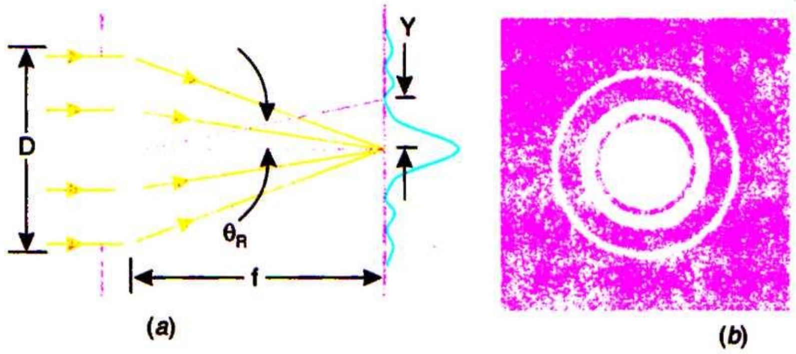
Introduction:

When two objects or their images are very close to each other, they appear as one and it may not be possible for the eye to see them as separate. If the objects are not seen separately, then we say that the details are *not resolved* by the eye. Optical instruments are used to assist the eye in resolving the objects or images. The method adapted to seeing the close objects as separate objects is called resolution. The ability of an optical instrument to produce distinctly separate images of two objects located very close to each other is called its *resolving power*. We use the term 'resolving power' in two different senses. In case of microscopes and telescopes, we talk of *geometrical resolution* where the geometrical positions between two nearby objects are to be resolved and in case of spectroscopes we refer to *spectral resolution* where differences of wavelengths of light in a given source are to be resolved. *Resolving power is normally defined as the reciprocal of the smallest angle subtended at the objective of optical instrument by two point objects, which can just be distinguished as separate*

Rayleigh's Criterion:

When a beam of light from a point object passes through the objective of a telescope, the lens acts like a circular aperture and produces a diffraction pattern instead of a point image. This diffraction pattern is a bright disc surrounded by alternate dark and bright rings (see Fig. 1).

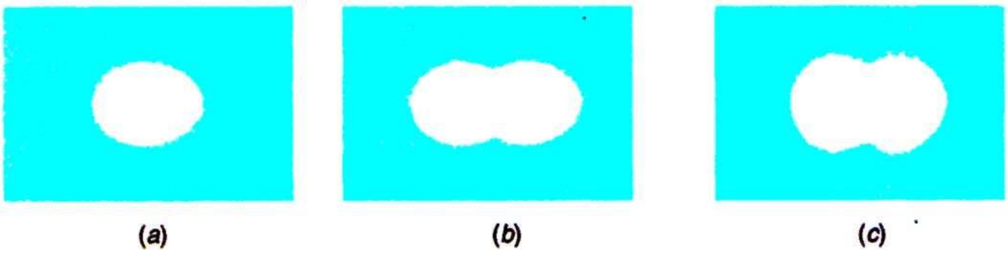


(a) The image of a point source of monochromatic light, formed by a lens of diameter *D*, is a diffraction pattern.
(b) The intensity pattern is a maximum at the center of the Airy disc.

Fig.1

It is known as *Airy's disc*. If there are two point objects lying close to each other, then two diffraction patterns are produced, which may overlap on each other and it may be difficult to distinguish them as separate (see Fig. 2a).

To obtain the measure of the resolving power of an objective lens Rayleigh suggested that the two images of such point-objects lying close to each other may be regarded as separated if the central maximum of one falls on the first minimum of the other. In other words, when the central bright image of one falls on the first dark ring of the other, the two images are said to be just resolved (see Fig. 2b). This is equivalent to the condition that the distance between the centers of the patterns shall **be** equal to the radius of the central disc. This is called the **Rayleigh criterion** for resolution and is also known as **Rayleigh's limit of resolution.**



(a) The angular separation of the sources is so small that the images are not resolved.

(b) The images are just resolved.

(c) The images are well resolved.

Fig. 2

Limit of Resolution of the Eye:

In Fig. 3, MN is the eye lens, A and B are two object points separated by a distance h and A' and B' are the corresponding image points at a distance h' formed on the retina, μ is the refractive index of the object medium and μ ' is the refractive index of the image medium. If the object is placed in air. μ = 1 and the image medium is vitreous humor whose refractive index is 1.33. If the object is situated at the least distance of distinct vision, u = 25 cm = 250 mm for a normal eye. If the diameter of the eye ball is about 2.5 cm, then m = 2.5 cm = 25 mm approximately. Taking the pupillary diameter of the eye as 2 mm, R = I mm.

Also, human eye is most sensitive to a wavelength $\lambda_o = 5500$ Å.

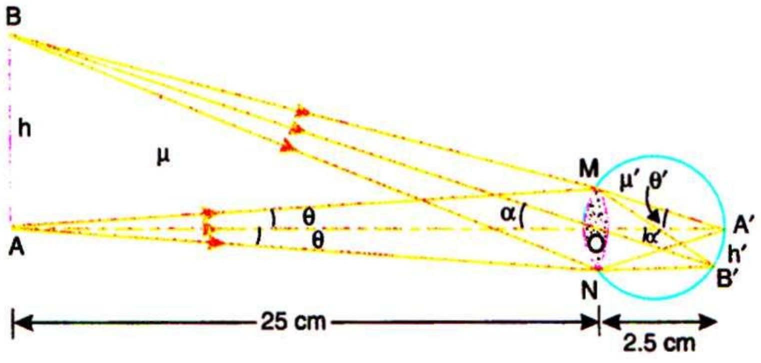


Fig.3

From the ΔAMO for small angles of θ,

$$\sin\theta = \tan\theta = \frac{R}{u} = \frac{1}{250} = 0.004$$

Numerical aperture $= \mu \sin\theta$

$$= 1 \times 0.004 = 0.004$$

Applying Rayleigh's criterion, the minimum distance (h) between two just resolvable object points of equal intensity is given by,

$$h = \frac{0.61\lambda_o}{\mu \sin\theta} = \frac{0.61 \times 5500 \times 10^{-8}\text{cm}}{0.004} = \frac{1}{100}\text{ cm} = \frac{1}{10}\text{ mm approximately.}$$

It means that if the object is situated at the least distance of distinct vision from the eye (25 cm), the minimum separation between two nearby object points should be of the order of 0.1 mm. If the object points are separated by a distance larger than 0.1 mm, they are clearly visible and are well resolved.

