Fraunhoffer Diffraction

CHAPTER OUTLINE

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Introduction:

To obtain a Fraunhoffer diffraction pattern, the incident wave front must be plane and the diffracted light is collected on the screen with the help of a lens. Thus, the source of light should either be at a large distance from the slit or a collimating lens must be used.

Fraunhoffer Diffraction at a Single Slit:

In Fig. 1 S is a narrow slit perpendicular to the plane of the paper and illuminated by monochromatic light. L_1 is the collimating lens and AB is a slit of width a. XY is the incident spherical wave front. The light passing through the slit AB is incident on the lens L_2 and the final refracted beam is observed on the screen MN. The screen is perpendicular to the plane of the paper. The line SP is perpendicular to the screen. L_1 and L_2 are achromatic lenses.

A plane wave front is incident on the slit AB and each point on this wave front is a source of secondary disturbance. The secondary waves traveling in the direction parallel to OP viz. AQ and BV come to focus at P and a bright central image is observed. The secondary waves from points equidistant from O and situated in the upper and lower halves OA and OB of the wave front travel the same distance in reaching P and hence the path difference is zero. The secondary waves reinforce one another and P will be a point of maximum intensity.

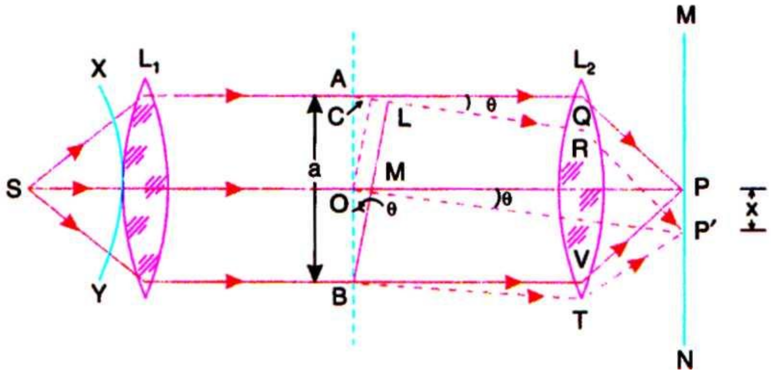

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Figure 1

Now, consider the secondary waves traveling in the direction AR, inclined at an angle θ to the direction OR All the secondary waves traveling in this direction reach the point P' on the screen. The point P' will be of maximum or minimum intensity depending on the path difference between the secondary waves originating from the corresponding points of the wave front. Let us draw OC and BL perpendicular to AR.

Then, in ΔABL

$$\sin\theta = \frac{AL}{AB} = \frac{AL}{a}$$

Or $$AL = a\sin\theta \qquad (1)$$

where a is the width of the slit and AL is the path difference between the secondary waves originating from A and B. If this path difference is equal to λ the wavelength of the light used, then P' will be a point of minimum intensity. The whole wave front can be considered to be of two halves OA and OB and if the path difference between the secondary waves from A and B is λ, then the path difference between the secondary waves from A and O will be $\lambda/2$. Similarly, for every point in the upper half OA, there is a corresponding point in the lower half OB, and the path difference between the secondary waves from these points is $\lambda/2$. Thus, destructive interference takes place and the point P' will be of minimum intensity. If the direction of the secondary waves is such that $AL = 2\lambda$, then also the point where they meet the screen will be of minimum intensity. This is so because the secondary waves from the corresponding points of the lower half differ in path by $\lambda/2$. and this again gives the position of minimum intensity. In general,

$$a\sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{n\lambda}{a} \qquad (2)$$

where θ_n gives the direction of the n^{th} minimum. Here n is an integer. If, however, the path difference is odd multiples of $\lambda/2$, the directions of the secondary maxima can be obtained. In this case,

$$a\sin\theta_n = (2n+1)\lambda/2$$

$$\sin\theta_n = \frac{(2n+1)\lambda}{2a} \qquad (3)$$

where n = 1,2.3,...

Thus, the diffraction pattern due to a single slit consists of a central bright maximum at P followed by secondary maxima and minima on both the sides, as shown in Fig. 2. P corresponds to the position of the central bright maximum and the points on the screen for which the path difference between the points A and B is λ, 2λ etc correspond to the position of secondary minima. The secondary maxima are of much less intensity. The intensity falls off rapidly from the point P outwards. If the lens L_2 is very near the slit or the screen is far away from the lens L_2, then,

$$\sin\theta = \frac{x}{f} \qquad (4)$$

where f is the focal length of the lens L_2.

But $\sin\theta = \frac{\lambda}{a}$

$$\therefore \qquad \frac{x}{f} = \frac{\lambda}{a} \quad \text{or} \quad x = \frac{f\lambda}{a} \qquad (5)$$

where x is the distance of the secondary minimum from the point P. Thus, the width of the central maximum W = 2x

$$W = \frac{2f\lambda}{a} \qquad (6)$$

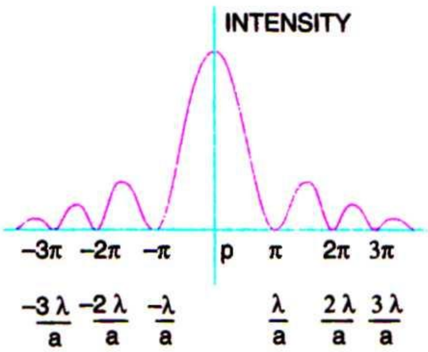


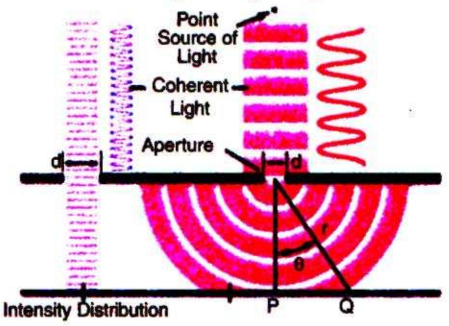
Figure 2

The width of the central maximum is proportional to the wavelength of the light. With red light (longer wavelength), the width of the central maximum is more than with violet light (shorter wavelength). With a narrow slit, the width of central maximum is more. The diffraction pattern consists of alternate bright and dark bands with monochromatic light. With white light, the central maximum is white and the rest of the diffraction bands are coloured. From equation (5) if the width a of the slit is large, $\sin\theta$ is small and hence θ is small. The maxima and minima are very close to the central maximum at P. But with a narrow slit, a is small and hence θ is large. This results in a distinct diffraction maxima and minima on both the sides of P.

Intensity Distribution in Diffraction Pattern Due to a Single Slit:

The intensity variation in the diffraction pattern due to a single slit can be investigated as follows. The incident plane wave front on the slit AB (Fig. 1) can be imagined to be divided into a large number of infinitesimally small strips. The path difference between the secondary waves emanating from the extreme points A and B is $a \sin\theta$ where a is the width of the slit and $\angle ABL = \theta$. For a parallel beam of incident light, the amplitude of vibration of the waves from each strip can be taken to be the same. As one considers the secondary waves in a direction inclined at an angle θ from the point B upwards, the path difference changes and hence the phase difference also increases. Let α be the phase difference between the secondary waves from the points B and A of the slit

Diffraction of Light Through an Aperture



Intensity distribution of light diffracted by single slit experiment

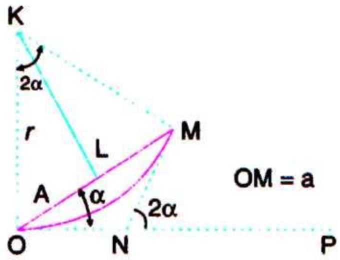


Figure 3

(See Fig. 1). As the wave front is divided into a large number of strips, the resultant amplitude due to all the individual small strips can be obtained by the vector polygon method. Here, the amplitudes arc small and the phase difference increases by infinitesimally small amounts from strip to strip. Thus, vibration polygon coincides with the circular arc OM (see Fig. 3). OP gives the direction of the initial vector and NM the direction of the final vector due to the secondary waves from A. K is the centre of the circular arc.

$\angle MNP = 2\alpha \qquad \therefore \quad \angle OKM = 2\alpha$

In ΔOKL $\qquad \sin\alpha = \dfrac{OL}{r};$

$$\therefore \quad OL = r\sin\alpha$$

where r is the radius of the circular arc.

$$\therefore \qquad \text{Chord } OM = 2\,OL = 2\,r\sin\alpha \tag{7}$$

The length of the arc OM is proportional to the width of the slit.

$$\text{Length of the arc } OM = Ka$$

where K is a constant and a is the width of the slit.

