5. Review on Concrete Bridges Design

5.1. Flexural Resistance

The factored flexure resistance (M_r) is the product of flexural resistance factor (ϕ_f) and nominal flexural resistance (M_n) that generally for strength limit states taken as:

$$M_r = \phi_f M_n$$

- $\phi_f = 0.9$ [reinforced concrete]
  $= 1.0$ [prestressed concrete]

5.1.1. Flexural Resistance of Reinforced Concrete Members

The moment capacity or nominal strength (M_n) of reinforced (nonprestressed) concrete beams can be obtained from the equivalent rectangular stress distribution, originally proposed in the 1930s by Charles Whitney (Figure 3.1). Both the equivalent compression block depth (a) and the neutral axis depth (c) measured from the extreme compression fibers are defined by:

$$a = \beta_1 c$$
$$\rightarrow c = a/\beta_1$$

- $\beta_1 = 0.85$ $[17 \leq f_c' \leq 28 \text{ MPa}]$
  $= 0.85 - 0.05(f_c' - 28)/7$ $[28 < f_c' \leq 55 \text{ MPa}]$
  $= 0.65$ $[f_c' > 55 \text{ MPa}]$

The value of the compression stress resultant at nominal strength (C) is product of the stress acting on the concrete area in the compression zone and can be expressed as:

$$C = 0.85 f_c' ab$$

If reinforcement yields $(f_s = f_y)$ when the concrete strain (ε_c) reaches its maximum value (0.003), the tensile stress resultant (T) can be expressed as:

$$T = A_s f_y$$

The equilibrium condition mandates $(C = T)$, so that:

$$0.85 f_c' ab = A_s f_y$$
$$\rightarrow a = A_s f_y / 0.85 f_c' b$$

The nominal strength (M_n) of the beam equals the moment provided by $(C\text{–}T)$ couple:

$$M_n = C\left(d_s - \frac{a}{2}\right) = 0.85 f_c' ab\left(d_s - \frac{a}{2}\right)$$
$$= T\left(d_s - \frac{a}{2}\right) = A_s f_y\left(d_s - \frac{a}{2}\right)$$

Because of structurally stipulated ductile failure, the reinforcing steel amount is limited (intentionally) to ensure its yielding before the onset of maximum concrete strain of (0.003); thus, tension-controlled section can be checked by considering the strains compatibility from the strain distribution diagram at nominal resistance and then calculate the flexural strength.

$$\frac{\varepsilon_s}{\varepsilon_c} = \frac{d_s - c}{c}$$

$$\rightarrow \varepsilon_s = \left(\frac{d_s - c}{c}\right)\varepsilon_c$$

If $\varepsilon_s \geq 0.005$

$\rightarrow$ tension-controlled section $\left(f_s = f_y \text{ and } \phi_f = 0.9\right)$

$$\therefore M_r = 0.9 A_s f_y \left(d_s - \frac{a}{2}\right)$$

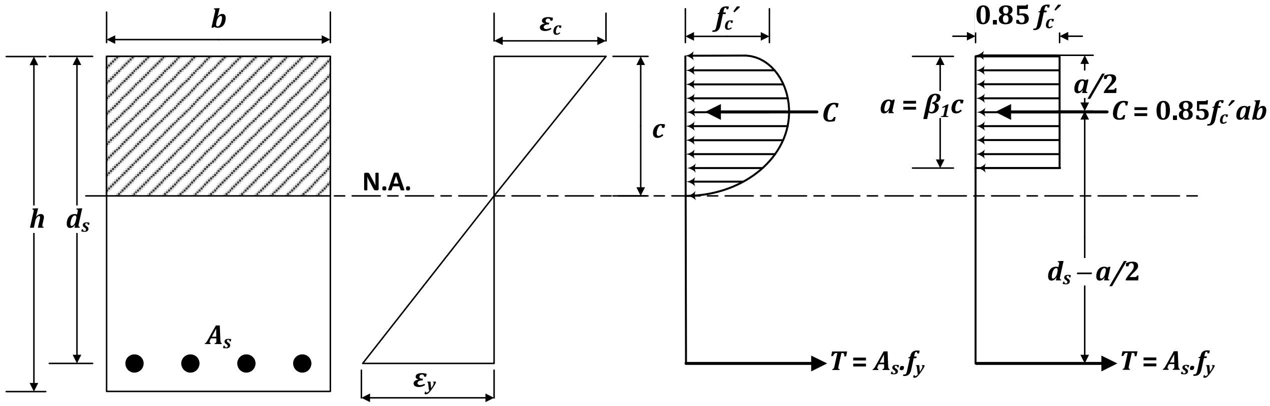

Figure 5-1: Equivalent Rectangular Stress Distribution for Strength Design Concept