Similarly, the distance h' between the centers of the two images is given by,

$$h' = \frac{0.61\lambda_o}{\mu' \sin\theta'} = \frac{0.61 \times 5500 \times 10^{-8}\text{cm}}{1.33 \times 0.04} = \frac{1}{1000}\text{ cm} = \frac{1}{100}\text{ mm approximately.}$$

$$(\because \sin\theta' = \frac{1}{25} = 0.04)$$

Also, $$\alpha = \sin\alpha = \frac{0.61\lambda_o}{\mu R} = \frac{0.61 \times 5500 \times 10^{-8}\text{cm}}{1 \times 0.1\text{ cm}} = 0.00034\text{ radian}$$

$$= 1\text{ minute of an arc (approx.)}$$

The value of h' (= 10^{-2}mm) is approximately equal to the distance between the cones in the fovea and thus the retinal structure is strikingly in accordance with the limit of resolution of the eye. Further, two point objects appear to be just resolved if the angle subtended by them at the eye is 1 minute of an arc. If the diameter of the pupil of the eye is smaller than 2 mm, the numerical aperture decreases and hence the value of h increases, i.e. two points will appear to be just resolved if the distance between the two is larger. Thus the resolving ability of the eye is decreased.

Resolving Power of Optical Instruments:

The magnifying power of a telescope or a microscope depends upon the focal lengths of the lenses used. By a proper choice of the lenses, it is possible to increase the size of the image, i.e. the image subtends a large angle at the eye. But it must be remembered that the increase in the size of the image, beyond a certain limit does not necessarily mean gain in *detail*. This is the case even if the lenses are free from all aberrations, chromatic and monochromatic. There is always a limit to the *useful magnification* of an optical instrument. This is due to the fact that for a wave surface, the laws of geometrical optics do not hold good. In the previous chapters dealing with diffraction of light, it has been shown that the image of a point source is not a point but it is a diffraction pattern. With a circular aperture kept in the path of incident light, the diffraction pattern of a point source of light consists of a central bright disc surrounded by alternatively dark and bright rings. This is called **Airy's disc.**

If the lens diameter or the size of the aperture is large, the diffraction pattern of a point source of light is small. If there are two nearby point sources, the diffraction discs of the two patterns may overlap and the two images may not be distinguished. An optical instrument like a telescope or a microscope is said to have resolved the two point sources when the two diffraction patterns are well separated from each other or when the diffraction patterns are small so that in both the cases, the two images are seen as separate ones. **The ability of an optical instrument, expressed in numerical measure, to resolve the images of two nearby points is termed as its resolving power.**

A telescope gives us geometric resolution between two far away objects subtending a very angle. There we define resolving limit of a telescope $= \theta_m$; where θ_m is the smallest angle resolved.

A microscope resolves the linear distance between two close objects. There we define, resolving limit of a microscope $= x_m$; where x_m is the smallest distance resolved.

In the case of a prism or a grating spectrograph, the term resolving power is referred to the ability of the prism or grating to resolve two nearby spectral lines so that the two lines can be viewed or photographed as separate lines.

Spectral resolving power $= \dfrac{\lambda}{\Delta\lambda_m}$ where $\Delta\lambda_m$ is the smallest difference of wavelength which can be resolved by the instrument at wavelength λ.

Criterion for Resolution According to Lord Rayleigh:

To express the resolving power of an optical instrument as a numerical value, Lord Rayleigh proposed an arbitrary criterion. According to him, two nearby images are said to be resolved if the position of the central maximum of one coincides with the first secondary minimum of the other and vice versa. The same criterion can be conveniently applied to calculate the resolving power of a telescope, microscope, grating, prism, etc. In Fig. 4, A and **B** are the central maxima of the diffraction patterns of two spectral lines of wavelengths λ_1 and λ_2. The difference in the angle of diffraction is large and the two images can be seen as separate ones. The angle of diffraction corresponding to the central maximum of the image **B** is greater than the angle of diffraction corresponding to the first minimum at the right of A. Hence the two spectral lines will appear to be well resolved (see Fig. 2c also).

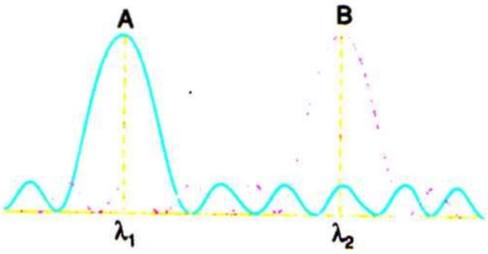


Fig.4

In Fig. 5, the central maximum corresponding to the wavelengths λ and $\lambda + d\lambda$ are very close. The angle of diffraction corresponding to the first minimum of A is greater than the angle of diffraction corresponding to the central maximum of **B.** The two images overlap and they cannot be distinguished as separate images. The resultant intensity curve gives the maximum as at C and the intensity of this maximum is higher than the individual intensities of A and B. Thus when the spectrograph is turned from A to B. the intensity increases, becomes maximum at C and then decreases. In this case, the two spectral lines are **not resolved** see Fig. 2(a) also.