Also,
$$2\alpha = \frac{\text{Arc } OM}{\text{radius}} = \frac{Ka}{r} \tag{8}$$

or
$$2r = \frac{Ka}{\alpha}$$

Substituting the value of $2r$ in equation (18.7), we get

$$\text{Chord OM} = \frac{Ka}{\alpha}.\sin\alpha$$

But, $OM = A$ where A is the amplitude of the resultant vibration.

$$\therefore \qquad A = (Ka)\,\frac{\sin\alpha}{\alpha}.$$

or
$$A = A_0\,\frac{\sin\alpha}{\alpha}.$$

(9)

Thus, the resultant amplitude of vibration at a point on the screen is given by $\mathbf{A}_{\circ}\frac{\sin\alpha}{\alpha}$ and the intensity I at the point is given by

$$I = A^2 = A_0^2\frac{\sin^2\alpha}{\alpha^2} = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \tag{10}$$

The intensity at any point on the screen is proportional to $\left(\frac{\sin\alpha}{\alpha}\right)^2$ A phase difference of 2π corresponds to a path difference of λ. Therefore, a phase difference of 2α is given

$$2\alpha = \frac{2\pi}{\lambda}a\sin\theta \tag{11}$$

where **a sinΘ** is the path difference between the secondary waves from A and B (Fig. 1)

$$\alpha = \frac{\pi}{\lambda}a\sin\theta \tag{12}$$

Thus, the value of α depends on the angle of the diffraction **Θ.** The value of $\left(\frac{\sin\alpha}{\alpha}\right)^2$ for different values of **Θ** gives the intensity at the point under consideration. Fig. 2 represents the intensity distribution. It is a graph of $\left(\frac{\sin\alpha}{\alpha}\right)^2$ (along the Y-axis), as a function of **α** or **sinΘ** (along the Y-axis).

(i) Central Maximum:

For the point P on the screen (Fig. 4) $\theta = 0$; and hence $\alpha = 0$.

The value of $\frac{\sin\alpha}{\alpha}$ when $\alpha \to 0$ is equal to 1. Hence, the intensity at **P** $= I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0$ which is a maximum.

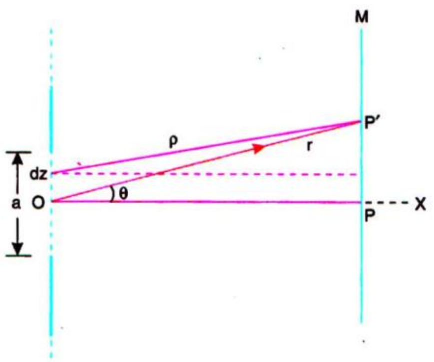


Figure 4

(ii) Secondary Maxima:

The directions of secondary maxima are given by the equation.

$$\sin\theta_n = \frac{(2n+1)\lambda}{2a}$$

Substituting this value of θ_n in equation (12), we get

$$\alpha = \frac{\pi}{\lambda}\cdot\frac{a(2n+1)\lambda}{2a} = \frac{(2n+1)\pi}{2} \qquad (13)$$

Substituting n = 1, 2, 3 etc. in equation (13). The values of α are given by

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \text{ etc.}$$

(a) For the first secondary maximum, $\alpha = \frac{3\pi}{2}$

and
$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left[\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right]^2 = I_0\left[\frac{-1}{\frac{3\pi}{2}}\right]^2 = \frac{4I_0}{9\pi^2} = \frac{I_0}{22}$$

(*b*) For the secondary maximum, $\alpha = \frac{5\pi}{2}$ and

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left[\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right]^2 = I_0\left[\frac{1}{\frac{5\pi}{2}}\right]^2 = \frac{4I_0}{25\pi^2} = \frac{I_0}{61}$$

Thus, the secondary maxima are of decreasing intensity and the directions of these maxima are obtained from the equation given above.

The intensity at P' is given by,

$$I' = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad (14)$$

$$dI' = I_o\left[\frac{\alpha^2 2\sin\alpha\ \cos\alpha - (\sin^2\alpha)2\alpha}{\alpha^4}\right]d\alpha$$

For I' to be a maximum, $\frac{dI'}{d\alpha} = 0$

$$\therefore \qquad \alpha^2(2\sin\alpha\ \cos\alpha) - (\sin^2\alpha)2\alpha = 0$$

$$\therefore \qquad \tan\alpha = \alpha$$

If graphs are plotted for $y = \alpha$ and $y = \tan \alpha$ .it will be found that the secondary maxima are not exactly midway between two minima. The positions of the secondary maxima are slightly towards the central maximum (Fig. 5)

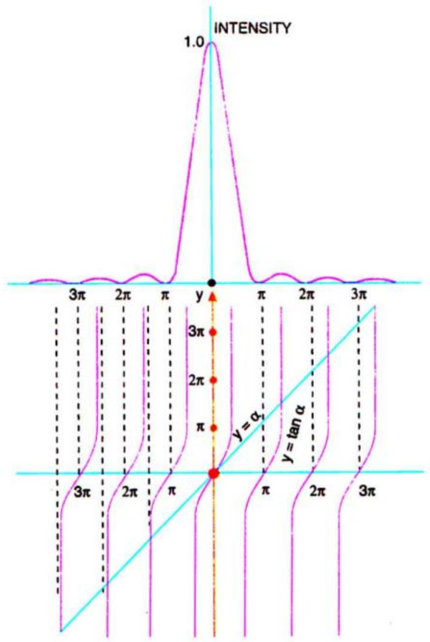


Figure 5

(iii) Secondary Minima:

The directions of the secondary minima are given by the equation

$a \sin \theta = n \lambda$

Substituting the value of $a \sin \theta$ in equation (12), we get

$$\alpha = \frac{\pi}{\lambda} n\lambda = n\pi \quad (15)$$

Substituting n = 1, 2, 3 etc. in equation (15), we obtain

$\alpha = \pi, 2\pi, 3\pi$ etc.

When these values of α arc substituted in the equation (14) for intensity, we get $I = 0$

In Fig. 5, the positions of the secondary minima are shown for values of

$\alpha = \pi, 2\pi, 3\pi$ etc.

$\frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a}$ etc. refer to the values of $\sin \theta$ for these positions.

Fraunhoffer Diffraction at a Circular Aperture:

In Fig.6, AB is a circular aperture diameter d. C is the centre of the aperture and P is a point on the screen. CP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. A plane wave front is incident on the circular aperture. The secondary wave traveling in the direction CO comes to the focus at P. Therefore, P corresponds to the position of the central maximum. Here, all the secondary waves emanating from points equidistant from O travel the same distance before reaching P and hence they all reinforce one another. Now, let us consider the secondary waves traveling in a direction inclined at an angle θ with the direction CP. All these secondary waves meet at P_1 on the screen. Let the distance PP_1 be x. The path difference between the secondary waves emanating from the points B and A (extremities of diameter) is AD.

From the ΔABD, $\quad AD = d \sin\theta$

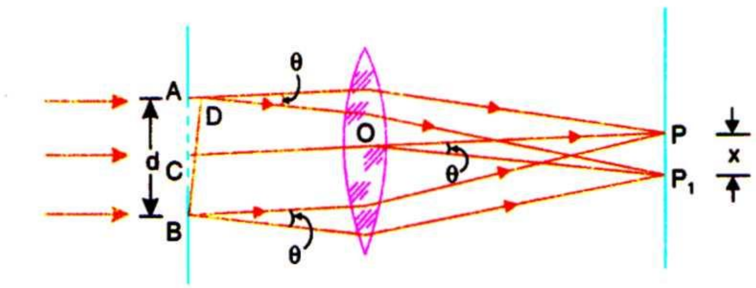


Figure 6

The point P_1 will be of minimum intensity if this path difference is equal to integral multiples λ i.e.

$$d \sin\theta_n = n\lambda \tag{16}$$

The point P_1 will be of maximum intensity if the path difference is equal to odd multiples of

$$d \sin\theta_n = \frac{(2n+1)\lambda}{2} \tag{17}$$

If P_1 is the point of minimum intensity, then all the points at the same distance from P as P_1 and lying on a circle of radius x will be of minimum intensity. Thus, the diffraction pattern due to a circular aperture consists of a central disc called the Airy's disc, surrounded by alternate dark and bright concentric rings called the Airy's rings. The intensity of the dark rings is zero and that of the bright rings decreases gradually outwards from P. Further, if the collecting lens is very near the slit or when the screen is at a large distance from the lens,

$$\sin\theta = \theta = \frac{x}{f} \tag{18}$$

Also, for the first secondary minimum, $d \sin\theta = \lambda$

$$\sin\theta = \theta = \frac{\lambda}{d} \tag{19}$$

From equation (18) and (19)

$$x = \frac{f\lambda}{d} \tag{20}$$

where x is the radius of the Airy's disc. But actually, the radius of the first dark ring is slightly more than that given by equation (20). According to Airy, it is given by

$$x = 1.22\frac{f\lambda}{d} \tag{21}$$

The discussion on the intensity distribution of the bright and dark rings is similar to the one given for a rectangular slit. With increase in diameter of the aperture, the radius of the central bright ring decreases.

<u>Fraunhoffer Diffraction at Double Slit:</u>

In Fig. 7, AB and CD are two rectangular slits parallel to one another and perpendicular to the plane of the paper. The width of each slit is a and the width of the opaque portion is b. L is a collecting lens and MN is a screen perpendicular to the plane of the paper. P is a point on the screen such that OP is perpendicular to the screen. Let a plane wave front be incident on the surface of XY. All the secondary waves traveling in a direction parallel to OP come to focus at P. Therefore, P corresponds to the position of the central bright maximum.

