Q1/ Find the general solution of the second order differential equation by using (variation of parameters method):

$\bar{\bar{y}} + 2\bar{y} + y = e^{-x} \boldsymbol{ln}(x)$

homognous Part:

$y'' + 2y' + y = 0$

let $y = e^{mx}$, $y' = me^{mx}$, $y'' = m^2 e^{mx}$

$m^2 e^{mx} + 2me^{mx} + e^{mx} = 0$

$(m^2 + 2m + 1) e^{mx} = 0 \qquad e^{mx} \neq 0$

$m^2 + 2m + 1 = 0$

$(m+1)(m+1) = 0$

either $m+1 = 0 \longrightarrow m_1 = -1$

or $m+1 = 0 \longrightarrow m_2 = -1$

$y_h = c_1 \underbrace{e^{-x}}_{u1} + c_2 \underbrace{x e^{-x}}_{u2}$

non homognenaus part:

$$w = \begin{vmatrix} u1 & u2 \\ u1' & u2' \end{vmatrix} = \begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & -xe^{-x} + e^{-x} \end{vmatrix} = e^{-2x}$$

$$w_2 = \begin{vmatrix} 0 & u_2 \\ f(x) & u_2' \end{vmatrix} = \begin{vmatrix} 0 & xe^{-x} \\ e^{-x}\ln x & -xe^{-x}+e^{-x} \end{vmatrix} = -xe^{-2x}\ln x$$

$$w_3 = \begin{vmatrix} u_1 & 0 \\ u_1' & f(x) \end{vmatrix} = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & e^{-x}\ln x \end{vmatrix} = e^{-2x}\ln x$$

$$v_1' = \frac{w_1}{w} = \frac{-x\cancel{e^{-2x}}\ln x}{\cancel{e^{-2x}}} = -x\ln x$$

$$v_1 = \int v_1'\,dx = \int -x\ln x\,dx$$

using integration by parts

$$u = \ln x \longrightarrow du = \frac{1}{x}dx$$

$$dv = -x\,dx \longrightarrow v = -\frac{x^2}{2}$$

$$\int u\cdot dv = v\cdot u - \int v\,du$$

$$\int -x\ln x\,dx = -\frac{x^2}{2}\ln x - \int \left(-\frac{x^2}{2}\right)\frac{1}{x}dx$$

$$v = -\frac{x^2}{2}\ln x + \frac{x^2}{4}$$

$$v_2' = \frac{w_2}{w} = \frac{\cancel{e^{-2x}} \ln x}{\cancel{e^{-2x}}} = \ln x$$

$$v_2 = \int v_2' \, dx = \int \ln x \, dx$$

using integration by part

$$u = \ln x \longrightarrow du = \frac{1}{x} dx$$

$$dv = dx \longrightarrow v = x$$

$$\int \ln x \, dx = x \ln x - \int x \frac{1}{x} dx$$

$$v_2 = x \ln x - x$$

$$y_p = u_1 v_1 + u_2 v_2$$

$$y_p = e^{-x}\left(-\frac{x^2}{2} \ln x + \frac{x^2}{4}\right) + x e^{-x}(x \ln x - x)$$

$$y_p = \left(\frac{x^2}{2} \ln x - \frac{3}{4} x^2\right) e^{-x}$$

$$y = y_h + y_p$$

$$y = c_1 e^{-x} + c_2 x e^{-x} + \left(\frac{x^2}{2} \ln x - \frac{3}{4} x^2\right) e^{-x}$$

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Q1/ Solve the following differential equation using (undetermined coefficient method):

$$3\bar{\bar{y}} + \bar{y} - 2y = 2cos(x) + 3e^{-x}$$

homogneous part

$$3y'' + y' - 2y = 0$$

let $y = e^{mx} \rightarrow y' = me^{mx} \longrightarrow y'' = m^2 e^{mx}$

$$3m^2 e^{mx} + me^{mx} - 2e^{mx} = 0$$

$$(3m^2 + m - 2)e^{mx} = 0 \qquad e^{mx} \neq 0$$

$$3m^2 + m - 2 = 0$$

$$(3m-2)(m+1) = 0$$

either $3m - 2 = 0 \longrightarrow m_1 = \frac{2}{3}$

or $m + 1 = 0 \longrightarrow m_2 = -1$

$$y_h = c_1 e^{2/3 x} + c_2 e^{-x}$$

non-homogneous part:

$$y_p = y_{p1} + y_{p2}$$

$$y_{p1} = a_0 \sin x + a_1 \cos x$$

$$y_{p1}' = a_0 \cos x - a_1 \sin x$$

$$y_{p1}'' = -a_0 \sin x - a_1 \cos x$$

$$+3(-a_0 \sin x - a_1 \cos x) + (a_0 \cos x - a_1 \sin x)$$
$$-2(a_0 \sin x + a_1 \cos x) = 2\cos x$$

$$-3a_0 \sin x - 3a_1 \cos x + a_0 \cos x - a_1 \sin x$$
$$-2a_0 \sin x - 2a_1 \cos x = 2\cos x$$

<u>$\cos x$</u>

$$a_0 - 5a_1 = 2 \quad (1)$$

$\sin x$

$$-5a_0 - a_1 = 0 \quad (2)$$

$$a_1 = -5a_0 \quad — (3) \qquad \text{sub in } (1)$$

$$a_0 - 5(-5a_0) = 2$$

$$26a_0 = 2 \quad \text{or} \quad a_0 = \frac{2}{26} = \frac{1}{13}$$

$$\therefore a_1 = -5\left(\frac{1}{13}\right) = -\frac{5}{13}$$

$$y_{p1} = \frac{1}{13}\sin x - \frac{5}{3}\cos x$$

$$y_{p2} = a_2 x e^{-x}$$

$$y_{p2}' = a_2 x(-e^{-x}) + a_2 e^{-x}$$

$$y_{p2}' = -a_2 x e^{-x} + a_2 e^{-x}$$

$$y_{p_2}'' = -a_2 x(-e^{-x}) + (-a_2 e^{-x}) - a_2 e^{-x}$$

$$y_{p_2}'' = a_2 x e^{-x} - 2a_2 e^{-x}$$

$$3(a_2 x e^{-x} - 2a_2 e^{-x}) + (-a_2 x e^{-x} + a_2 e^{-x})$$

$$-2a_2 x e^{-x} = 3e^{-x}$$

$$3a_2 x e^{-x} - 6a_2 e^{-x} - a_2 x e^{-x} + a_2 e^{-x}$$

$$-2a_2 x e^{-x} = 3e^{-x}$$

$$-6a_2 + a_2 = 3 \longrightarrow a_2 = -\frac{3}{5}$$

$$y_{p2} = \frac{-3}{5} x e^{-x}$$

$$y_p = y_{p1} + y_{p2}$$

$$y_p = \frac{1}{13}\sin x - \frac{5}{13}\cos x - \frac{3}{5} x e^{-x}$$

$$y = y_h + y_p$$

$$y = c_1 e^{2/3 x} + c_2 e^{-x} + \frac{1}{13}\sin x - \frac{5}{13}\cos x - \frac{3}{5} x e^{-x}$$

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