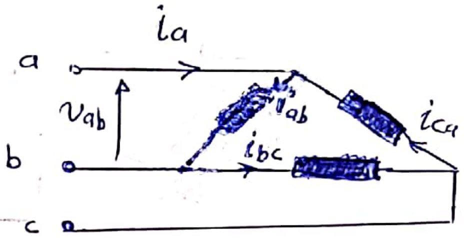
Example 5-16 Lander P214

A three-phase bridge inverter is fed from a d.c. source of 240V. If the control is by 180° firing, quasi-square-wave output, determine the load line-current waveform at 50 Hz output given the load is (i) delta-connected, with each phase 6Ω resistance and 0.012 H inductance, (ii) star-connected, with each phase 2Ω resistance and 0.004 H inductance.

solution

(i)



$i_a = i_{ab} - i_{ca}$

①

first 120°: $V_{ab} = 240\,V = V_s$

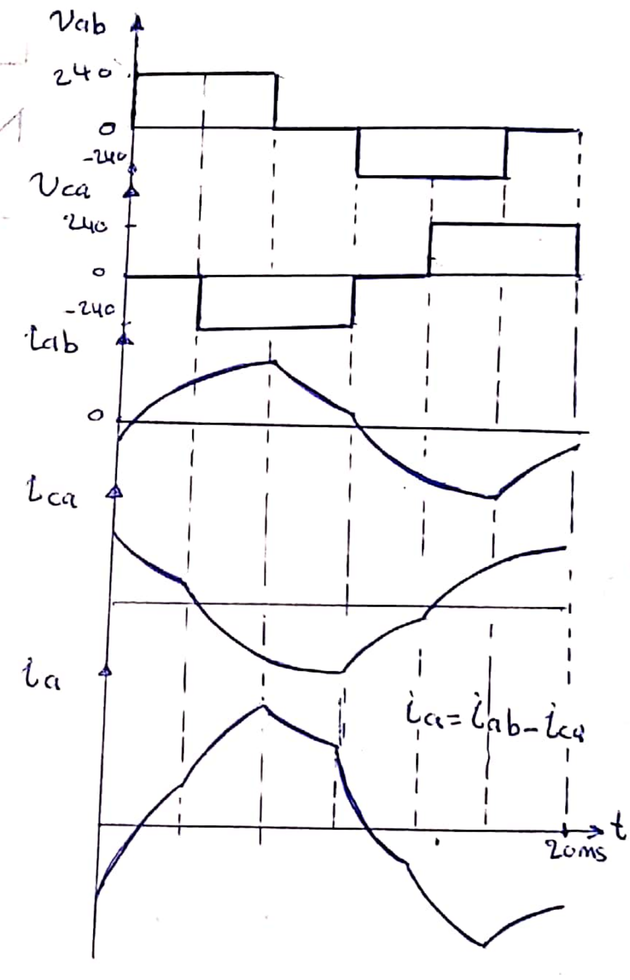
$$i_{ab} = \frac{V_s}{R} - \left(\frac{V_s}{R} - I_o\right) e^{-\frac{R}{L}t} \quad , I_o = 0$$

$$= \frac{240}{6} - \left(\frac{240}{6} - 0\right) e^{-500t}$$

$$= 40 - 40e^{-500t}$$

at the end of this period $i_{ab} = 40 - 40e^{-500 \times (20 \times 10^{-3} \times \frac{2}{6})}$

$$= 38.573\ A$$



②

For the next 60°: $V_{ab} = 0$

$$i_{ab} = I_1 e^{-\frac{R}{L}t}$$

$$= 38.573 e^{-500t}$$

at the end of this period:

$$i_{ab} = 38.573 e^{-500 \times (20 \times 10^{-3} \times \frac{1}{6})}$$

$$= 7.285\ A$$

③ For the next 120° $V_{ab} = -V_s = -240V$

$$i_{ab} = -\frac{V_s}{R} + \left(\frac{V_s}{R} + I_2\right) e^{-\frac{R}{L}t}$$

$$= -40 + (40 + 7.285)\, e^{-500t}$$

at the end of this period $i_{ab} = -40 + 47.285 e^{-500 \times (20 \times 10^{-3} \times \frac{2}{6})} = -38.313\,A$

(4) for the next 60° $V_{ab} = 0$

$$i_{ab} = -38.313\, e^{-500t}$$

at the end of this period: $i_{ab} = -38.313\, e^{-500 \times (20\times10^{-3} \times \frac{1}{6})} = -7.24\ A$

For the next cycle the expression for the four periods will be:

(1) $i_{ab} = 40 - (40 + 7.24)\, e^{-500t}$ ending with $40 - 47.24\, e^{-500\times20\times10^{-3}\times\frac{2}{6}} = 38.31\ A$

(2) $i_{ab} = 38.31\, e^{-500t}$ ending with $38.31\, e^{-500\times20\times10^{-3}\times\frac{1}{6}} = 7.24\ A$

(3) $i_{ab} = -40 + (40 + 7.24)\, e^{-500t}$ ending with $-40 + 47\, e^{-500\times20\times10^{-3}\times\frac{2}{6}} = -38.31\ A$

(4) $i_{ab} = -38.31\, e^{-500t}$ ending with $-38.31\, e^{-500\times20\times10^{-3}\times\frac{1}{6}} = -7.24\ A$

i.e steady-state conditions have been reached in the second cycle.