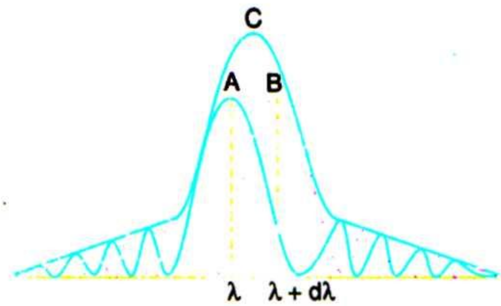


Fig.5

In Fig. 6, the position of the central maximum of A (wavelength λ) coincides with the position of the first minimum of **B**(wavelength $\lambda + d\lambda$). Similarly, the position of the central maximum of **B** coincides with the position of the first minimum of A. Further, the resultant intensity curve shows a **dip** at C i.e. in the middle of the central maximum of A and B (here it is assumed that the two spectral lines are of the same intensity). The intensity at C is approximately 20 % less than that at A or B. If a spectrograph is turned from the position corresponding to the central image of A to the one corresponding to the image of B, there is noticeable decrease in intensity between the two central maxima. The spectral lines can be distinguished from one another and according to Rayleigh they are said to be **just resolved** (see Fig. 19.2 b also). Rayleigh's condition can also be stated as follows. **Two images are said to be**

just resolved if the radius of the central disc of either pattern is **equal to the distance between the centers of the two patterns.**

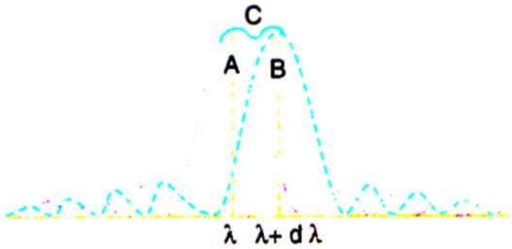


Fig.6

Resolving Power of a Telescope:

Let *a* be the diameter of the objective of the telescope (Fig. 7). Consider the rays of light from two neighboring points of a distant object. The image of each point object is a Fraunhofer diffraction pattern.

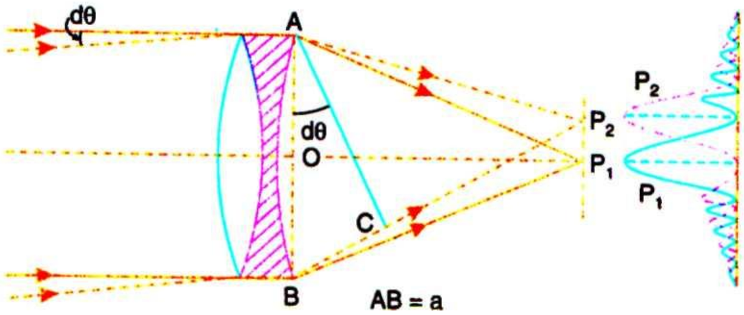


Fig.7

Let P_1 and P_2 be the positions of the central maxima of the two images. According to Rayleigh, these two images are said to be resolved if the position of the central maximum of the second image coincides with the first minimum of the first image and vice versa. The path difference between the secondary waves traveling in the directions AP_1 and BP_1 is zero and hence they reinforce with one another at P_1 Similarly, all the secondary waves from the corresponding points between A and B will have zero path difference. Thus P_1 corresponds to the position of the central maximum of the first image.

The secondary waves traveling in the directions AP_2 and BP_2 will meet at P_2 on the screen. Let the angle P_2AP_1 be dϴ. The path difference between the secondary waves traveling in the directions BP_2 and AP_2 is equal to BC (Fig. 7).

From the Δ ABC, BC = AB sin $d\theta$ = AB.$d\theta$ = $a.d\theta$ (for small angles)

If this path difference $a.d\theta = \lambda$, the position of P_2 corresponds to the first minimum of the first image. But P_2 also is the position of the central maximum of the second image. Thus, Rayleigh's condition of resolution is satisfied if,

$$a.d\theta = \lambda \quad \text{or} \quad d\theta = \frac{\lambda}{a} \tag{1}$$

The whole aperture AB can be considered to be made of two halves AO and OB. The path difference between the secondary waves from the corresponding points in the two halves will be $\frac{\lambda}{2}$. All the secondary waves destructively interfere with one another and hence P_2 will be the first minimum of the first image.
The equation $d\theta = \frac{\lambda}{a}$ holds good for rectangular apertures. For circular apertures, this equation, according to Airy, can be written as,

$$d\theta = \frac{1.22\,\lambda}{a} \tag{2}$$

where λ is the wavelength of light and a is the aperture of the telescope objective. The aperture is equal to the diameter of the metal ring which the objective lens is mounted. Here refers to the **limit of resolution** of the telescope.
The reciprocal of $d\theta$ measures the resolving power of the telescope.

$$\therefore \qquad \frac{1}{d\theta} = \frac{a}{1.22\lambda} \tag{3}$$

$d\theta$ is also the angle subtended by the two distant object points at the objective.

From equation (3), it is clear that a telescope with large diameter of the objective has higher resolving power,

Thus, resolving power of a telescope can be defined as the reciprocal of the angular separation that two distant object points must have, so that their images will appear just resolved according to Rayleigh's criterion.

I f f is the focal length of the telescope objective, then,

$$d\theta = \frac{r}{f} = \frac{1.22\,\lambda}{a}$$

$$\text{or} \qquad r = \frac{1.22\,f\lambda}{a} \tag{4}$$

where r is the radius of the central bright image. The diameter of the first dark ring is equal to the diameter of the central image. The central bright disc is the Airy's disc.

From equation (4), if the focal length of the objective is small, the wavelength is small and the aperture is large, then the radius of the central bright disc is small. The diffraction patterns will appear sharper and the angular separation between the two just resolvable point objects will be smaller. Correspondingly, the resolving power of the telescope will be higher.

Let two distant stars subtend an angle of 1 second of an arc at the objective of the telescope.