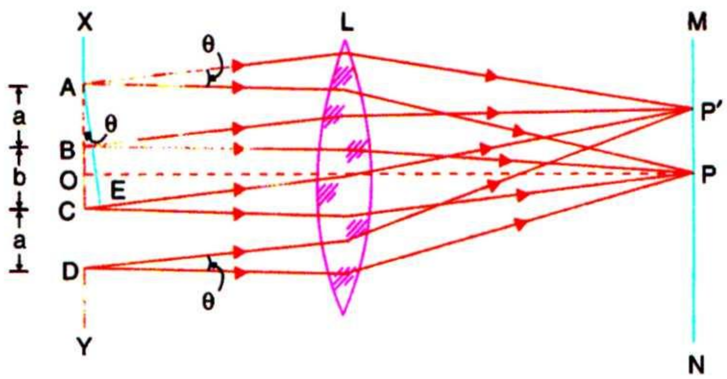


Figure 7

In this case, the diffraction pattern has to be considered in two parts (i) the interference phenomenon due to the secondary waves emanating from the corresponding points of the two slits and *(ii)* the diffraction pattern due to the secondary waves from the two slits individually. For calculating the positions of interference maxima and minima, the diffracting angle is denoted as θ and for the diffraction maxima and minima it is denoted as ϕ.. Both the angles θ and ϕ refer to the angle between the direction of the secondary waves and the initial direction of the incident light.

(i) **Interference maxima and minima:**

Let us consider the secondary waves traveling in a direction inclined at an angle θ with the initial direction.

In Δ CAN the (Fig. 8) $\sin\theta = \frac{CN}{AC} = \frac{CN}{a+b}$

$$CN = (a+b)\sin\theta$$

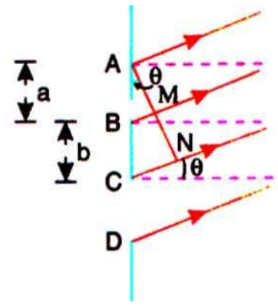


Figure 8

If this path difference is equal to odd multiples of gives the direction of minima due to interference of the secondary waves from the two slits.

$$CN = (a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2} \qquad (22)$$

Putting $n = 1$, 2, 3, etc., the values of $\theta_1, \theta_2, \theta_3$, etc, corresponding to the directions of minima can be obtained.

From equation (22)

$$\sin\theta_n = \frac{(2n+1)\lambda}{2(a+b)} \qquad (23)$$

On the other hand, if the secondary waves travel in a direction θ' such that the path difference is even multiples of $\lambda/2$, then θ' gives the direction of the maxima due to interference of light waves emanating from the two slits.

$$\therefore \quad CN = (a+b)\sin\theta'_n = 2n\frac{\lambda}{2}$$

$$\sin\theta'_n = \frac{n\lambda}{(a+b)} \tag{24}$$

Putting n = 1, 2, 3 etc. $\theta'_1, \theta'_2, \theta'_3$ etc corresponding to the directions of the maxima can be obtained. From equation (23), we get

$$\sin\theta_1 = \frac{3\lambda}{2(a+b)}$$

and

$$\sin\theta_2 = \frac{5\lambda}{2(a+b)}$$

$$\therefore \quad \sin\theta_2 - \sin\theta_1 = \frac{\lambda}{(a+b)} \tag{25}$$

Thus, the angular separation between any two consecutive minima (or maxima) is equal to $\frac{\lambda}{a+b}$. The angular separation is inversely proportional to (a + b), the distance between the two slits.

***(ii)* Diffraction maxima and Minima:**

Let us consider the secondary waves traveling in a direction inclined at an angle ϕ with the initial direction of the incident light.

If the path difference BM is equal to λ the wavelength of the light used, then ϕ will give the direction of the diffraction minimum (Fig. 8). That is, the path difference between secondary waves emanating from the extremities of a slit (i.e., points A and B) is equal to λ. Considering the wave front on AB to be made up of the two halves, the path difference between the corresponding points of the upper and lower halves is equal to $\lambda/2$. The effect at P' due to the wave front incident on AB is zero. Similarly, for the same direction of the secondary waves, the effect at P' due to the wave front incident on the slit CD is also zero. In general,

$$a\sin\phi_n = n\lambda.$$

Putting n =1, 2, 3, etc., the values of ϕ_1, ϕ_2, ϕ_3 etc. corresponding to the directions of diffraction minima can be obtained.

<u>Fraunhoffer Diffraction at Double Slit (Calculus Method):</u>

The intensity distribution due to fraunhoffer diffraction at double slit (two parallel slits) can be obtained by integrating the expression for dy (vide single slit) for both the slits.

$$y = K\left[\int_{-a/2}^{a/2} \sin\left\{2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{z\sin\theta}{\lambda}\right)\right\}dz + \int_{d-a/2}^{d+a/2} \sin\left\{2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{z\sin\theta}{\lambda}\right)\right\}dz\right] \tag{26}$$

$$\therefore \quad y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right) - \frac{K\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left[\frac{t}{T} - \frac{r}{\lambda} + \frac{z\sin\theta}{\lambda}\right]\right]_{d-a/2}^{d+a/2}$$

$$\therefore \quad y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)$$

$$-\frac{K\lambda}{2\pi \sin\theta}\left[\cos 2\pi\left[\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}+\frac{a\sin\theta}{2\lambda}\right]-\cos 2\pi\left[\frac{t}{T}-\frac{r}{\lambda}-\frac{d\sin\theta}{\lambda}-\frac{a\sin\theta}{2\lambda}\right]\right]$$

$$y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)+\frac{K\lambda}{\pi\sin\theta}\left[\sin 2\pi\left[\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right]\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)\right]$$

But $\alpha = \dfrac{\pi a\sin\theta}{\lambda}$

$$\therefore \quad y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)+\sin 2\pi\left[\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right]\right]$$

$$y = 2Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right).\cos\frac{\pi d\sin\theta}{\lambda}$$

Let $\dfrac{\pi d\sin\theta}{\lambda} = \beta.$

$$\therefore \quad y = 2Ka\left(\frac{\sin\alpha}{\alpha}\right)\cos\beta\,\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right) \qquad (27)$$

The intensity at the point P′ is given by

$$I = 4K^2a^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\cos^2\beta$$

$$\text{But} \quad I_0 = K^2a^2 \therefore I = 4I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right)\cos^2\beta \qquad (28)$$

The intensity of the central maximum = $4I_0$ when $\alpha = 0$ and $\beta = 0$. In Fig. 10, the dotted curve represents the intensity distribution due to diffraction pattern due to double slit and the thick line curve represents the intensity distribution duo to the interference between the light from both the slits. The pattern consists of interference maxima within each diffraction maximum.

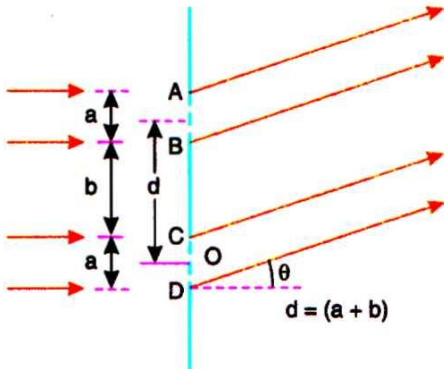


Figure 9

Intensity distribution due to the Fraunhoffer diffraction at two parallel slits is shown in Fig. 10. The full line represents equally spaced interference maxima and minima and the dotted curve represents the diffraction maxima and minima. In the region originally occupied by the central maximum of the single slit diffraction pattern, equally spaced interference maxima and minima are observed. The intensity of the central interference maximum is four times the intensity of the central maximum of the single slit diffraction pattern. The intensity of other interference maxima on the two sides of the central maximum gradually decreases. In the region of the secondary maxima due to diffraction at a single slit, equally spaced interference maxima of low intensity are observed. The intensity distribution shown in Fig. 10 corresponds to 2a = b where a is the width of each slit and b is the opaque spacing between the two slits (see Fig. 9). Thus, the pattern due to diffraction at a double slit consists of a diffraction pattern due to the individual slits of width a each and the interference maxima and minima of equal spacing. The spacing of the interference maxima and minima is dependent on the values of a and b.

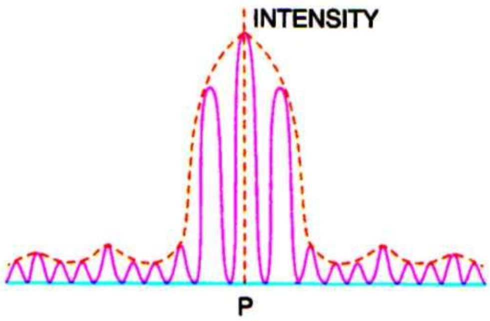
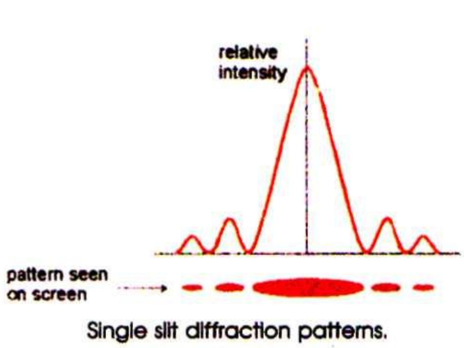


Figure 10

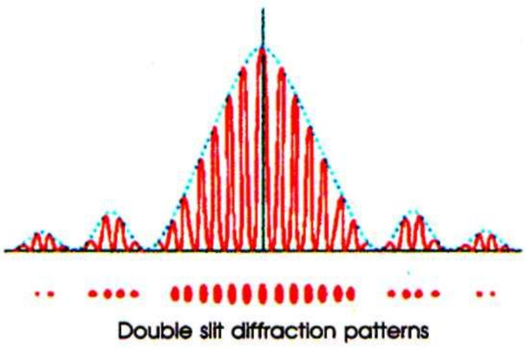
<u>Distinction Between Single Slit and Double Slit Diffraction Patterns:</u>

The single slit diffraction pattern consists of a central bright maximum with secondary maxima and minima of gradually decreasing intensity. The double slit diffraction pattern consists of equally spaced interference maxima and minima with in the central maximum. The intensity of the central maximum in diffraction pattern due to a double slit is four times that of the central maximum in the diffraction pattern due to diffraction at a single slit. In the above arrangement, if one of the slits is covered with opaque screen, the pattern observed is similar to the one observed with a single slit.