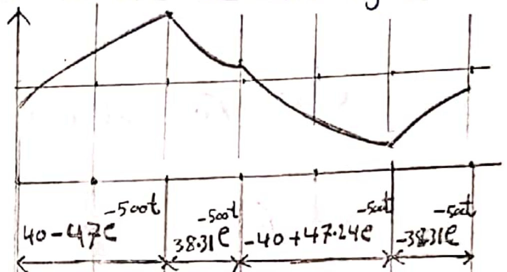
i_{ab} for four periods is:

first 120°: $i_{ab} = 40 - 47.24\, e^{-500t}$

next 60°: $i_{ab} = 38.31\, e^{-500t}$

next 120°: $i_{ab} = -40 + 47.24\, e^{-500t}$

next 60°: $i_{ab} = -38.31\, e^{-500t}$



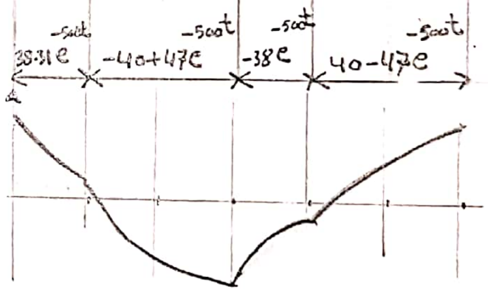
i_{ca} is identical to i_{ab} by displaced by 240:

first 60°: $i_{ca} = 38.31\, e^{-500t}$

next 120°: $i_{ca} = -40 + 47.24\, e^{-500t}$

next 60°: $i_{ca} = -38.31\, e^{-500t}$

next 120°: $i_{ca} = 40 - 47.24\, e^{-500t}$



$i_a = i_{ab} - i_{ca}$ hence:-

first 60°: $i_a = 40 - 47.24\, e^{-500t} - 38.31\, e^{-500t}$

next 60°: $i_a = 40 - 47.24\, e^{-(500t + \frac{500}{50\times6})} + 40 - 47.24\, e^{-500t}$

$= 40 - 47.24\, e^{-500t} \cdot e^{-\frac{500}{300}} + 40 - 47.24\, e^{-500t}$

$= 80 - 8.922\, e^{-500t} - 47.24\, e^{-500t}$

next 60°: $i_a = 38.31e^{-500t} + 40 - 47.24\,e^{-500(t+\frac{1}{300})}$

$= 38.31\,e^{-500t} + 40 - 8.922\,e^{-500t}$

next 60° : $i_a = -40 + 47.24\,e^{-500t} + 38.31\,e^{-500t}$

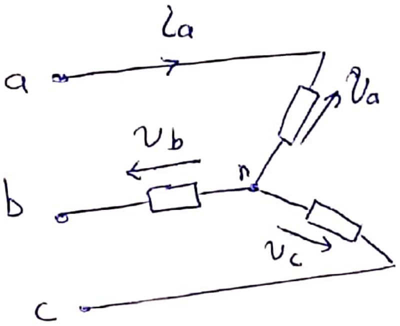
next 60° : $i_a = -40 + 47.24\,e^{-500(t+\frac{1}{300})} - 40 + 47.24\,e^{-500t}$

$= -40 + 8.922\,e^{-500t} - 40 + 47.24\,e^{-500t}$

next 60° : $i_a = -38.31\,e^{-500t} - 40 + 47.24\,e^{-500(t+\frac{1}{300})}$

$= -38.31\,e^{-500t} - 40 + 8.922\,e^{-500t}$

(b) For star load:

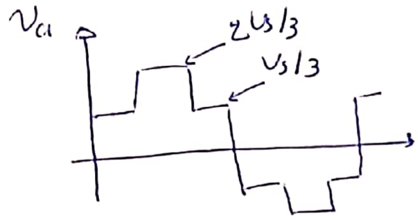


① first 60°

$i_a = \frac{V_s/3}{R} - \left(\frac{V_s/3}{R} - I_o\right)e^{-\frac{R}{L}t}$

$= \frac{80}{2} - \frac{80}{2}e^{-500t}$

$= 40 - 40e^{-500t}$



ending with $i_a = 40 - 40e^{-500\times\frac{1}{300}} = 32.445\,A$

② next 60°

$i_a = \frac{2V_s/3}{R} - \left(\frac{2V_s/3}{R} - I_o\right)e^{-500t}$

$= \frac{160}{2} - \left(\frac{160}{2} - 32.445\right)e^{-500t}$

ending with $i_a = \cancel{7}1$

next 60°:

$$i_a = \frac{80}{2} - \left(\frac{80}{2} - 71\right)e^{-500t} \quad \text{ending with } i_a = 45.855 \text{ A}$$

next 60°:

$$i_a = -\frac{V_s/3}{R} + \left(\frac{V_s/3}{R} + I_2\right)e^{-\frac{R}{L}t}$$

$$= -\frac{80}{2} + \left(\frac{80}{2} + 45.855\right)e^{-500t}$$

ending with $i_a = -23.78$ A

next 60°:

$$i_a = -\frac{160}{2} + \left(\frac{160}{2} - 23.78\right)e^{-500t}$$

ending with $i_a = -69.38$ A

next 60°:

$$i_a = -\frac{80}{2} + \left(\frac{80}{2} - 69.38\right)e^{-500t}$$

ending with $i_a = -45.55$ A

For the second cycle :-

first 60°: $i_a = 40 - (40 + 45.55)e^{-500t}$ ending with 23.84 A

next 60°: $i_a = 80 - (80 - 23.84)e^{-500t}$ ending with 69.39

next 60°: $i_a = 40 - (40 - 69.39)e^{-500t}$ ending with 45.55 A

next 60°: $i_a = -40 + (40 + 45.55)e^{-500t}$ ending with −23.84 A

next 60°: $i_a = -80 + (80 - 23.84)e^{-500t}$ ending with −69.39 A

next 60°: $i_a = -40 + (40 - 69.39)e^{-500t}$ ending with −45.55 A

steady-state condition have been reached.