1 second of an arc $= 4.85 \times 10^{-6}$ radian. Let the wavelength of light be 5500 Å. Then, the diameter of the objective required for just resolution can be calculated from the equation,

$$d\theta = \frac{1.22\,\lambda}{a}$$

or

$$a = \frac{1.22\,\lambda}{d\theta} = \frac{1.22 \times 5500 \times 10^{-8}\text{cm}}{4.85 \times 10^{-8}} = 13.9 \text{ cm (approximately)}$$

The resolving power of a telescope increases with increase in the diameter of the objective. With the increase in the diameter of the objective, the effect of the spherical aberration becomes appreciable. So, in the case of large telescope objectives, the central portion of the objective is covered with a stop so as to minimize the effect of spherical aberration. This, however, does not affect the resolving power of the telescope.

Relation Between Magnifying Power and Resolving Power of a Telescope:

The magnifying power of a telescope is given by,

$$M = \frac{D}{d} \tag{5}$$

where D is the diameter of the objective (entrance pupil) and d is the diameter of the exit pupil. The magnification of a telescope is said to be normal if d, the diameter of the exit pupil is equal to d_e the diameter of the pupil of the eye. Therefore, the normal magnification of the telescope is given by,

$$M = \frac{D}{d_e} \tag{6}$$

Further, the limit of resolution of a telescope is given by, $d\theta = \frac{1.22\,\lambda}{D}$ (7)

And the limit of resolution of eye is given by, $d\theta' = \frac{1.22\,\lambda}{d_e}$ (8)

From equation (7) and (8), we get

$$\frac{d\theta'}{d\theta} = \frac{\text{Limit of resolution of the eye}}{\text{Limit of resolution of the telescope}} = \frac{1.22\lambda}{de} \Big/ \frac{1.22\lambda}{D} = \frac{D}{de} = M$$

Thus, the product of normal magnifying power of a telescope and its limit of resolution is equal to the limit of resolution of the unaided eye.

Taking a pupil diameter of the eye as 2 mm and wavelength of light as 5500 Å, the angular separation $(d\theta')$ between two distant object points resolvable by the eye is given by,

$$d\theta' = \frac{1.22\,\lambda}{d_e} = \frac{1.22 \times 5500 \times 10^{-8}\ \text{cm}}{0.2\ \text{cm}}$$

$$= \frac{1.22 \times 5500 \times 10^{-8}\ \text{cm}}{0.2\ \text{cm}} \times \frac{180}{\pi} \times 60 \text{ minutes} = 1 \text{ minute of an arc (approx.)}$$

Similarly, the angular separation between two distant stars just resolvable by a telescope objective of diameter 254 cm is approximately $\frac{1}{20}$ th second of an arc.

i.e. $$d\theta = \frac{1}{20} \text{th second of an arc}$$

$$d\theta' = \text{one minute of an arc} = 60 \text{ seconds of an arc}$$

Normal magnifying power of a telescope objective of diameter 254 cm

$$= \frac{d\theta'}{d\theta} = \frac{60}{1/20} = 1200$$

If the normal magnifying power is 1200, full advantage of the high resolving power of the telescope can be taken.

If two telescope objectives have the same focal length, the magnifying power will be the same in the two cases. But, the telescope with an objective of larger aperture has high resolving power than the one with a smaller aperture. With increase in size of the diameter of the objective of a telescope, the resolving power increases. Also with a large diameter objective, the radius of the central disc of the diffraction pattern is smaller and consequently the image obtained is sharp and more intense.

Resolving Power of a Microscope:

The minimum distance by which two points in the object are separated from each other so that their images as produced by the microscope are just seen separate is called the *limit of resolution*. The reciprocal of limit of resolution is known as the *resolving power*.

In Fig. 8, MN is the aperture of the objective of a microscope and A and B are two object points at a distance d apart. A' and B' correspond to diffraction patterns due to A and B. A' and B' are surrounded by alternate dark and bright diffraction rings. The two images are said to be just

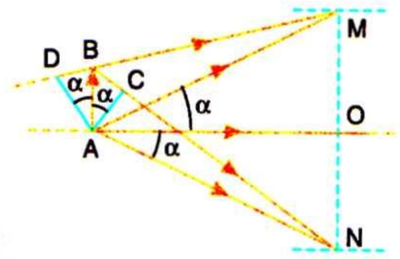


Fig.8

resolved if the position of the central maximum of B' also corresponds to the first minimum of the image of A'.

The path difference between the extreme rays from the point B and reaching A' is given by,

$$(BN + NA') - (BM + MA')$$

$$\text{But} \quad NA' = MA'$$

$$\therefore \ \text{Path difference} = BN - BM$$

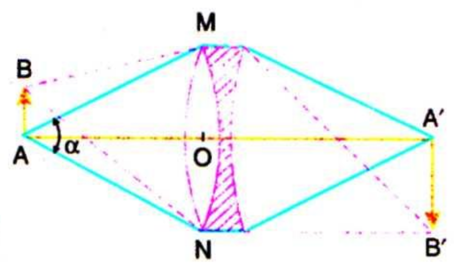


Fig.9

In Fig. 9, AD is perpendicular to DM and AC is perpendicular to BN.

$$\therefore \quad BN - BM = (BC + CN) - (DM - DB)$$

$$\text{But} \quad CN = AN = AM = DM$$

$$\therefore \ \text{Path difference} = BC + DB$$

From the Δ^S ACB and ADB,

$$BC = AB \sin\alpha = d \sin\alpha$$

and

$$DB = AB \sin\alpha = d \sin\alpha$$

$$\therefore \ \text{Path difference} = 2d \sin\alpha$$

If this path difference $2\,d\,\sin\alpha = 1.22\,\lambda$, then, A' corresponds to the first minimum of the image B' and the two images appear just resolved.