The spacing of diffraction maxima and minima depends on a, the width of the slit and the spacing of the interference maxima and minima depends on the value of a and b where b is opaque spacing between the two slits. The intensities of the interference maxima are not constant but decrease to zero on either side of the central maximum. These maxima reappear two or three times before the intensity becomes too low to be observed.



Single slit diffraction patterns.


Double slit diffraction patterns

<u>Missing Orders in a Double Slit Diffraction Pattern:</u>

In the diffraction pattern due to a double slit discussed earlier, the slit width is taken as a and the separation between the slits as b. If the slit width a is kept constant, the diffraction pattern remains the same. Keeping a constant, if the spacing b is altered the spacing between the interference maxima changes. Depending on the relative values of a and b certain orders of interference maxima will be missing in the resultant pattern.

The directions of interference maxima are given by the equation,

$$(a + b) \sin \theta = n \lambda \qquad (29)$$

The direction of diffraction minima are given by the equation,

$$a \sin \theta = p\lambda \qquad (30)$$

In equations (29) and (30) n and ρ are integers. If the value of a and b are such that both the equations are satisfied simultaneously for the same value of ϴ then the positions of certain interference maxima correspond to the diffraction minima at the same position on the screen.

(*ii*) If $2a = b$, then $3a \sin\theta = n\lambda$

and $a \sin\theta = p\lambda$

$\therefore$ $n/p = 3$ or $n = 3p$

If $p = 1, 2, 3$, etc, $n = 3, 6, 9$, etc.

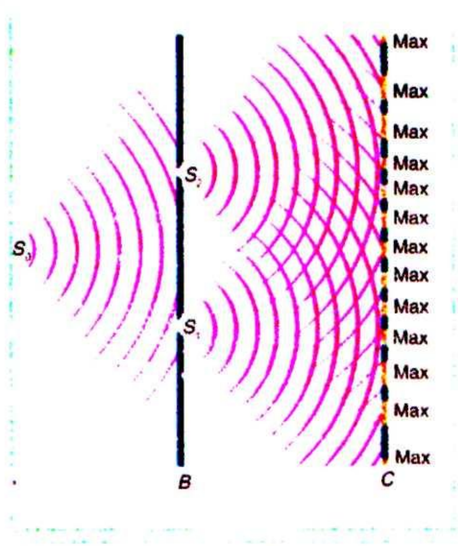
Thus the orders 3, 6, 9 etc. of the interference maxima will be missing in the diffraction pattern. On both sides of the central maximum, the number of interference maxima is 2 and hence there will be five interference maxima in the central diffraction maximum. The position of the third interference maximum also corresponds to the first diffraction minimum.

(*iii*) If $a + b = a$ i.e., $b = 0$

The two slits join and all the orders of the interference maxima will be missing. The diffraction pattern observed on the screen is similar to that due to a single slit of width equal to 2a.

Interference and Diffraction:

It is clear from the double slit diffraction pattern that interference takes place between the secondary waves originating from the corresponding points of the two slits and also that the intensity of the interference maxima and minima is controlled by the amount of light reaching the screen due to diffraction at the individual slits. The resultant intensity at any point on the screen is obtained by multiplying the intensity function for the interference and the intensity function for the diffraction at the two slits. The values of the intensity functions are taken for the same direction of the secondary waves. But the interference of all the secondary waves originating from the whole wave front is termed as diffraction. Hence the pattern obtained on the screen may be called an interference pattern or a diffraction pattern. The term interference may be used for those cases in which the resultant amplitude at a point is obtained by the superposition. of two or more beams. Diffraction can be defined as the phenomenon in which the resultant amplitude at any point on the screen is obtained by integrating the effect of infinitesimally small number of elements in to which the whole wave front can be divided. Thus, the resultant diffraction pattern obtained with a double slit can be taken as a combination of the effect of both interference and diffraction.



Interference and Diffraction.

Fraunhoffer Diffraction at N Double Slits:

Fraunhoffer diffraction at two slits consists of diffraction maxima and minima given by $\frac{\sin^2\alpha}{\alpha^2}$ and the sharp interference maxima and minima, in each diffraction maximum governed by $\cos^2\beta$ term.

To derive an expression for the intensity distribution due to diffraction at N slits, the expression for *dy* has to be integrated for N slits.

For a single slit, $dy = K\int_{-a/2}^{+a/2} \sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)dz$

Let $\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)$ be equal to $\phi(z)$ (i.e. function of z).

For N slits

$$dy = K\left[\int_{-a/2}^{+a/2}\phi(z)dz+\int_{d-\frac{a}{2}}^{d+\frac{a}{2}}\phi(z)dz+\int_{2d-\frac{a}{2}}^{2d+\frac{a}{2}}\phi(z)+\ldots\ldots+\int_{(N-1)d-\frac{a}{2}}^{(N-1)d+\frac{a}{2}}\phi(z)dz\right] \tag{31}$$

On simplification

$$Y = Ka\frac{\sin\alpha}{\alpha}\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)+\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right)+\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{2d\sin\theta}{\lambda}\right)+ \ldots\ldots\ldots\ldots\ldots+\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{(N-1)d\sin\theta}{\lambda}\right)\right]$$

Here $\alpha = \frac{\pi a\sin\theta}{\lambda}$. For a general trigonometric summation

$$\sum_{p=0}^{p=n}\sin(x+pm)=\frac{\sin\left(x+\frac{nm}{2}\right)\sin\left[\left(\frac{n+1}{2}\right)m\right]}{\sin\left(\frac{m}{2}\right)}$$

Here $x=2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right);\ m=\frac{2\pi d\ \sin\theta}{\lambda}=2\left[\frac{\pi d\ \sin\theta}{\lambda}\right]=2\beta$

where $\beta=\frac{\pi\ d\sin\theta}{\lambda}$ and $n=(N-1)$

$$\therefore\quad y=\frac{Ka\left(\frac{\sin\alpha}{\alpha}\right)\left[\sin\left(x+\frac{(N-1)m}{2}\right)\sin\left(\frac{Nm}{2}\right)\right]}{\sin\left(\frac{m}{2}\right)}$$

$$y=Ka\left(\frac{\sin\alpha}{\alpha}\right)\frac{(\sin N\beta)}{\sin\beta}\left[\sin 2\pi\left\{\frac{t}{T}-\frac{r}{\lambda}+\frac{(N-1)d\ \sin\ \theta}{2\lambda}\right\}\right]\tag{32}$$

The intensity at a point P′ is given by

$$I=K^2a^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)\tag{33}$$

The maximum intensity, when $\alpha=0$ and $\beta=0$, is $I_0=K^2a^2$

$$\therefore\quad I=I_0\left(\frac{\sin^2\alpha}{\alpha}\right)\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)\tag{34}$$

The expression $\frac{\sin^2\alpha}{\alpha^2}$ represents the diffraction pattern due to a single slit. The additional $\frac{\sin^2 N\beta}{\sin^2\beta}$ represents the interference effects due to the secondary waves from the N slits.

The numerator will be zero when, $N\beta=0,\pi,2\pi,3\pi$.....etc $=k\pi$

Denominator is also zero when, $\beta=0,\pi,2\pi,3\pi$.....etc.

Since the quotient $\frac{0}{0}$ is indeterminate, $N\beta=k\pi$ gives the condition for minimum intensity for all values of k other than $k=0,N,2N,3N$ etc.

The directions of principal maxima correspond to the values of $k=0,N,2N$ etc.

$$N\beta=\frac{N\pi d\sin\theta}{\lambda}\quad\text{or}\quad k\pi=\frac{N\pi\ d\sin\theta}{\lambda}$$

For directions of principal maxima, $k=0,1N,2N,3N$, etc. $=n\,N$

When $n=1,2,3$ etc., $n\,\pi\,N=\frac{N\pi\ d\sin\theta}{\lambda}$

$\therefore\quad d\sin\theta=n\lambda\qquad(n=1,2,3,\ldots$ etc)

If the width of the slit is a and the width of the opaque spacing is b, $d=(a+b)$ and $(a+b)\sin\theta=n\lambda$

Putting n = 1,2,3 etc. the directions of principal maxima $\theta_1, \theta_2, \theta_3$ ....etc can be determined.

For values of k in between 0 and N between N and 2N, etc., there are (N-l) secondary minima and (N-2) secondary maxima.

The intensity distribution due to diffraction and N slits is shown in Fig. 11.

