

Tutorial Sheet No. 1

Q1) A 115V, 50Hz, four-pole split-phase ^{to the} induction motor has the following data: $r_1 = 1.8 \Omega$, $x_1 = 2 \Omega$, $r_2 = 3 \Omega$, $x_2 = 2 \Omega$ and $X_M = 50 \Omega$. At a slip of 0.05, motor rotational losses are 42W. At the indicated slip, find the following quantities: P_m , P_g , P_{in} , P_{out} , T , T_L , η and power factor.

Q2) A 200V, 50Hz, 4-pole, 1425 rpm split-phase motor has the following parameters: $Z_m = (2 + j4) \Omega$, $Z_2 = (4 + j3) \Omega$ at $s = 1$, $X_M = 80 \Omega$. Rotational losses in Watts are given by $P_{rot} = 50 + K(1-s)$ and equal 88W at rated speed. The motor is permanently coupled to a load whose torque in (Nm) is given by $T_L = 2.2 \times 10^{-6} n^2$; $n =$ motor speed in rpm.

At starting and during the acceleration period, the auxiliary winding is open-circuited at 450 rpm. Determine whether the motor continues acceleration or it will start deceleration. Find the accelerating or the decelerating torque at 450 rpm.

Q3) A 220V, 2.25 kW, 50Hz, 6-pole capacitor start induction motor has the following impedances referred to the main winding: $r_1 = 1.3 \Omega$, $x_1 = 2.01 \Omega$, $r_2 = 1.73 \Omega$, $x_2 = 2.01 \Omega$, and $X_M = 105 \Omega$. At a slip

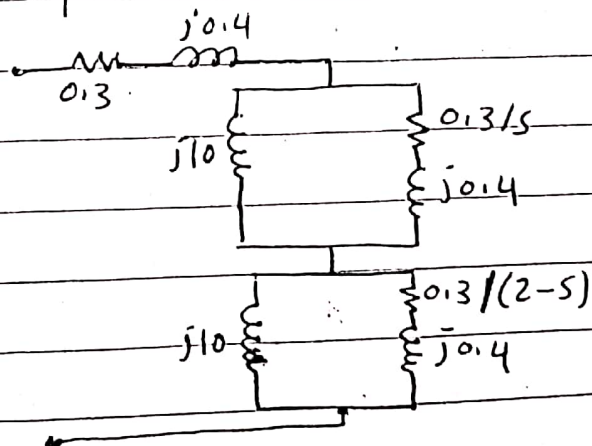
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of 0.05, the motor's rotational losses are 290 W. Find the followings at 5 percent slip: P_g , P_m , P_{in} , P_{out} , induced torque, load torque and efficiency.

Q4) Assume that at a given motor speed the variation of rotational losses with small variation in source voltage is almost negligible. Find the developed torque of the motor described in question three if it is operating at the same slip and its terminal voltage is: (a) 231 V, (b) 209 V, (c) calculate and comment on the values of the "torque/voltage" ratio at 231, 220, and 209 V.

Q5) The adjacent figure shows the equivalent circuit of a 60 Hz, 120 V, 2-pole single phase induction motor. All values are in ohms and "s" is the per unit slip. When the motor is running at a slip of 5%, determine:

- The input current and power.
- The developed torque in Nm.



Q6) The following details apply to the main winding of a capacitor motor when operating under balanced conditions: 250V input voltage at 50Hz, 12.5A, 1.25kW, 600 turns. Calculate the number of turns of the auxiliary winding and the VA rating of the capacitor under the balanced conditions.

Q7) A) Consider a single-phase capacitor motor with turns ratio "a". Draw motor equivalent circuits through main and auxiliary windings for operation with both windings, assuming the slip is "s". Simplify the circuits for $s=1$.

B) A capacitor start motor has $Z_m = (5 + j6) \Omega$, $Z_a = (3 + j3) \Omega$, $X_M = j40 \Omega$, $a = 0.8$ and rotor main impedance referred to the main winding is $Z_2 = (4 + j4) \Omega$. Find the capacitive reactance, which has to be connected in series with the auxiliary to put stator currents in quadrature at starting.

(C) For the motor in (B), determine the required auxiliary circuit impedance to completely eliminate the backward torque at starting.

