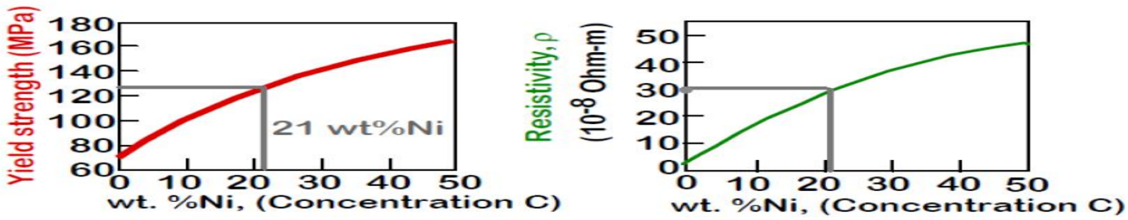


Figure (4): The copper – nickel phase diagram.

Example – 4: ***Estimate the electrical conductivity of a Cu-Ni alloy that has a yield strength of 125 MPa.***

Example – 4Solution:



$$\rho = 30x10^{-8}\ \text{Ohm}-\text{m}$$

$$\sigma = \frac{1}{\rho} = 3.3x10^{6}\ (\text{Ohm}-\text{m})^{-1}$$

Example 5: The electrical resistivity of a beryllium alloy containing 5 at% of an alloying element is found to be 50×10^{-6} ohm.cm at 400°C. Determine the contributions to resistivity due to temperature and due to impurities by finding the expected resistivity of pure beryllium at 400°C, the resistivity due to impurities, and the defect resistivity coefficient. What would be the electrical resistivity if the beryllium contained 10 at% of the alloying element at 200°C?

Solution

From the data in Table 4, the ρ of Be at 400°C should be:

$\boldsymbol{\rho_{th} = \rho_{RT}\,(1 + \alpha\Delta T)}$

$\boldsymbol{\rho_{400} = (4\times10^{-6}\ \Omega.cm)[1 + (0.025)(400 - 25)] = 41.5\times10^{-6}\ \Omega.cm}$

Consequently the resistance due to impurities is

$\boldsymbol{\rho_{total} = \rho th + \rho d}$

$\boldsymbol{50 \times10^{-6}\ (\Omega.cm) = 41.5\times10^{-6}\ (\Omega.cm) + \rho_d = 8.5\times10^{-6}(\Omega.cm)}$

Since there are 5 at% impurities present, cimp = 0.05, and the defect (impurities) resistivity coefficient A is

$\rho_{imp} = A c_{imp}(1 - cimp)$ **or** $A = \rho_{imp} / c_{imp}(1 - c_{imp})$

$A = 8.5\times10^{-6} / (0.05)(1 - 0.05) = 178.9\times10^{-6} (\Omega.cm)$

The resistivity at 200°C in an alloy containing 10 at% impurities is:

$P_{200} = \rho th + \rho d$

$= (4\times10^{-6}\ \Omega.cm)[1 + (0.025)(200 - 25)] + 178.9\times10^{-6}(0.1)(1 - 0.1)$

$= 21.5\times10^{-6} + 16.1\times10^{-6} = 37.6\times10^{-6}\ (\Omega.cm)$

- ***Influence of processing and/or Plastic Deformation***

Plastic deformation and/or metal processing techniques affect the electrical properties of a metal in different ways, Table (2). Where, raising the electrical resistivity as a result of increased numbers of electron-scattering dislocations. The effect of deformation on resistivity is also represented in Figure (5). Furthermore, its influence is much weaker than that of increasing temperature or the presence of impurities. For example; solid solution strengthening is not a good way to obtain high strength in metals intended to have high conductivities. The mean free paths are very short due to the random distribution of the interstitial or substitution atoms. Figure (5) shows the defect of zinc and other alloying elements on the conductivity of copper; as the amount of alloying element increases, the conductivity decreases substantially.