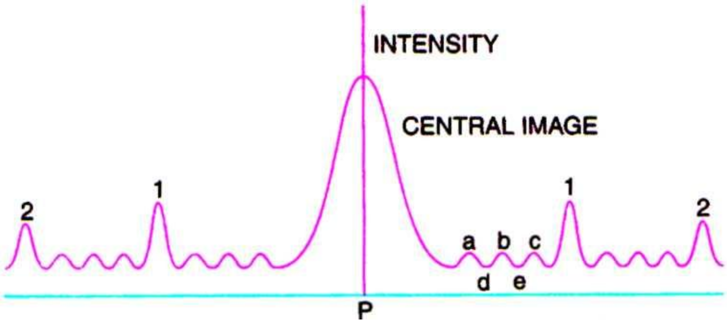


Figure 11

Intensity of Principal Maxima:

In a diffraction grating there are about 6000 narrow slits in one cm. For values of $\beta = k\pi$ and $\beta = 0, \pi, 2\pi, 3\pi$ etc.

$$\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$$

which is indeterminate.

To find the value of this limit, the numerator and the denominator are differentiated. Thus, we get

$$\underset{\beta\to n\pi}{Lt}\frac{\sin N\beta}{\sin\beta} = \underset{\beta\to n\pi}{Lt}\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)} = \underset{\beta\to n\pi}{Lt}\frac{N\cos N\beta}{\cos\beta} = \pm N$$

Thus, the resultant amplitude is proportional to N and resultant intensity is proportional to N^2.

$$I = N^2 I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right) \qquad (35)$$

These maxima are intense and are called **principal maxima.**

Plane Diffraction Grating:

A diffraction grating is an extremely useful device and in one of its forms it consists of a very large number of narrow slits side by side. The slits are separated by opaque spaces. When a wave front is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions. Such a grating is called a **transmission grating.** The secondary waves from the positions of the slits interfere with one another, similar to the interference of waves in Young's experiment. Joseph Fraunhoffer used the first grating which consisted of a large number of parallel fine wires stretched on a frame. Now, gratings are prepared by ruling equidistant parallel lines on a glass surface. The lines are drawn with a fine diamond point. The space in between any two lines is transparent to light and the lined portion is opaque to light. Such surfaces act as transmission gratings. If, on the other hand, the lines are drawn on a silvered surface (plane or concave) then light is reflected from the positions of the mirror in between any two lines and such surfaces act as *reflection gratings*.

If the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of the light is produced. Gratings used for the study of the visible region of the spectrum contain **10,000** lines per cm. Gratings, with originally ruled surfaces are only few. For practical purposes, replicas of the original grating are prepared. On the original grating surface a thin layer of collodion solution is poured and the solution is allowed to harden. Then, the film of collodion is removed from the grating surface and then fixed between two glass plates. This serves as a plane transmission grating. A large number of replicas are prepared in this way from a single original ruled surface.

Theory of Transmission Grating:

In Fig. 12, XY is the grating surface and MN is the screen, both perpendiculars to the plane of the paper. The slits are all parallel to one another and perpendicular to the plane of the paper. Here

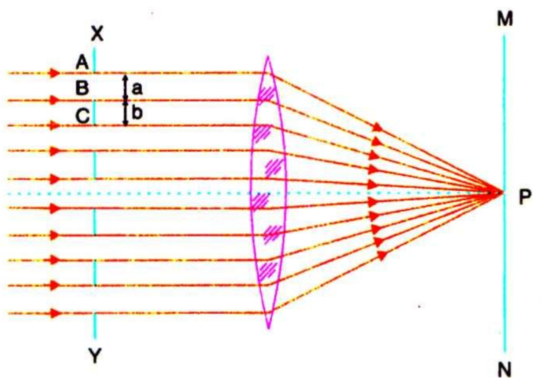


Figure 12

AB is a slit and BC is an opaque portion. The width of each slit is a and the opaque spacing between any two consecutive slits is b. Let a plane wave front be incident on the grating surface. Then all the secondary waves traveling in the same direction as that of the incident light will come to focus at the point P on the screen. The screen is placed at the focal plane of the collecting lens. The point P where all the secondary waves reinforce one another corresponds to the position of the central bright maximum.

Now, consider the secondary waves traveling in a direction inclined at an angle θ with the direction of the incident light (Fig. 13). The collecting lens also is suitably rotated such that the axis of the lens is parallel to the direction of the secondary waves. These secondary waves come to focus at point P_1 on the screen. The intensity at P_1 will depend on the path difference between the secondary waves originating from the corresponding points A and C of two neighbouring slits. In Fig. 13, AB = *a* and BC = *b*. The path difference between the secondary waves starting from A and C is equal to AC $\sin\theta$.

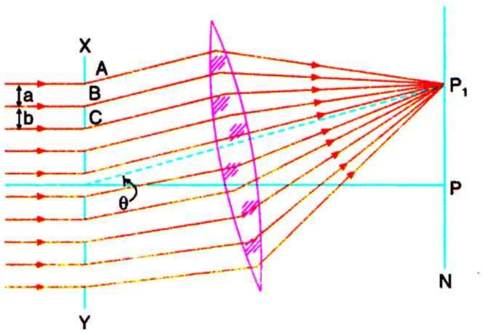


Figure 13

But $$AC = AB + BC = a + b$$

Path difference $$= AC \sin\theta = (a + b)\sin\theta$$

The point P_1 will be of maximum intensity if this path difference is equal to integral multiples of λ where λ is the wavelength of light. In this case, all the secondary waves originating from the corresponding points of the neighbouring slits reinforce one another and the angle θ gives the direction of maximum intensity. In general

$$(a + b)\sin\theta_n = n\lambda \tag{36}$$

where θ_n is the direction of the n^{th} principal maximum. Putting *n* = 1. 2, 3, etc., the angles etc. corresponding to the directions of the principal maxima can be obtained. θ_1, θ_2, θ_3.

If the incident light consists of more than one wavelength, the beam gets dispersed and the angles of diffraction for different wavelengths will be different. Let λ and $\lambda + d\lambda$ be two nearby wavelengths present in the incident light and θ and $(\theta + d\theta)$ be the angles of diffraction corresponding to these two wavelengths. Then, for the first order principal maxima

$(a + b) \sin \theta = \lambda$

and $(a + b) \sin (\theta + d\theta) = \lambda + d\lambda$

Thus, in any order, the number of principal maxima corresponds to the number of wavelengths present. A number of parallel slit images corresponding to the different wavelengths will be observed on the screen. In equation (36), $n = 1$ gives the direction of the first order image, $n = 2$ gives the direction of the second order image and so on. When white light is used, the diffraction pattern on the screen consists of a white central bright maximum and on both sides of this maximum a spectrum corresponding to the different wavelengths of light present in the incident beam will be observed in each order.

Secondary maxima and minima:

The angle of diffraction θ_n corresponding to the direction of the n^{th} principal maximum is given by the equation

$(a + b) \sin \theta_n = n \lambda$

In this equation, $(a + b)$ is called the grating constant. For a grating with 15,000 lines/inch, the value of

$$(a+b) = \frac{2.54}{15000} \text{cm}$$

Now, let the angle of diffraction be increased by a small amount $d\theta$ such that the path difference between the secondary waves from the points A. and C increases by λ / N (See Fig. 13). Here N is the total number of lines on the grating surface. Then, the path difference between the secondary waves from the extreme points of the grating surface will be $(\lambda/N)N = \lambda$. Assuming the whole wave front to be divided into two halves, the path difference between the corresponding points of the two halves will be $\lambda/2$ and all the secondary waves cancel one another's effect. Thus, $(\theta_n + d\theta)$ will give the direction of the first secondary minimum after the n^{th} primary maximum. Similarly, if the path difference between the secondary waves from the points A and C is $2\lambda / N$, $3\lambda / N$ etc. for gradually increasing values of $d\theta$, these angles correspond to the directions of 2^{nd}, 3^{rd} etc secondary minima after the n* primary maximum. If the value is $2\lambda / N$, then the path difference between the secondary waves from the extreme points of the grating surface is $(2\lambda / N)N = 2\lambda$ and considering the wave front to be divided into 4 portions, the concept of the 2^{nd} secondary minimum can be understood. The number of secondary minima in between any two primary maxima is (N-1) and the number of secondary maxima is (N-2).

The intensity distribution on the screen is shown in Fig. 11. P corresponds to the position of the central maxima and 1, 2, etc., on the two sides of P represent the 1^{st}, 2^{nd}, etc. principal maxima, a, *b*, etc. are secondary maxima and *d*, *e* etc. are the secondary minima. The intensity as well as angular spacing of the secondary maxima and minima is so small in comparison to the principal maxima that they cannot be observed. It results in uniform darkness between any two principal maxima.

Width of Principal Maxima:

The direction of the n^{th} principal maximum is given by

$$(a+b)\sin\theta_n = n\lambda$$

Let $\theta_n + d\theta$ and $\theta_n - d\theta$ give the directions of the first secondary minima on the two sides of the n^{th} primary maxima (see Fig. 14). Then

$$(a+b)\sin[\theta_n \pm d\theta] = n\lambda \pm \frac{\lambda}{N} \qquad (37)$$

where N is the total number of the lines on the grating surface.

