Example 3.9

Determine the values of bending moment and shear force used to design and evaluate the reinforced concrete slab bridge shown below. Take the effective span length of 25 m, future wearing surface load of 1.2 kN/m² and barrier (curbs and parapet) load of 7.5 kN/m each side.

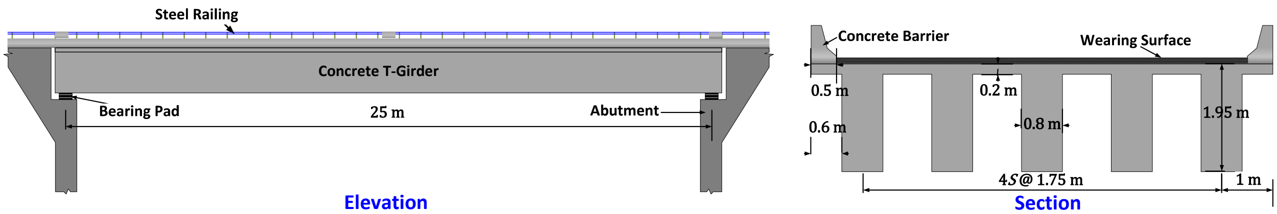


Figure 3-17: Details for Example 3.9

Solution 3.9

Deck Slab

Unfactored Loads Per Unit Width:

- Permanent loads:

$$w_d = h_d b \Upsilon_c = (0.2)(1)(24) = 4.8 \text{ kN/m}$$
$$M_{DC} = w_{DC} L^2/24 = (4.8)(1.75)^2/24 = 0.61 \text{ kN.m}$$
$$M_{DC}^- = w_{DC} L^2/12 = (4.8)(1.75)^2/12 = 1.23 \text{ kN.m}$$
$$w_{ws} = q_{ws} b = (1.2)(1) = 1.2 \text{ kN/m}$$
$$M_{DW} = w_{DW} L^2/24 = (1.2)(1.75)^2/24 = 0.15 \text{ kN.m}$$
$$M_{DW}^- = w_{DW} L^2/12 = (1.2)(1.75)^2/12 = 0.31 \text{ kN.m}$$

- Live loads:

$$S = 1.75 \text{ m}$$
$$S = 1.7 \text{ m} \rightarrow M_{LL} = 21.44 \text{ kN.m}$$
$$S = 1.8 \text{ m} \rightarrow M_{LL} = 21.79 \text{ kN.m}$$
$$M_{LL} = (21.44 + 21.79)/2 = 21.62 \text{ kN.m}$$
$$0.5 b_w = 400 \text{ mm}$$
$$0.5 b_w = 300 \text{ mm} \rightarrow M_{LL}^- = (9.71 + 10.44)/2 = 10.08 \text{ kN.m}$$
$$0.5 b_w = 450 \text{ mm} \rightarrow M_{LL}^- = (6.06 + 6.27)/2 = 6.17 \text{ kN.m}$$
$$\frac{450 - 300}{400 - 300} = \frac{6.17 - 10.08}{M_{LL}^- - 10.08} \rightarrow M_{LL}^- = 7.47 \text{ kN.m}$$

Strength I Limit State:

$$\eta_i = 1.0$$
$$M_u = \eta_i [1.25 M_{DC} + 1.50 M_{DW} + 1.75 M_{LL+IM}]$$
$$M_u = 1.25(0.61) + 1.50(0.15) + 1.75(21.62) = 38.82 \text{ kN.m}$$
$$M_u^- = 1.25(1.23) + 1.50(0.31) + 1.75(7.47) = 15.08 \text{ kN.m}$$

Interior Girder

Unfactored Loads:

$$b_f = S = 1.75 \text{ m}$$
$$A_g = A_f + A_w = (1.75)(0.2) + (0.8)(1.75) = 1.75 \text{ m}^2$$