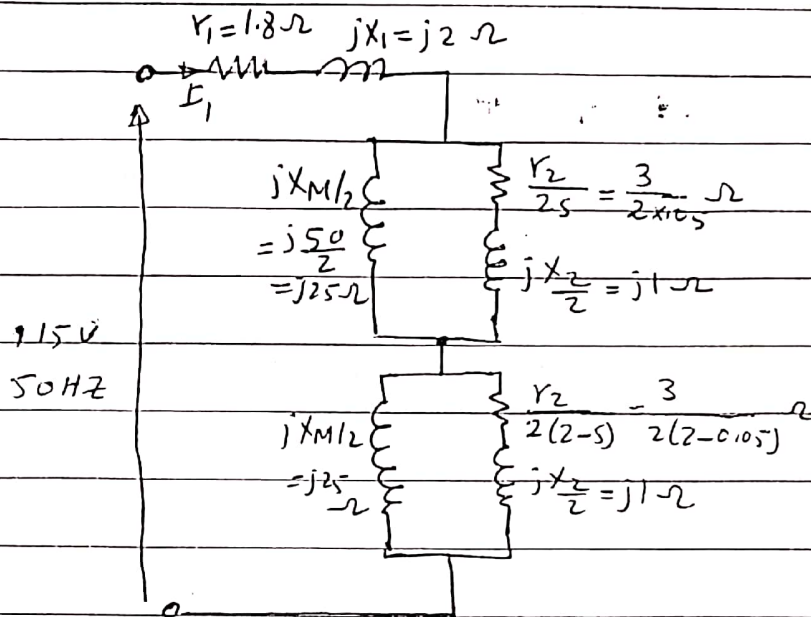
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Q10) A single-phase 220V, 50 Hz, 1410 rpm, 4 pole induction motor has the following parameters, $r_1 = 2.3 \Omega$, $r_2 = 3.6 \Omega$, $x_1 = x_2 = 2.5 \Omega$ and $x_M = 60 \Omega$. The motor drives its fan load at rated speed when rated voltage is applied (Fan loads are characterized by $T_L \propto n^2$; where n is the speed). What resistance should be connected in series with the motor to reduce its speed to 1190 rpm?

Q11) A $\frac{1}{4}$ hp, 50 Hz, 4-pole, 110V, two values capacitor motor has the following constants, $r_1 = 2.02 \Omega$, $x_1 = 2.79 \Omega$, $r_2 = 4.12 \Omega$, $x_2 = 2.12 \Omega$, $r_a = 7.4 \Omega$, $x_a = 3.22 \Omega$, $x_M = 66.8 \Omega$, $a = 1.18$, starting capacitor: $r_c = 3 \Omega$, $x_c = -14.5 \Omega$, running capacitor: $r_c = 9 \Omega$, $x_c = -17.2 \Omega$. At standstill conditions, find the current in each winding, the line current, P_f, the voltage across the capacitor and torque.

Q12) Repeat (Q.11) but taking $s = 0.05$.

Q1)



$$Z_f = \left(\frac{r_2}{2s} + j\frac{X_2}{2} \right) // j\frac{X_M}{2}$$

$$Z_f = \left(\frac{3}{2 \times 0.05} + j1 \right) // j25 = \frac{(30 + j1)(j25)}{30 + j1 + j25} = \frac{30 \angle 1.9^\circ \times 25 \angle 90^\circ}{39.6 \angle 40.9^\circ}$$

$$Z_f = 18.94 \angle 51^\circ = 11.9 + j14.71 \Omega$$

$$Z_b = \left(\frac{r_2}{2(2-s)} + j\frac{X_2}{2} \right) // j\frac{X_M}{2}$$

$$Z_b = \left(\frac{3}{2(2-0.05)} + j1 \right) // j25 = \frac{(0.769 + j1)(j25)}{0.769 + j1 + j25}$$

$$Z_b = \frac{1.26 \angle 52.4^\circ \times 25 \angle 90^\circ}{26 \angle 88.3^\circ} = 1.21 \angle 54.1^\circ$$

$$Z_b = 0.709 + j0.98 \Omega$$

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$$I_1 = \frac{V}{r_1 + jX_1 + Z_f + Z_b} = \frac{115 \angle 0^\circ}{1.8 + j2 + 11.9 + j14.7 + 0.709 + j0.98}$$

$$I_1 = \frac{115 \angle 0^\circ}{14.40 + j17.68} = \frac{115 \angle 0^\circ}{22.8 \angle 50.8^\circ}$$

$$I_1 = 5.04 \angle -50.8^\circ \text{ A}$$

$$P_f = \cos(-50.8^\circ) = 0.632 \text{ Lagging}$$

$$P_g = I_1^2 (R_f - R_b) = 5.04^2 (11.9 - 0.709)$$

$$P_g = 284.26 \text{ W}$$

$$P_m = (1-s)P_g = (1-0.05) \times 284.26 = 270 \text{ W}$$

$$\omega_s = \frac{120f}{60p} \times 2\pi = \frac{120 \times 50}{60 \times 4} \times 2\pi$$

$$T = \frac{P_g}{\omega_s} = \frac{284.26 \times 4 \times 60}{120 \times 50 \times 2\pi} = 1.8 \text{ Nm}$$

$$P_{out} = P_m - P_{rot} = 270 - 42 = 228 \text{ W}$$

$$T_L = \frac{P_{out}}{\omega_p} = \frac{P_{out}}{(1-s)\omega_s} = \frac{228}{(1-0.05) \times \frac{120 \times 50 \times 2\pi}{60 \times 4}} = 1.52 \text{ Nm}$$

$$P_{in} = V I_1 \cos\phi = 115 \times 5.04 \times 0.6327 = 366.7 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{228}{366.7} \times 100 = 62\%$$