Dividing (37) by (36), we get

$$\frac{(a+b)\sin[\theta_n \pm d\theta]}{(a+b)\sin\theta_n} = \frac{n\lambda \pm \frac{\lambda}{N}}{n\lambda}$$

$$\sin[\theta_n \pm d\theta] \qquad 1$$

Expanding this equation, we get $\dfrac{\sin\theta_n . \cos d\theta \pm \cos\theta_n \sin d\theta]}{\sin\theta_n} = 1 \pm \dfrac{1}{nN}$ (38)

For small values of $d\theta$; $\cos d\theta = 1$ and $\sin d\theta = d\theta$.

$$\therefore\ 1 \pm \cot\theta_n\, d\theta = 1 \pm \frac{1}{nN} \quad \text{or} \quad \cot\theta_n\, d\theta = \frac{1}{nN} \quad \therefore\ d\theta = \frac{1}{nN\cot\theta_n} \qquad (39)$$

In equation (39), $d\theta$ refers to half the angular width of the principal maximum. The half width $d\theta$ is (i) inversely proportional to N, the total number of lines and (ii) inversely proportional to $n\ \cot\theta_n$. The value of $n\ \cot\theta_n$ is more for higher orders because the increase in the value of $\cot\theta_n$ is less than the increase in the order. Thus, the half width of the principal maximum is less for higher orders. Also, the larger the number of lines on the grating surface, the smaller is the value of $d\theta$. Further, the value of θ_n is higher for longer wavelengths and hence the spectral lines are sharper towards the violet than the red end of the spectrum.

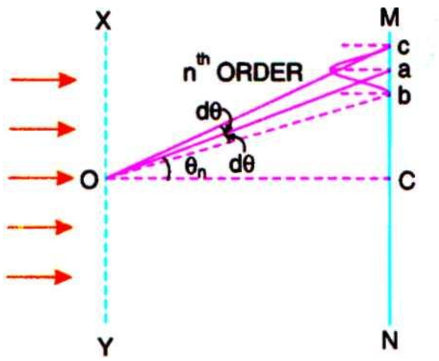


Figure 14

Oblique Incidence:

Let a parallel beam of light be incident obliquely on the grating surface at an angle of incidence i (Fig. 15). Then the path difference between the secondary waves passing through the points A and C = FC + CE

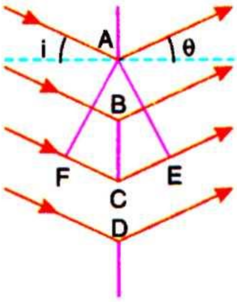


Figure 15

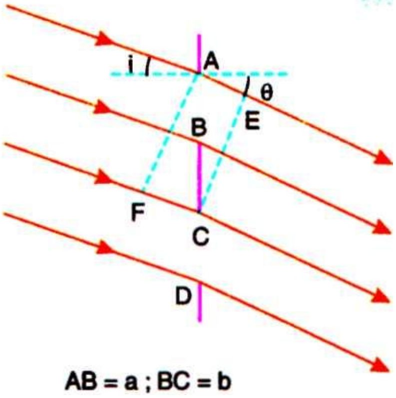


Figure 16

From the Δ^{le} AFC it is seen that $FC = (a+b)\sin i$

and from Δ^{le}ACE $CE = (a+b)\sin\theta$

$$\therefore \quad FC + CE = (a+b)[\sin\theta + \sin i] \tag{40}$$

The equation (40) holds good if the beam is diffracted upwards. Fig. 16 illustrates the diffraction of the beam downwards. In this case the path difference

$$= (a+b)[\sin\theta - \sin i] \tag{41}$$

For the nth primary maximum
$$(a+b)[\sin\theta_n + \sin i] = n\lambda \tag{42}$$

or
$$(a+b)\left[2\sin\frac{\theta_n + i}{2}.\cos\frac{\theta_n - i}{2}\right] = n\lambda$$

or
$$\sin\frac{\theta_n + i}{2} = \frac{n\lambda}{2(a+b)\cos\frac{\theta_n - i}{2}} \tag{43}$$

The deviation of the diffraction beam $= \theta_n + i$.

For the deviation $\theta_n + i$ to be a minimum, $\sin\frac{\theta_n + i}{2}$ must be minimum. This is possible if the value of $\cos\frac{\theta_n - i}{2}$ is maximum, i.e., $\frac{\theta_n - i}{2} = 0$ or $\theta_n = i$.

Thus, the deviation produced in the diffracted beam is a minimum when the angle of incidence is equal to the angle of diffraction. Let D_n be the angle of minimum deviation.

Then $D_m = \theta_n + i$

But $\theta_n = i$

$$\theta_n = \frac{D_m}{2} \quad \text{and} \quad i = \frac{D_m}{2}$$

$$(a+b)\left[\sin\frac{D_m}{2} + \sin\frac{D_m}{2}\right] = n\lambda$$

or
$$2(a+b)\sin\frac{D_m}{2} = n\lambda$$

(44)

Equation (44) refers to the principal maximum of the n^th order for a wavelength λ.

Absent Spectra with a Diffraction Grating:

In the equation $(a+b)\sin\theta = \lambda$, if $(a+b) < \lambda$, then $\sin\theta > 1$. But this is not possible. Hence the first order spectrum is absent. Similarly, the second, the third, etc order spectra will be absent if $(a+b) < 2\lambda$, $(a+b) < 3\lambda$ etc. In general, $(a+b) < n\lambda$, then the n^{th} order spectrum will be absent.

The condition for absent spectra can be obtained from the following considerations. For the n^{th} order principal maximum

$$(a+b)\sin\theta_n = n\lambda \qquad (45)$$

Further, if the value of a and θ_n are such that

$$a\sin\theta_n = \lambda \qquad (46)$$

then, the effect of the wave front from any particular slit will be zero. Considering each slit to be made up of two halves, the path difference between the secondary waves from the corresponding points will be $\lambda/2$ and they cancel one another's effect. If the two conditions given by equations (45) and (46) are simultaneously satisfied, then dividing (45) by (46), we get

$$\frac{(a+b)\sin\theta_n}{a\sin\theta_n} = \frac{n\lambda}{\lambda}$$

or

$$\frac{a+b}{a} = n \qquad (47)$$

In equation (47), the values of n =1, 2, 3 etc. refer to the order of the principal maxima that are absent in the diffraction pattern.

(i) If $\frac{a+b}{a} = 1; b = 0$

In this case, the first order spectrum will be absent and the resultant diffraction pattern is similar to that due to the single slit.

(ii) If $\frac{a+b}{a} = 2;\ a = b$

i.e., the width of the slit is equal to the width of the opaque spacing between any two consecutive slits. In this case, the second order spectrum will be absent.

Overlapping of Spectral Lines:

If the light incident on the grating surface consists of a large range of wavelengths, then the spectral lines of shorter wavelength and of higher order overlap on the spectral lines of longer wavelength and of lower order. Let the angle of diffraction θ be the same for (*i*) the spectral line of wavelength λ_1 in the first order, *(ii)* the spectral line of wavelength λ_2 in the second order and *(iii)* the spectral line of wavelength λ_3 in the third order. Then

$$(a+b)\sin\theta = 1.\lambda_1 = 2\lambda_2 = 3\lambda_3 = \ldots\ldots$$

The red line of wavelength 7000 Å in the third order, the green line of wavelength 5250 Å in the fourth order and the violet line of wavelength **4200A°** in the fifth order are all formed at the same position of the screen because.

$$\begin{aligned}(a+b)\sin\theta &= 3 \times 7000 \times 10^{-8}\text{cm}\\ &= 4 \times 5250 \times 10^{-8}\text{cm}\\ &= 5 \times 4200 \times 10^{-8}\text{cm}\end{aligned}$$

For the visible region of the spectrum, there is no overlapping of the spectra) lines. The range of wavelengths for the visible part of the spectrum is 4000 Å to 7200 Å. Thus, the diffracting angle for the red end of the spectrum in the first order is less than the diffracting angle for the violet end of the spectrum in the second order. If, however, the observations are made with a photographic plate, the spectrum recorded may extend up to 2000 Å in the ultra violet region. In this case, the spectral line corresponding to a wavelength of **4000A°** in the first order and a spectral line of wavelength 2000 Å in the second order overlap. Suitable filters are used to absorb those wavelengths of the incident light which will overlap with the spectral lines in the region under investigation.

Determination of Wavelength of a Spectral Line Using the Transmission Grating:

In the laboratory, the grating spectrum of a given source of light is obtained by using a spectrometer. Initially ail the adjustments of the spectrometer are made and it is adjusted for parallel rays by Schuster's method. The slit of the collimator is illuminated by monochromatic light (say light from sodium lamp) and the position of the telescope is adjusted such that the image of the slit is obtained at the position of the vertical cross-wire in the field of view of the telescope. Now the axes of the collimator and the telescope are in the same line. The position of the telescope is noted on the circular scale and $90°$ is added to this reading. The telescope is turned to this position. In this position the axis of the telescope is perpendicular to the axis of collimator. The position of the telescope is fixed. The given transmission grating is mounted at the centre of the prism table such that the grating surface is perpendicular to the prism table. The prism table is suitably rotated such that the image of the slit reflected from the grating surface is obtained in the centre of field of view of the telescope. This means that the parallel rays of light from the collimator are incident at an angle $45°$ on the grating surface because the axis of the collimator and the telescope are perpendicular to each other. The reading of the prism table is noted and adding $45°$ to this reading, the prism table is suitably rotated to the new position so that the grating surface is normal to the incident light.