$$\therefore \quad 2\,d \sin\alpha = 1.22\,\lambda$$

or $$d = \frac{1.22\,\lambda}{2\sin\alpha} \qquad (9)$$

Equation (9) derived above is based on the assumption that the object points A and B arc self-luminous. But actually, the objects viewed with a microscope are not self-luminous but are illuminated with light from a condenser. It is found that the resolving power depends upon the mode of illumination. According to Abbe, the least distance between two just resolvable object points is given by.

$$d = \frac{\lambda_0}{2\mu\sin\alpha} \qquad (10)$$

where λ_0 is the wavelength of light in vacuum and μ is the refractive index of the medium between the object and the objective. The space between the objective and the object is filled with oil (cedar wood oil) in microscopes of high resolving power. This has two advantages. Firstly the loss of light by reflection at first lens surface is decreased and secondly the resolving power of the microscope is increased.

The expression μ sinα in equation (10) is called the **numerical aperture** of the objective of the microscope and is a characteristic of the particular objective used. The highest value of numerical aperture obtained in practice is about 1.6. Taking the effective wavelength of white light as:5500 Å and μ sinα = 1.6,

$$d = \frac{5500\times10^{-8}\text{cm}}{2\times1.6} = 1.72\times10^{-5}\ \text{cm}$$

where d is the linear distance between two just resolvable object points.

Ways of Increasing Resolution:

From equation (10), we see that we can achieve increased resolution in two ways, namely by (i) increasing the numerical aperture of the objective and (ii) decreasing the wavelength λ of the light used. Thus, by using ultraviolet light and quartz lenses, the resolving power of the microscope can be increased further. Such a microscope is called an **ultra-microscope.**

The magnifying power of a microscope is said to be normal if the diameter of the exit pupil is equal to the diameter of the pupil of the eye. If the magnifying power is higher than the normal, it does not correspondingly help in observing better details of the object. If the magnifying power of the microscope is less than the normal, then this means that full advantage of the available resolving power of the microscope objective is not taken.

In an electron microscope, the wavelength of electron beam used is of the order of 0.12 A°, which is more than thousand times smaller than the wavelength of visible light. Hence, the resolving power of an electron microscope is much higher than that of an ordinary microscope. However, the numerical aperture of an electron microscope is smaller than that of an ordinary microscope.

Magnification Versus Resolution:

The student should now be careful to notice the significance of resolution. In any optical instrument, the first magnification is done by the objective (because it faces the object). If the diffraction patterns of the different parts of the object are not well resolved at this stage, they can *never* be resolved further by any amount of magnification at the later stages. The patterns may get *magnified but not resolved*. High magnification alone will not reveal any further details of the object. Thus, for a given optical system, there will be a limit to useful magnification.

For the standard observer, the near point is about 25cm from the eye, and for many purposes one can assume that at this distance the limit of resolution is about 0.01 cm. It means that objects can be resolved by the eye when they subtend an angle of about one minute. Thus, for two optical images to be resolved by the observer, they must have an angular separation not less than this. Therefore, the magnifying power of the instrument should be such that the physically resolved images by the instrument subtend an angle of at least one minute at the observer's eye. Thus, for any visual instrument, there is a *minimum magnifying power* below which the eye will be unable to resolve the objects even though the images as formed by the instrument itself may clearly be resolved. This minimum magnifying power is usually called the **necessary magnifying power.** A magnifying power excessively greater than this is of no advantage, since the eye can never resolve images that the instrument does not resolve. This additional magnification is a disadvantage because it renders visible the diffuse nature of the Airy discs and one gets an impression that image quality is poor. Such additional magnification, greatly in excess of the necessary value, is called **empty magnification.**

Resolving Power of a Prism:

The term resolving power applied to the spectrographic devices (using a grating or a prism) signifies the ability of the instrument to form two separate spectral images of two neighboring wavelengths, λ and $\lambda + d\lambda$ in the wavelength region λ.

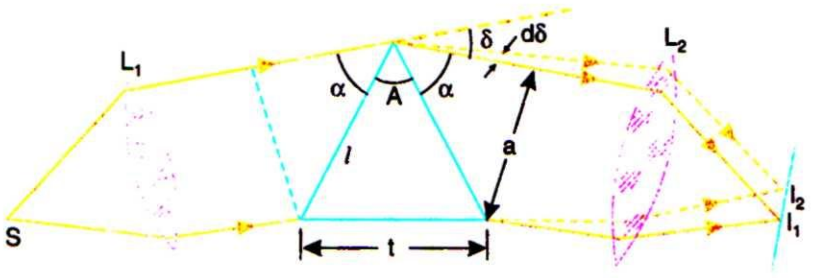


Fig.10

In Fig. 10, S is a source of light, L_1 is a collimating lens and L_2 is the telescope objective. As two wavelengths λ and $\lambda + d\lambda$ are very close, if the prism is set in minimum deviation position would hold good for both the wavelengths. The final image I_1 corresponds to the principal maximum for the wavelength λ and I_2 corresponds to the principal maximum for the wavelength $\lambda + d\lambda$.. 1_1 and I_2 are formed at the focal plane of the telescope objective L_2. The face of the prism limits the incident beam to a rectangular section of width a. Hence, the Rayleigh criterion can be applied in the case of a rectangular aperture.