Also, age hardening and dispersion strengthening reduce the conductivity to an extent that is less than solid – solution strengthening, since there is a longer mean free path between precipitates, as compared with the path between point defects. Strain hardening and grain – size control has even less effect on conductivity, Figure (5) and Table (2).

TABLE 18-4 ■ ***The effect of alloying, strengthening, and processing on the electrical conductivity of copper and its alloys***

Alloy	$\frac{\sigma_{alloy}}{\sigma_{Cu}} \times 100$	Remarks
Pure annealed copper	100	Few defects to scatter electrons; the mean free path is long.
Pure copper deformed 80%	98	Many dislocations, but because of the tangled nature of the dislocation networks, the mean free path is still long.
Dispersion-strengthened Cu-0.7% Al_2O_3	85	The dispersed phase is not as closely spaced as solid-solution atoms, nor is it coherent, as in age hardening. Thus, the effect on conductivity is small.
Solution-treated Cu-2% Be	18	The alloy is single phase; however, the small amount of solid-solution strengthening from the supersaturated beryllium greatly decreases conductivity.
Aged Cu-2% Be	23	During aging, the beryllium leaves the copper lattice to produce a coherent precipitate. The precipitate does not interfere with conductivity as much as the solid-solution atoms.
Cu-35% Zn	28	This alloy is solid-solution strengthened by zinc, which has an atomic radius near that of copper. The conductivity is low, but not as low as when beryllium is present.

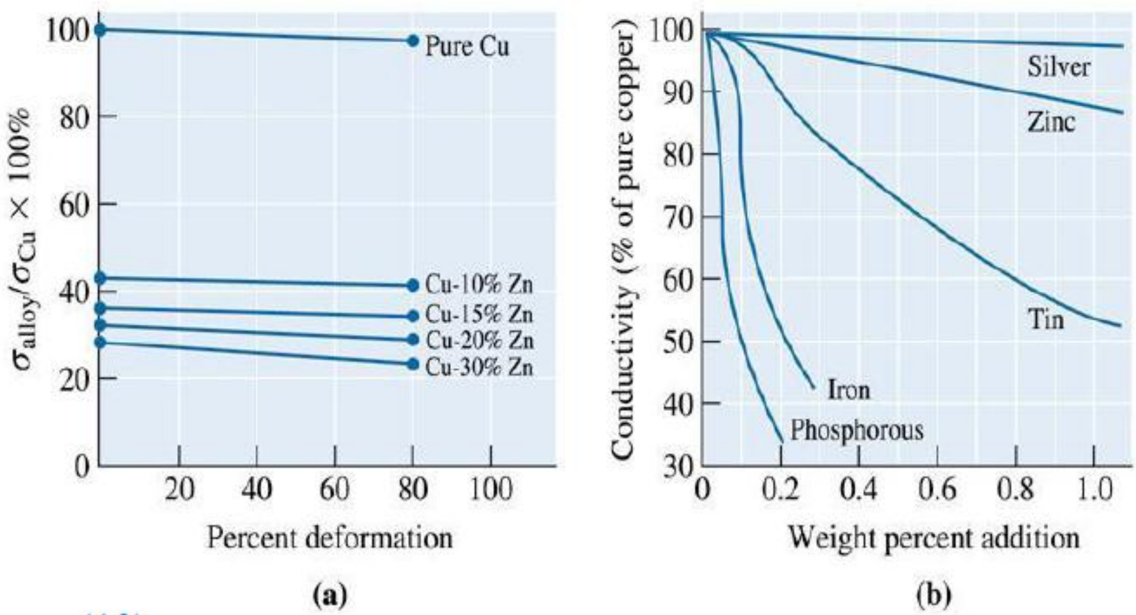


Figure (5): (a) the effect of solid-solution strengthening and cold working on the electrical conductivity of copper, and (b) the effect of addition of selected elements on the electrical conductivity of copper.