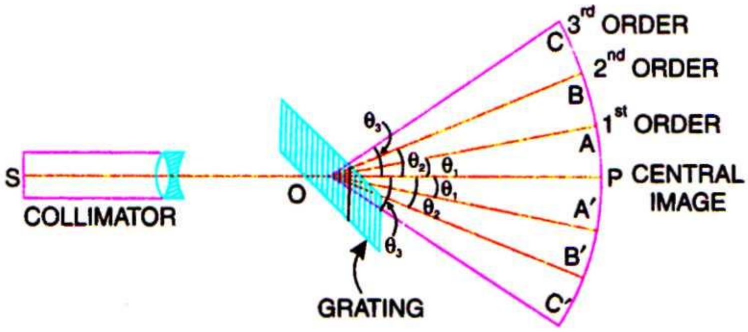


Figure 17

If the wavelength of the sodium light is to be determined, then the angles of diffraction θ_1 and θ_2 corresponding to the first and second order principal maxima are determined (Fig. 18.18). OA, OB etc.. give the directions of the telescope corresponding to the first and second order images, A', B' etc refer to the positions of these images towards the left of the central maximum. The angles AOA' and BOB' are measured and half of these angles measure θ_1 and θ_2. . Then

$$(a+b)\sin\theta_1 = 1\lambda \tag{48}$$

$$(a+b)\sin\theta_2 = 2\lambda \tag{49}$$

Then the value of λ is calculated from equations (48) and (49) and the mean value is taken, $(a + b)$ is the grating element and it is equal to the reciprocal of the number of lines per cm. If the number of lines on the grating surface is 15,000 per inch then

$$(a+b) = \frac{2.54}{15000}\,\text{cm}$$

If the source of light emits radiations of different wavelengths, then the beam gets dispersed by the grating and in each order a spectrum of the constituent wavelengths is observed. To find the wavelength of any spectral line, the diffracting angles are noted in the first and second orders and using the equations given above, the wavelength of the spectral line can be calculated. Overlapping spectral orders can be avoided by using suitable colour filters so that the wavelengths beyond the range of study are eliminated,

With a diffraction grating, the wavelength of the spectral line can be determined very accurately. The method involves only the accurate measurement of the angles of diffraction.

Taking $\lambda = 6000\ \text{Å} = 6000 \times 10^{-8}\ \text{cm}$ and $(a+b) = \dfrac{2.54}{15000}\ \text{cm}$

$$(a+b)\sin\theta_1 = 1\lambda$$
$$(a+b)\sin\theta_2 = 2\lambda$$
$$\theta_1 = 20^\circ - 45'$$

and $$\theta_2 = 45^\circ - 7'$$

As the angles are large they can be measured accurately with a properly calibrated spectrometer. The number of lines per inch (or cm), is given on the grating by the manufacturing company and hence $(a + b)$ can be calculated. As the method does not involve measurements of very small distances (as in the case of interference experiments) an accurate value of λ can be obtained.

Dispersive Power of Grating:

Dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in wavelength between the two spectral lines. It can also be defined as ihc difference in the angle of diffraction per unit change in wavelength. The diffraction of the n^{th} order principal maximum for a wavelength λ, is given by the equation

$$(a+b)\sin\theta = n\lambda$$

Differentiating this equation with respect to θ and λ. we get

$$(a+b)\cos\theta\, d\theta = n d\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} = \frac{nN'}{\cos\theta} \qquad (50)$$

From equation (50) it is clear that the dispersive power of the grating is *(i)* directly proportional to the order of the spectrum, n *(ii)* directly proportional to the number of lines per cm, N' and *(iii)* inversely proportional to $\cos\theta$. Thus, the angular spacing of any two spectral lines is double in the second order spectrum than that in the first order. Secondly, the angular dispersion of the lines is more with a grating having a larger number of lines per cm. Thirdly, the angular dispersion is a minimum when $\theta = 0$. If the value of θ is not large, the value of $\cos\theta$ can be taken as unity and the influence of this factor can be neglected. Then it is clear that the angular dispersion of any two spectral lines is directly proportional to the difference in wavelength of the spectral lines. A spectrum of this type is called **a normal spectrum**.

If the linear spacing of two spectral lines of wavelengths λ and $\lambda + d\lambda$ is dx in the focal plane of the telescope objective or photographic plate, then

$$dx = f\,d\theta$$

where f is the focal length of the objective. The linear dispersion is

$$\frac{dx}{d\lambda} = f\frac{d\theta}{d\lambda} = \frac{fnN'}{\cos\theta}$$

$$dx = \frac{fnN'}{\cos\theta}\cdot d\lambda \tag{51}$$

The linear dispersion is useful in studying the photographs of a spectrum.

Prism and Grating Spectra:

For dispersing a given beam of light and for studying the resultant spectrum, a diffraction grating is mostly used instead of a prism. The grating and prism spectra differ in the following points.

(i) With a grating, a number of spectra of different orders can be obtained on the two sides of the central maximum whereas with a prism only one spectrum is obtained.

(ii) The spectra obtained with a grating are comparatively purer than those with a prism.

(iii) Knowing the grating element $(a + b)$ and measuring the diffraction angle, the wavelength of any spectral line can be measured accurately. But in case of a prism, the angles of deviation are not directly related to the wavelength of the special line. The angles of deviation are dependent on the refractive index of the material of the prism, which depends on the wavelength of light.

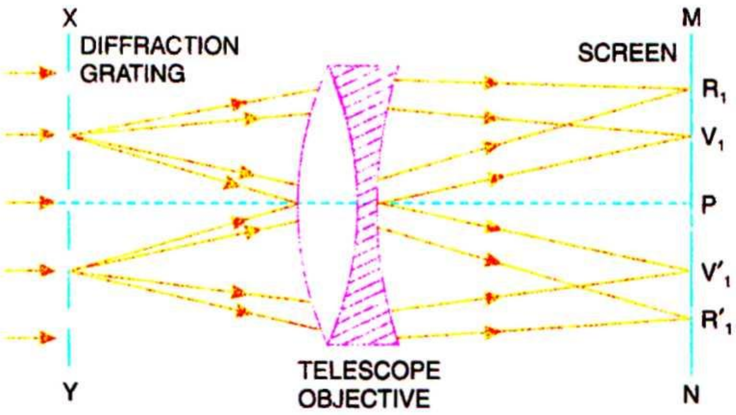


Figure 18

(iv) With a grating, the diffraction angle for violet end of die spectrum is less than for red. In Fig. 18, V_1R_1 and $V_1' R_1'$ refer to the first order spectra on the two sides of the central maximum P. With a prism (Fig. 19), the angle of deviation for the violet rays of light is more than for the red rays of light.

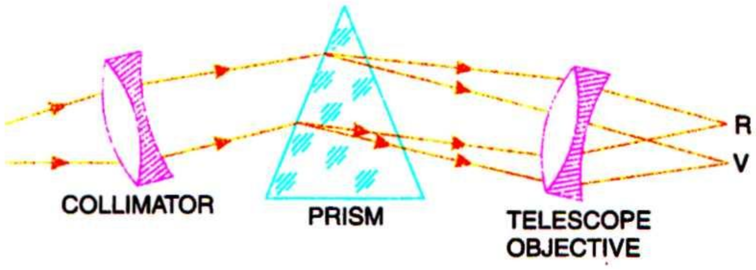


Figure 19

(v) The intensities of the spectral lines with a grating are much less than with a prism. In a grating spectrum, most of the incident light energy is associated with the undispersed central bright maximum and the rest of the energy is distributed in the different order spectra on the two sides of the central maximum. But in a prism most of the incident light energy is distributed in a single spectrum and hence brighter spectral lines are obtained.

(v) The dispersive power of a grating is given by

$$\frac{d\theta}{d\lambda} = \frac{nN'}{\cos\theta}$$

and this is constant for a particular order. Thus, the spectral lines are evenly distributed. Hence, the spectrum obtained with a grating is said to be rational (Fig. 20). The refractive index of the material of a prism changes more rapidly at the violet end than at the red end of the spectrum. The dispersive power of a prism is given by

$\frac{d\mu}{\mu-1}$ and this has higher value in the violet region than in the red region. Hence, there will be more spreading of the spectral lines towards the violet and the spectrum obtained with a prism is said to be irrational (see Fig. 20).

(vii) The resolving power of a grating is given by nN whereas the resolving power of a prism is given by $t\frac{d\mu}{d\lambda}$ where t is the base of the prism. The resolving power of a grating is much higher than that of a prism. Hence the same two nearby spectral lines appear better resolved with a grating than with a prism.

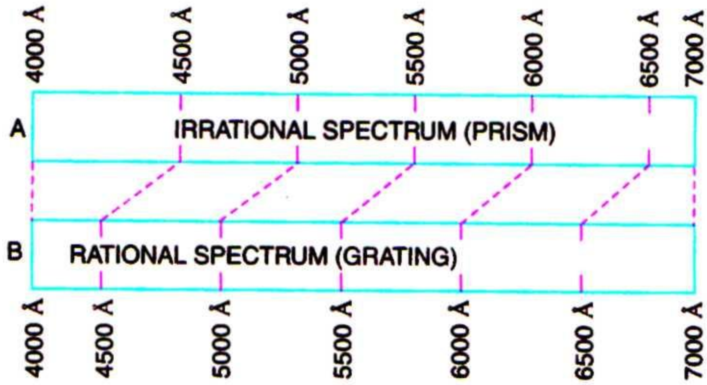


Figure 20

(viii) Lastly, the spectra obtained with different gratings are identical because the dispersive power and the resolving power of a grating do not depend on the nature of the material of the grating. But the spectra obtained with prisms made of different materials are never identical because both dispersive and resolving powers depend on the nature of the material of the prism.