In the case of diffraction at a rectangular aperture, the position of I_2 will correspond to the first minimum of the image I_1 for wavelength λ, provided

$$a.d\delta = \lambda \text{ or } d\delta = \frac{\lambda}{a} \qquad (11)$$

Here δ is the angle of minimum deviation for wavelength λ. From the Fig. 10,

$$\alpha + A + \alpha + \delta = \pi$$

$$\therefore \quad \alpha = \left[\left(\frac{\pi}{2}\right) - \left(\frac{A+\delta}{2}\right)\right]$$

$$\therefore \sin\alpha = \sin\left[\left(\frac{\pi}{2}\right) - \left(\frac{A+\delta}{2}\right)\right] = \cos\left(\frac{A+\delta}{2}\right)$$

$$\text{But } \sin\alpha = \frac{a}{l}; \qquad \therefore \cos\left(\frac{A+\delta}{2}\right) = \frac{a}{l} \qquad (12)$$

$$\text{Also,} \qquad \sin\frac{A}{2} = \frac{t}{2l} \qquad (13)$$

$$\text{In the case of a prism,} \quad \mu = \frac{\sin\frac{A+\delta}{2}}{\sin\frac{A}{2}}$$

$$\therefore \quad \sin\frac{A+\delta}{2} = \mu \sin\frac{A}{2} \qquad (14)$$

Here μ and δ are dependent on wavelength of light λ. Differentiating equation (14) with respect to λ, we get

$$\frac{1}{2}\cos\left(\frac{A+\delta}{2}\right)\frac{d\delta}{d\lambda}=\frac{d\mu}{d\lambda}\left(\sin\frac{A}{2}\right) \tag{15}$$

Substituting the values of $d\delta$ from equations (12) and (13), we obtain

$$\frac{1}{2}\left(\frac{a}{l}\right)\frac{d\delta}{d\lambda}=\frac{d\mu}{d\lambda}\left(\frac{t}{2l}\right)$$

or

$$a.\frac{d\delta}{d\lambda}=t.\frac{d\mu}{d\lambda} \tag{16}$$

Substituting the value of $d\delta$ equation (11), we get

$$\frac{\lambda}{d\lambda}=t\cdot\frac{d\mu}{d\lambda} \tag{17}$$

The expression $\frac{\lambda}{d\lambda}$ measures the resolving power of the prism. It is defined as the ratio of the wavelength λ to the smallest difference in wavelength $d\lambda$ between this line and a neighboring line such that the two lines appear just resolved, according to Rayleigh's criterion.

$$\text{So, resolving power of a prism} = t\cdot\frac{d\mu}{d\lambda} \tag{18}$$

It means that the resolving power (i) is directly proportional to the length of the base of the prism and (ii) rate of change of refractive index with respect to wavelength for that particular material.

Resolving Power of a Plane Transmission Grating:

The resolving power of a grating is defined as the ratio of the wavelength λ of any spectral line to the smallest difference in wavelength $d\lambda$, between this line and a neighboring line such that the two lines appear just resolved, according to Rayleigh's criterion.

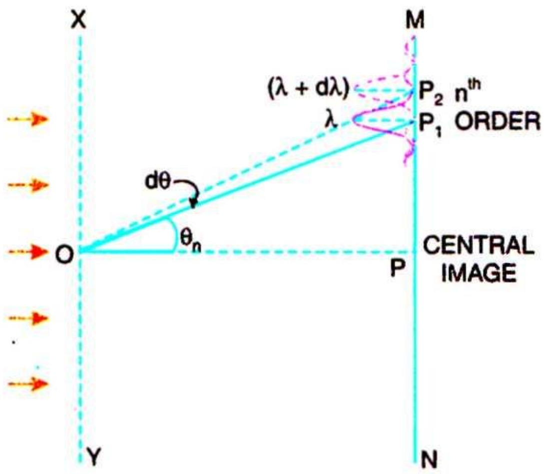


Fig.11

So resolving power of a grating $= \frac{\lambda}{d\lambda}$

In Fig. 11, XY is the grating surface and MN is the field of view of the telescope, P, is n^{th} primary maximum of a spectral line of wavelength λ at an angle of diffraction θ_n.P_2 is the $\mathbf{n^{th}}$ primary maximum of a second spectral line of wavelength $\lambda + d\lambda$ at a diffracting angle $\theta_n + d\theta$. P_1 and P_2 are the spectral lines in the $\mathbf{n^{th}}$ order. These two spectral lines according to Rayleigh, will appear just resolved if the position of P_2 also corresponds to the first minimum of P_1.

The direction of the $\mathbf{n^{th}}$ primary maximum for a wavelength λ is given by,

$$(a + b) \sin \theta_n = n\lambda \qquad (19)$$

The direction of the $\mathbf{n^{th}}$ primary maximum for a wavelength $\lambda + d\lambda$ is given by,

$$(a + b) \sin (\theta_n + d\theta) = n(\lambda + d\lambda) \qquad (20)$$

These two lines will appear just resolved if the angle of diffraction $(\theta_n + d\theta)$ also corresponds to the direction of the first secondary minimum after the n_{th} primary maximum at P_1 (corresponding to the wavelength λ).This is possible if the extra path difference introduced is $\frac{\lambda}{N}$ where N is the total number of lines on the grating surface

$$\therefore \qquad (a + b) \sin (\theta_n + d\theta) = n\lambda + \frac{\lambda}{N}$$

Equating the right hand sides of equations (19.25) and (19.26),

$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N} \quad \text{or} \quad nd\lambda = \frac{\lambda}{N} \quad \text{or} \quad \frac{\lambda}{d\lambda} = nN \qquad (21)$$

The quantity $\frac{\lambda}{d\lambda} = nN$ measures the resolving power of a grating. Thus, the resolving power of a grating is independent of the grating constant. The resolving power is directly proportional to (i) the order of the spectrum and (ii) the total number of lines on the grating surface. For a given grating, the distance between the spectral lines is double in the second order spectrum than that in the first order spectrum.

The dispersive power of a grating is given by, $\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} = \frac{nN'}{\cos\theta}$; and the resolving power of a grating is given by, $\frac{\lambda}{d\lambda} = nN$, where n is the order of the spectrum and N is the total number of lines on the grating surface. N' is the number of lines per cm on the grating surface. Here t) gives the direction of the $\mathbf{n^{th}}$ principal maximum corresponding to a wavelength λ. From the above equation, it is clear that the dispersive power increases with increase in the number of lines per cm and the resolving power increases with increases in the total number of lines on the grating surface (i.e. the width of the grating surface).