$w_{DC} = w_g = A_g \Upsilon_c = (1.75)(24) = 42 \text{ kN/m}$

$M_{DC} = w_{DC} L^2/8 = (42)(25)^2/8 = 3281.25 \text{ kN.m}$

$V_{DC} = w_{DC} L/2 = (42)(25)/2 = 525 \text{ kN}$

$b_{ws} = b_f = 1.75 \text{ m}$

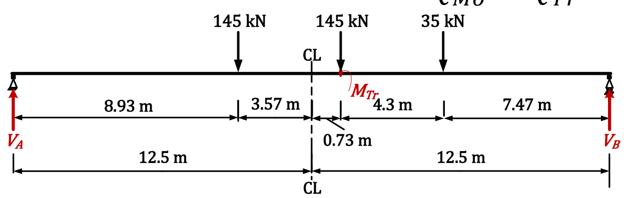
$w_{ws} = q_{ws} b_{ws} = (1.2)(1.75) = 2.1 \text{ kN/m}$

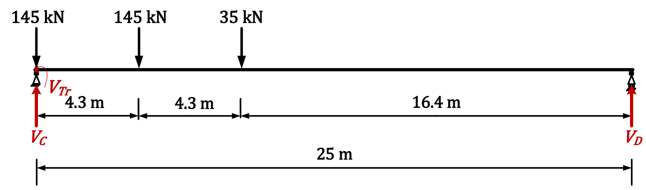
$M_{DW} = w_{DW} L^2/8 = (2.1)(25)^2/8 = 164.06 \text{ kN.m}$

$V_{DW} = w_{DW} L/2 = (2.1)(25)/2 = 26.25 \text{ kN}$

- Live loads:

$L = 25 \text{ m} > 12 \text{ m} \rightarrow Q_{Mo} = Q_{Tr}$





$\Sigma M_B = 0 \curvearrowright^{+}$

$V_A (25) = (35)(7.47) + (145)(11.77 + 16.07)$

$\therefore V_A = 171.93 \text{ kN}$

$M_{Tr} = (171.93)(13.23) - (145)(4.3) = 1651.13 \text{ kN.m}$

$\Sigma M_D = 0 \curvearrowright^{+}$

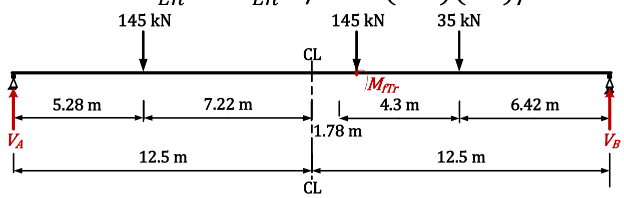
$V_{Tr} (25) = (35)(16.4) + (145)(20.7 + 25)$

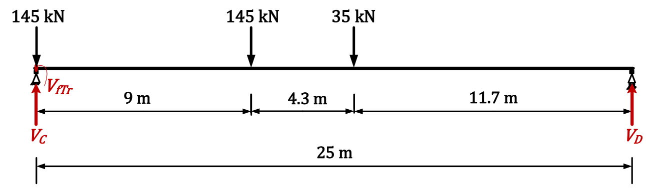
$\therefore V_{Tr} = 288 \text{ kN}$

$w_{Ln} = 9.3 \text{ kN/m}$

$M_{Ln} = w_{Ln} L^2/8 = (9.3)(25)^2/8 = 726.56 \text{ kN.m}$

$V_{Ln} = w_{Ln} L/2 = (9.3)(25)/2 = 116.25 \text{ kN}$





$\Sigma M_B = 0 \curvearrowright^{+}$

$V_A (25) = (35)(6.42) + (145)(10.72 + 19.72)$

$\therefore V_A = 185.54 \text{ kN}$

$M_{fTr} = (185.54)(14.28) - (145)(9) = 1344.51 \text{ kN.}$

Live load distribution factors:

$N_g \geq 4$	$N_g = 5$	$\therefore$ OK
$6 \leq L \leq 73$	$L = 25 \text{ m}$	$\therefore$ OK
$1.1 \leq S \leq 4.9$	$S = 1.75 \text{ m}$	$\therefore$ OK
$110 \leq h_d \leq 300$	$h_d = 200 \text{ mm}$	$\therefore$ OK
$4\text{x}10^9 \leq K_g \leq 3\text{x}10^{12}$	$K_g = 2.02\text{x}10^{12} \text{ mm}^4$	$\therefore$ OK