WORKED OUT EXAMPLES

Example 1: In Fraunhoffer diffraction pattern due to a narrow slit a screen is placed 2m away from the lens to obtain the pattern. If the slit width is 0.2 mm and the first minima lie 5 mm on either sides of die central maximum, find the wavelength of light.

Solution: Here, $a = 0.2$ mm $= 0.02$ cm , $x = 5$ mm $= 0.5$ cm, $D = 2$ m $= 200$ cm

In the case of Fraunhofer diffraction at a narrow rectangular aperture,

$$a \sin\theta = n\lambda$$

Here $n = 1$ $\quad\therefore\ a \sin\theta = \lambda$

$$\sin\theta = \frac{x}{D} \quad \therefore \lambda = \frac{a\,x}{D}$$

or

$$\lambda = \frac{0.02\text{ cm} \times 0.5\text{cm}}{200\text{cm}} = 5 \times 10^{-5}\text{ cm} = \mathbf{5000\ Å}$$

Example 2: Diffraction pattern of a single slit of width 0.5 cm is formed by a lens of focal length 40 cm. Calculate the distance between the first dark and the next bright fringe from the axis. Wavelength of light used is **4890 Å**.

Solution: Here, $a = 0.5$ cm $= 5 \times 10^{-3}$ m , $\quad f = 40$ cm $= 0.4$ m,

$$\lambda = 4890\text{ Å} = 4890 \times 10^{-10}\text{ m}$$

For a minima, $\quad a \sin\theta = n\,\lambda.$ Also $\sin\theta = \dfrac{x_1}{f}$

As $\quad n = 1$, we get $\dfrac{x_1}{f} = \dfrac{\lambda}{a}$

or

$$x_1 = \frac{f\,\lambda}{a} = \frac{0.4 \times 4890 \times 10^{-10}}{5 \times 10^{-3}}\text{ m}$$

$\therefore$

$$x_1 = 3.912 \times 10^{-5}\text{ m.}$$

Now for a maximum, $a \sin\theta = \dfrac{(2n+1)\,\lambda}{2}$. $\quad$ Also $\sin\theta = \dfrac{x_2}{f}$

As $\quad n = 1$, we get $\dfrac{x_2}{f} = \dfrac{3\lambda}{2a}$ or $x_2 = \dfrac{3\lambda f}{2a}$

$\therefore$

$$x_2 = \frac{3 \times 4890 \times 10^{-10} \times 0.4}{2 \times 5 \times 10^{-3}}\text{ m} = 5.868 \times 10^{-5}\text{ m}$$

$\therefore$

$$x_2 - x_1 = 5.868 \times 10^{-5}\text{ m} - 3.912 \times 10^{-5}\text{ m}$$
$$= 1.956 \times 10^{-5}\text{ m} = \mathbf{1.9 \times 10^{-2}\ mm.}$$

Example 3: Deduce the missing orders for a double slit Fraunhofer diffraction pattern, if the slit widths are 0.16 mm and they are 0.8 mm apart.

Solution: Here $a = 0.16$ mm $= 0.016$ cm and $b = 0.8$ mm $= 0.08$ cm

Equation for interference maxima is, $(a + b)\sin\theta = n\,\theta$

Equation for diffraction minima is, $a\sin\theta = p\,\lambda$

$$\therefore \quad \frac{(a+b)}{a} = \frac{n}{p}$$

$$\therefore \quad \frac{n}{p} = \frac{(0.016 + 0.080)}{0.016} = 6$$

$$\therefore \quad n = 6p$$

For the values of $p = 1, 2, 3$ etc.

$n = 6, 12, 18$ etc.

Thus the orders 6, 12, 18 etc. of the interference maxima will be missing from the diffraction pattern.

Example.4: In a diffraction phenomenon using double slit, calculate (i) the distance between the central maximum and the first minimum of the fringe envelope and (ii) the distance between any two consecutive double slit dark fringes.

Given data: Wavelength of light = 5000 Å, slit width = 0.02 mm,

Spacing between two slits = 0.10 mm, screen to slits distance = 100 cm

Given data : Wavelength of light = 5000 Å, slit width = 0.02 mm,

Spacing between two slits = 0.10 mm, screen to slits distance = 100 cm

Solution: Here, $a = 0.02$ mm $= 2 \times 10^{-5}$ m, $b = 0.1$ mm $= 10^{-4}$ m, $(a + b) = 1.2 \times 10^{-4}$ m,

$\lambda = 5000$ Å, $d = 100$ cm $= 1$ m

(*i*) The angular separation between the central maximum and the first minimum is,

$$\sin\theta_1 = \theta_1 = \frac{\lambda}{2(a+b)} \quad \text{and} \quad \theta_1 = \frac{x_1}{D}$$

$$\therefore \quad \frac{x_1}{D} = \frac{\lambda}{2(a+b)} \quad \therefore \; x_1 = \frac{\lambda D}{2(a+b)}$$

$$\therefore \quad x_1 = \frac{5\times10^{-7}\,\text{m} \times 1\,\text{m}}{2\left(1.2\times10^{-4}\ \text{m}\right)} = 2.08 \times 10^{-3}\ \text{m} = \mathbf{2.08\ mm}$$

The separation between the central maximum and the first minimum is **2.08 mm.**

(*ii*) The angular separation between two consecutive dark fringes,

$$\sin\theta_1 - \sin\theta_2 = \theta_1 - \theta_2 = \theta = \frac{3\lambda}{2(a+b)} - \frac{\lambda}{2(a+b)}$$

$$\theta = \frac{\lambda}{(a+b)}; \quad \text{Also} \quad \theta = \frac{x_2}{D}$$

$$\therefore \quad x_2 = \frac{\lambda D}{(a+b)} \quad \therefore \; x_2 = \frac{5\times10^{-7}\,\text{m}\times1\text{m}}{1.2\times10^{-4}\,\text{m}}$$

$$= 4.16 \times 10^{-3}\ \text{m} = \mathbf{4.2\ mm}$$

Example 5: A parallel beam of light of wavelength **5460 Å** is incident at an angle of **30^o** on a plane transmission grating which has **6000 lines /cm** .Find the highest order spectrum that can be observed.

Solution : Here $\theta = 30^o, \lambda = 5460\ \text{Å} = 5460 \times 10^{-10}$ m,

$$(a+b) = \frac{1}{6\times10^3}\ \text{cm} = \frac{1}{6\times10^5}\ \text{m}$$

Now, $(a+b)[\sin\theta_n + \sin i] = n\lambda$

But here $\theta_n = i$

$\therefore$ $(a+b)\,[2\sin i\,] = n\,\lambda$

$\therefore$ $$n = \frac{(a+b)[2\sin i]}{\lambda} = \frac{1}{6\times10^5\times5460\times10^{-10}} = 3.05$$

or $n = 3$

Example 6: Calculate the possible order of spectra with a plane transmission grating having 18,000 lines per inch when light of wavelength 4500 A$^\circ$ is used.

Solution: Order of spectra, $n = \dfrac{d\sin\theta}{\lambda}$. The highest order occurs when $\sin\theta = 1$;

$\therefore$ $$n = \frac{d}{\lambda} = \frac{1}{N\lambda} = \frac{1}{(7.09\times10^5\,\text{lines/m})\,(4500\times10^{-10}\,\text{m})} = 3$$

Example **7:** Light which is a mixture of two wavelengths 5000 A and 5200 A is incident normally on a plane transmission grating having 10000 lines per cm. A lens of focal length 150 cm is used to observe the spectrum on the screen. Calculate the separation in cm of the two lines in the first order spectrum.

Solution: Here $(a+b) = 10^{-4}$ cm, $\lambda_1 = 5000\ \text{Å} = 5\times10^{-5}$ cm;

$\lambda_2 = 5200\ \text{Å} = 5.2\times10^{-5}$ cm and $n = 1$

Now $$\sin\theta_1 = \frac{n\lambda_1}{a+b} = \frac{1\times5\times10^{-5}\,\text{cm}}{10^{-4}\,\text{cm}}$$

$\therefore$ $\theta_1 = \sin^{-1}(0.5) = 30^o$

Similarly, $$\sin\theta_2 \frac{n\lambda_2}{a+b} = \frac{1\times5.2\times10^{-5}\,\text{cm}}{10^{-4}\,\text{cm}}$$

$= 0.52$

$\theta_2 = \sin^{-1}(0.52) = 31.3^o$

Further, $\tan\theta_1 = \dfrac{x_1}{f}$ and $\tan\theta_2 = \dfrac{x_2}{f}$

$\therefore$ $(x_2 - x_1) = f\,[\tan\theta_2 - \tan\theta_1]$

$= 150\ [0.0687 - 0.5774]$

$= 4.7$ cm