High dispersive power refers to wide separation of the spectral lines whereas high resolving power refers to the ability of the instrument to show nearby spectral lines as separate ones.

Michelson`s Stellar Interferometer:

The smallest angular separation (Θ) between two distant point sources for viewing the two images of the sources as separate with a telescope, is given by ,

$$\theta = \frac{1.22\,\lambda}{D} \tag{22}$$

where λ is the wavelength of light and D is the diameter of the objective of the telescope. Let the telescope objective be covered with a screen which is pierced with two parallel slits. Let the slit separation (d) be almost equal to the diameter of the telescope;
a suitable value for $d = \frac{D}{1.22}$. Now let the telescope be directed towards a distant double star so that the line joining the two stars is perpendicular to the length of either slit. Interference fringes due to the double slit will be observed in the focal plane of the objective. The condition for the first appearance of fringes is given by,

$$\alpha = \frac{\lambda}{2d} = \frac{1.22\,\lambda}{2D} = \frac{\theta}{2} \tag{23}$$

where α is the angular separation between the two stars when the first disappearance of the fringes takes place. Similarly, for values of a given by the multiples of $\frac{\lambda}{2d}$, disappearance of the fringes can be observed. If the double slit is avoided and the observations are made directly, the multiples can be ruled out. The angular separation α is half the angle Θ, where Θ is the minimum angle of resolution of the telescope objective.

The method employing the double slit interference is used to measure the angular separation between two stars. Michelson in 1920 successfully used this method to find the diameters of stars. The arrangement is known as Michelson's stellar interferometer (Fig. 12).

It consists of four mirrors M_1 M_2 M_3 and M_4 arranged as shown in the figure. L is the objective of the telescope and the two slits are kept in the paths of light reflected from the mirrors M_3 and M_4 Let S_1 and S_2 be the ends of a diameter of the star. The paths of the rays of light from these two points S_1 and S, are shown in the figure. The mirrors M_1 and M_3 are parallel. The mirrors M_1 and M_2 are mounted on a girder and by sliding these mirrors, the distance D between the mirrors can be altered. The silvered faces of M_1 and M_3 (and M_3 and M_4) face each other. Interference fringes will be observed in the field of view of the telescope. The path difference between the rays of light from to L and M_2 to L is zero.

In the side figure, A is the point of incidence of the rays of light on the mirror M_2 and B is the point of incidence of the rays of light on the mirror M_1. The path difference between the rays traveling from S_2 (one end of the diameter of the disc of the star) and reaching A and B is equal to the distance BC.

From the ΔABC, $\theta = \frac{BC}{D}$ or $BC = D\theta$.

For the first disappearance of the fringes, this path difference must be equal to 1.22 λ.

or $$D\theta = 1.22\,\lambda$$

or $$\theta = \frac{1.22\,\lambda}{D} \qquad (24)$$

In equation (24), θ measures the angular diameter of the star. In one of the experiments of Michelson, using a 250 cm reflecting telescope at Mount Wilson observatory, the disappearance of the fringes was observed when the distance between the mirrors M_1 and M_2 was 306.5 cm. If the average wavelength of light from the star is assumed to be 5750 **A,** the angular diameter of the star can be calculated from the equation

$\theta = 1.22\lambda/D$

$\lambda = 5750$ Å =

$\theta = \frac{1.22 \times 575}{306}$

$= 0.04718$ s

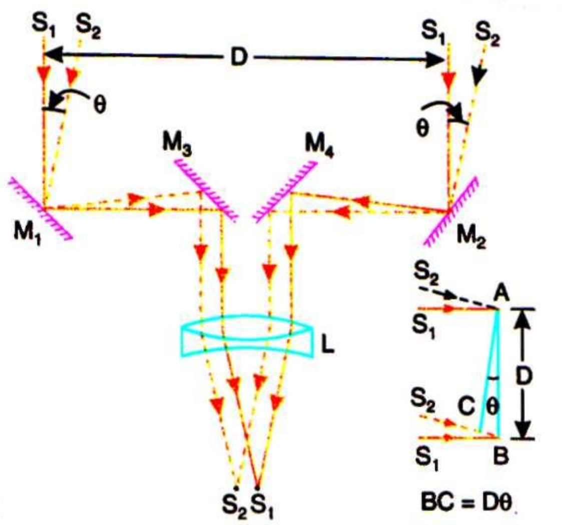


Fig. 12

WORKED OUT PROBLEMS

Example 1: Find the separation of two points on the moon that can be resolved by a 500 cm telescope. The distance of the moon is 3.8 x 10^{s} km. The eye is most sensitive to light of wavelength 5500 **A.**

Solution: The limit of resolution of a telescope is given by $d\theta = \frac{1.22\,\lambda}{a}$

Here $\lambda = 5500\ \text{Å} = 5500 \times 10^{-8}$ cm, a = 500 cm

$$\therefore \quad d\theta = \frac{1.2 \times 5500 \times 10^{-8}\ \text{cm}}{500\ \text{cm}}$$

$$\therefore \quad d\theta = 13.42 \times 10^{-8}$$

Let the distance between the two points be x.

$$\therefore \quad d\theta = \frac{x}{R}$$

Here $R = 3.8 \times 10^{10}$ cm

$$x = R.\,d\theta = 3.8 \times 10^{10}\ \text{cm} \times 13.42 \times 10^{-8}$$

$$= \mathbf{51.0\ m.}$$

Example 2: Two pin holes 1.5 mm apart are placed in front of a source of light of wavelength 5.5 x 10^{-5} cm and seen through a telescope with its objective stopped down to a diameter of 0.4 cm. Find the maximum distance from the telescope at which the pin holes can be resolved.