5.1.2. Flexural Resistance of Prestressed Concrete Members

Prestressing steel in beams is provided in the form of tendons. In a beam, the tendons can be bonded, unbonded or both.

5.1.2.1. Components with Bonded Tendons

For flanged sections (T-section behavior) subjected to flexure about one axis, the neutral axis depth (c) from the extreme compression fibers can be defined by:

$$c = \frac{A_{ps} f_{pu} + A_s f_s + A_s' f_s' - 0.85 f_c' (b - b_w) h_f}{0.85 f_c' \beta_1 b_w + k A_{ps} \dfrac{f_{pu}}{d_{ps}}}$$

Similarly, (c) for rectangular section behavior can be simply determined from the same equation but substituting $(b - b_w)$:

$$c = \frac{A_{ps} f_{pu} + A_s f_s + A_s' f_s'}{0.85 f_c' \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_{ps}}}$$

For fanged and rectangular sections subjected to flexure about one axis in which the effective prestressing stress $(f_{pe} \geq 0.5 f_{pu})$, the average stress in prestressing steel (f_{ps}) may be taken as:

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_{ps}}\right)$$

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right)$$

where:

A_{ps}: area of prestressing steel (mm^2).

f_{pu}, f_{py}, f_{ps}: tensile strength, yield strength and average stress at prestressing steel (MPa).

A_s, A_s': area of nonprestressed (mild) tension and compression steel (mm^2).

f_s, f_s': stress in tension and compression reinforcing steel at nominal flexural resistance (MPa).

b, b_w: width of compression face of the and web width/or diameter of a circular section (mm).

h_f: depth of compression flange (mm).

d_{ps}: distance from the extreme compression fiber to the centroid of the prestressing force (mm).

5.1.2.2. *Components with Unbonded Tendons*

For fanged and rectangular sections subjected to flexure about one axis and for biaxial flexure with axial loads, the average stress in prestressing steel (f_{ps}) may be taken as:

$$f_{ps} = f_{pe} + 6300\left(\frac{d_{ps} - c}{l_e}\right) \le f_{py}$$

$$= f_{pe} + 103 \qquad [c \text{ is yet unkown}]$$

$$l_e = \frac{2l_l}{2 + N_s}$$

For flanged sections subjected to flexure about one axis and for biaxial flexure with axial loads, the neutral axis depth (c) from the extreme compression fibers can be defined by:

$$c = \frac{A_{ps}f_{ps} + A_s f_s + A_s' f_s' - 0.85 f_c'(b - b_w)h_f}{0.85 f_c' \beta_1 b_w}$$

Similarly, (c) for rectangular section behavior can be simply determined:

$$c = \frac{A_{ps}f_{ps} + A_s f_s + A_s' f_s'}{0.85 f_c' \beta_1 b}$$

where:

f_{pe}, f_{py}: effective stress after all losses in and yield strength of prestressing steel (MPa).

l_e, l_l: effective and between anchorages lengths of tendon (mm).

N_s: number of plastic hinges at supports in an assumed failure crossed by the tendon between anchorages or discretely bonded points.

- $N_s = 0$ [simple spans]
  $= 1.0$ [end spans of continuous units]
  $= 2.0$ [interior spans of continuous units]

5.1.2.3. *Components with Combined Bonded and Unbonded Tendons*

For fanged and rectangular sections with combined bonded and unbonded tendons subjected to flexure, the average stress in prestressing steel (f_{ps}) may be taken as:

$$f_{ps} = \frac{A_{psb}f_{pu} + A_{psu}f_{pe}}{A_{ps}}$$

where:

A_{psb}, A_{psu}: area of bonded and unbonded prestressing steel (mm^2).