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$$Q_2 \rightarrow n_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ RPM}$$

$$s = \frac{n_s - n_r}{n_s} = \frac{1500 - 450}{1500} = 0.7$$

$$\text{The slip at rated speed, } s = \frac{1500 - 1425}{1500} = 0.05, k = \frac{88 - 50}{1 - 0.05} =$$

$$P_{rot} = 50 + 40(1 - 0.7) = 62 \text{ W (losses at 450 rpm)} \quad k = 40$$

$$T_L = 2.2 \times 10^{-6} \times 450^2 = 0.4455 \text{ Nm}$$

$$Z_f = \left(\frac{r_2}{s} + j \frac{x_2}{2} \right) \parallel j X_M$$

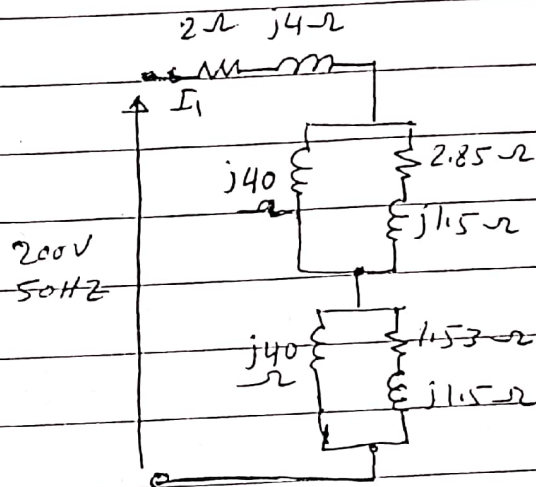
$$= \left(\frac{2}{0.7} + j 1.5 \right) \parallel (j 40)$$

$$Z_f = (2.85 + j 1.5) \parallel j 40$$

$$Z_f = \frac{(2.85 + j 1.5)(j 40)}{2.85 + j 1.5 + j 40}$$

$$Z_f = \frac{3.22 \angle 27.75^\circ \times 40 \angle 90^\circ}{41.59 \angle 86^\circ}$$

$$Z_f = 3 \angle 31.75^\circ$$



$$Z_f = 2.55 + j 1.57 \Omega$$

$$Z_b = \left(\frac{r_2}{2(2-s)} + j \frac{x_2}{2} \right) \parallel j X_M$$

$$= \left(\frac{2}{2-0.7} + j 1.5 \right) \parallel j 40$$

$$Z_b = \frac{(1.53 + j 1.5)(j 40)}{1.53 + j 1.5 + j 40} = \frac{2.14 \angle 44.43^\circ \times 40 \angle 90^\circ}{41.52 \angle 87.88^\circ}$$

$$Z_b = 2.06 \angle 46.55^\circ = 1.41 + j 1.49 \Omega$$

$$I_1 = \frac{V}{Z_m + Z_f + Z_b} = \frac{200 \angle 0^\circ}{2 + j 4 + 2.55 + j 1.57 + 1.41 + j 1.49}$$

$$I_1 = \frac{200 \angle 0^\circ}{5.96 + j 7.06} = \frac{200 \angle 0^\circ}{9.23 \angle 49.8^\circ} = 21.66 \angle -49.8^\circ \text{ A}$$

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$$P_g = I_1^2 (R_f - R_b)$$

$$P_g = 21.66^2 (2.55 - 1.41) = 534.83 \text{ W}$$

$$P_m = (1 - s) P_g = (1 - 0.17) \times 534.83 = 160.4 \text{ W}$$

$$P_{out} = P_m - P_{rot} = 160.4 - 62 = 98.4 \text{ W}$$

$$T_L = \frac{P_{out}}{\omega_r} = \frac{P_{out}}{(1-s)\omega_s} = \frac{98.4}{(1-0.17) \times \frac{1500 \times 2\pi}{60}} = 2.08 \text{ Nm}$$

The shaft developed torque is more than the applied load (0.4455 Nm), therefore the motor under acceleration.

The accelerating torque is

$$2.08 - 0.4455 = 1.634 \text{ Nm}$$