Solution: Here, $\lambda = 5.5 \times 10^{-5}$ cm; a = 0.4 cm; x = 1.5 mm = 0.15 cm

Now, $$d\theta = \frac{1.22\,\lambda}{a}$$

$$d\theta = \frac{x}{d} \quad \therefore \quad d = \frac{x\,a}{1.22\,\lambda} = \frac{0.15 \times 0.4}{1.22 \times 5.5 \times 10^{-5}}\ \text{cm} = 894.2\ \text{cm}$$

$$= \mathbf{8.9\ m}$$

Example 3: Calculate the useful magnifying power of a telescope of 10 cm objective, assuming that the limit of resolution of the eye is 2 minutes of an arc. Wavelength of light used is 6000 **A°.**

Solution: Here diameter of the objective D = 10 cm, $\lambda = 6000\ \text{Å} = 6 \times 10^{-5}$ cm.

Limit of resolution of the telescope, $d\theta = \frac{1.22\,\lambda}{D}$

$$= \frac{1.22 \times 6 \times 10^{-5}\ \text{cm}}{10} = 7.32 \times 10^{-6}\ \text{radian}$$

Limit of resolution of the eye, $d\theta'$ = 2 minutes of an arc

$$= \frac{2}{60} \times \frac{22}{7 \times 180} = 582 \times 10^{-6}\ \text{radian}$$

Useful magnifying power of the telescope $= \frac{d\theta}{d\theta'} = \frac{582 \times 10^{-6}}{7.32 \times 10^{-6}} = \mathbf{79.5}$

Example 4: Sodium light of wavelength 5890 A is used to view an object under a microscope. The aperture of the objective has a diameter of 0.9 cm.

(a) Calculate the limiting angle of resolution

(b) Using violet light, what is the maximum limit of resolution for this

Solution: (*a*) Limiting angle of resolution, $\theta_m = 1.22\left(\frac{\lambda}{d}\right)$

Here, $\lambda = 5890$ Å $= 5.89 \times 10^{-7}$ m, $d = 0.9$ cm $= 9 \times 10^{-3}$ m

$$\theta_m = 1.22\left(\frac{5.89 \times 10^{-7}}{9 \times 10^{-3}}\right) = \mathbf{7.98 \times 10^{-5}\ radian}$$

(*b*) The wavelength of violet light is 4000 Å.

The maximum limit of resolution for the microscope corresponds to the smallest angle.

Here, $\lambda = 4000$ Å $= 4 \times 10^{-7}$ m

$$\theta_m = 1.22\left(\frac{\lambda}{d}\right)$$

$$\therefore \quad \theta_m = 1.22\left(\frac{4 \times 10^{-7}}{9 \times 10^{-3}}\right) = \mathbf{5.42 \times 10^{-5}\ radian}$$

Example 5: Calculate the minimum thickness of the base of a prism which will just resolve the D_1 and D_2 lines of sodium. The refractive index of glass is 1.6545 for λ = **6563 A**$^\circ$ and 1.6635 for λ = 5270**A**$^\circ$**.**

Solution: Resolving power of the prism $= \frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}$

$$\frac{d\mu}{d\lambda} = \left[\frac{1.6635 - 1.6545}{(6563 - 5270) \times 10^{-10}\text{m}}\right]$$

and $$\frac{\lambda}{d\lambda} = \frac{5893}{6}$$

now $$t = \frac{\lambda/d\lambda}{d\mu/d\lambda}$$

$$\therefore \quad t = \frac{5893 \times 1293 \times 10^{-10}\text{ m}}{6 \times 0.0090} = \mathbf{14\ mm}$$

Example 6: Light is incident normally on a grating of total ruled width 5 x 10^{-3} m with 2500 lines in all. Calculate the angular separation of the two sodium lines in the first order spectrum. Can they be seen distinctly?

Solution: (*i*) Here, $N = 2500$, width of ruling $= 5 \times 10^{-3}$ m, $n = 1$,

$\lambda_1 = 5890 \times 10^{-10}$ m, $\quad \lambda_2 = 5896 \times 10^{-10}$ m

Number of lines per meter $= \frac{2500}{5 \times 10^{-3}} = 5 \times 10^5$

$$\therefore \quad (a+b) = \frac{1}{5 \times 10^5} = 2 \times 10^{-6}\text{ m}$$

For the first order ($n = 1$),

$$\sin\theta_1 = \frac{n\lambda_1}{(a+b)} = \frac{\lambda_1}{(a+b)} = \frac{5890 \times 10^{-10}\text{ cm}}{2 \times 10^{-6}\text{cm}} = 0.2945$$

$$\therefore \quad \theta_1 = 17°8'$$

Home work:

1. Calculate the resolving power of a Telescope whose objective lens has a diameter of 508 cm and λ=6000 A˚.
2. Find the number of lines a grating should have to resolve the second order doublet having wavelength difference 6 x 10^{-10} at 5893 x 10^{-10} m
3. Find the limit of resolution of a Laboratory Microscope having numerical aperture 1.2 when used for light of wavelength λ = 6 x 10^{7} m.
4. Calculate the aperture of the objective of a telescope which can resolve two stars separated by an angular distance of 4.84 x 10^{6} m. radians. The wavelength of light is 5000 A˚.
5. Calculate the number of lines that a grating must have to resolve D_1 and D_2lines of sodium in the second order. (λ_1= 5890 **A** and λ_2 = 5896 A).
6. A telescope of aperture 3 cm. is focused on a window 80 m away fitted with a wire mesh of spacing 2 mm. Will the telescope able to observe the wire- mesh?
7. Calculate the minimum spectral width for two wavelengths which can just be resolved in first order with a grating having 1100 lines when a light of wavelength 680 mm is used.