$n = E_g/E_d = 1.0$

$I_g = b_w {h_w}^3/12 = (800)(1750)^3/12 = 357.29\text{x}10^9\ \text{mm}^4$

$A_g = 1.75\text{x}10^6\ \text{mm}^2$

$e_g = h_f/2 + h_w/2 = 200/2 + 1750/2 = 975\ \text{mm}$

$K_g = n\left(I_g + A_g {e_g}^2\right) = (1.0)[357.29\text{x}10^9 + 1.75\text{x}10^6(975)^2] = 2.02\text{x}10^{12}\ \text{mm}^4$

$w = 9\ \text{m}$

$N_L = \text{INT}(\text{w}/3.6) = \text{INT}(9.0/3.6) = 2 \therefore \text{check both } g_{i1} \text{ and } g_{i2}$

Live load distribution factor for bending moment:

$g_{im1} = 0.06 + (S/4.3)^{0.4}(S/L)^{0.3}\left(K_g/L{h_d}^3\right)^{0.1}$

$= 0.06 + (1.75/4.3)^{0.4}(1.75/25)^{0.3}[2.02\text{x}10^{12}/(25\text{x}10^3)(200)^3]^{0.1} = 0.46$

$g_{im2} = 0.075 + (S/2.9)^{0.6}(\text{S}/\text{L})^{0.2}\left(K_g/L{h_d}^3\right)^{0.1}$

$= 0.075 + (1.75/2.9)^{0.6}(1.75/25)^{0.2}[2.02\text{x}10^{12}/(25\text{x}10^3)(200)^3]^{0.1} = 0.62$

$\therefore DF_{im} = 0.62$

Live load distribution factor for shear:

$g_{iv1} = 0.36 + S/7.6$

$= 0.36 + 1.75/7.6 = 0.59$

$g_{iv2} = 0.2 + S/3.6 - (S/10.7)^2$

$= 0.2 + 1.75/3.6 - (1.75/10.7)^2 = 0.66$

$\therefore DF_{iv} = 0.66$

Live load distribution factor for deflection:

$DF_\Delta = mN_L/N_g = (1.0)(2)/5 = 0.40$

Live load distribution factor for fatigue:

$DF_{ifm} = g_{im1}/1.2 = 0.46/1.2 = 0.38$

$DF_{ifv} = g_{iv1}/1.2 = 0.59/1.2 = 0.49$

Total live load:

$M_{LL+IM} = [M_{Tr}(1 + IM) + M_{Ln}]DF_{im}$

$= [1651.13(1.33) + 726.56](0.62) = 387.26\ \text{kN.m}$

$V_{LL+IM} = [V_{Tr}(1 + IM) + V_{Ln}]DF_{iv}$

$= [288(1.33) + 116.25](0.62) = 309.56\ \text{kN}$

$M_{f+IM} = \left[M_{fTr}(1 + IM)\right]DF_{ifm}$

$= [1344.51(1.15)](0.38) = 587.55\ \text{kN.m}$

Strength Limit State:

$\eta_i = 1.0$

$M_{u1} = \eta_i[1.25M_{DC} + 1.50M_{DW} + 1.75M_{LL+IM}]$

$= 1.25(3281.25) + 1.50(164.06) + 1.75(387.26) = 2025.36\ \text{kN.m}$

$V_{u1} = \eta_i[1.25V_{DC} + 1.50V_{DW} + 1.75V_{LL+IM}]$

$= 1.25(525) + 1.50(26.25) + 1.75(117.61) = 726.44\ \text{kN}$

Service Limit State:

$M_{s1} = 1.0M_{DC} + 1.0M_{DW} + 1.0M_{LL+IM}$

$= 3281.25 + 164.06 + 387.26 = 3832.57$ kN.m

Fatigue Limit State:

$M_{f1} = 1.75 M_{f+IM}$

$= 1.75(587.55) = 1028.21$ kN.m

<u>Exterior Girder</u>

Unfactored Loads:

- Permanent loads:

$b_f = S/2 + w_o = 1.75/2 + 1.0 = 1.88$ m

$A_g = (1.88)(0.2) + (0.8)(1.75) = 1.78\ \text{m}^2$

$w_g = (1.78)(24) = 42.75$ kN/m

$w_{ba} = 7.5$ kN/m

$w_{DC} = w_g + w_{ba} = 42.75 + 7.5 = 50.25$ kN/m

$M_{DC} = w_{DC}L^2/8 = (50.25)(25)^2/8 = 3925.78$ kN.m

$V_{DC} = w_{DC}L/2 = (50.25)(25)/2 = 628.13$ kN

$b_{ws} = b_f - W_e = 1.88 - 0.5 = 1.38$ m

$w_{ws} = (1.2)(1.38) = 1.66$ kN/m

$M_{DW} = w_{DW}L^2/8 = (1.66)(25)^2/8 = 129.69$ kN.m

$V_{DW} = w_{DW}L/2 = (1.66)(25)/2 = 20.753$ kN

- Live loads:

$M_{eTr} = M_{Tr}/2 = 1651.13/2 = 825.57$ kN.m

$V_{eTr} = V_{Tr}/2 = 288/2 = 144$ kN

$w_{Ln} = (3.1)(1.38) = 4.28$ kN/m

$M_{Ln} = w_{Ln}L^2/8 = (4.28)(25)^2/8 = 334.38$ kN. m

$V_{Ln} = w_{Ln}\,L/2 = (4.28)(25)/2 = 53.5$ kN

Distribution factors:

$-0.3 \le d_e \le 1.7$ $\qquad d_e = 0.5$ $\qquad \therefore$ OK

$\because N_L = 2 \rightarrow$ check both g_{e1} and g_{e2}

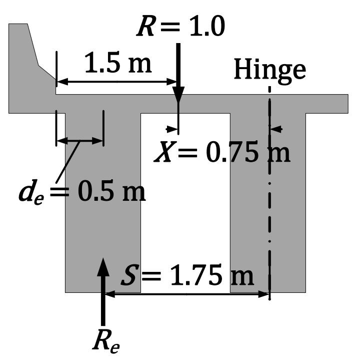


Figure 3-18: Level Rule for Exterior Girder

Live load distribution factor for bending moment:

$R_e = X/S = 0.75/1.75 = 0.43$

$g_{em1} = mR_e = (1.2)(0.43) = 0.51$

$e_m = 0.77 + d_e/2.8 = 0.77 + 0.5/2.8 = 0.95$
$g_{em2} = e_m g_{im2} = (0.95)(0.62) = 0.59$
$\therefore DF_{em} = 0.59$

Live load distribution factor for shear:

$g_{ev1} = g_{em1} = 0.51$
$e_v = 0.6 + d_e/3 = 0.6 + 0.5/3 = 0.77$
$g_{ev2} = e_v g_{iv2} = (0.77)(0.66) = 0.51$
$\therefore DF_{ev} = 0.51$

Live load distribution factor for fatigue:

$DF_{efm} = DF_{efv} = 0.43$

Live load distribution factor for deflection:

$DF_{\Delta} = 0.40$

Total live load:

$M_{LL+IM} = [M_{Tr}(1 + IM) + M_{Ln}]DF_{em}$
$= [825.57(1.33) + 334.38](0.59) = 845.11 \text{ kN.m}$
$V_{LL+IM} = [V_{Tr}(1 + IM) + V_{Ln}]DF_{ev}$
$= [144(1.33) + 53.5](0.51) = 124.96 \text{ kN}$

Strength Limit State:

$\eta_i = 1.0$
$M_{u1} = \eta_i[1.25M_{DC} + 1.50M_{DW} + 1.75M_{LL+IM}]$
$= 1.25(3925.78) + 1.50(129.69) + 1.75(845.11) = 6580.7 \text{ kN.m}$
$V_{u1} = \eta_i[1.25V_{DC} + 1.50V_{DW} + 1.75V_{LL+IM}]$
$= 1.25(628.13) + 1.50(20.75) + 1.75(124.96) = 1034.97 \text{ kN}$

Service Limit State:

$M_{s1} = M_{DC} + M_{DW} + M_{LL+IM}$
$= 3925.78 + 129.69 + 845.11 = 4900.58 \text{ kN.m}$