Chapter Two

Waves

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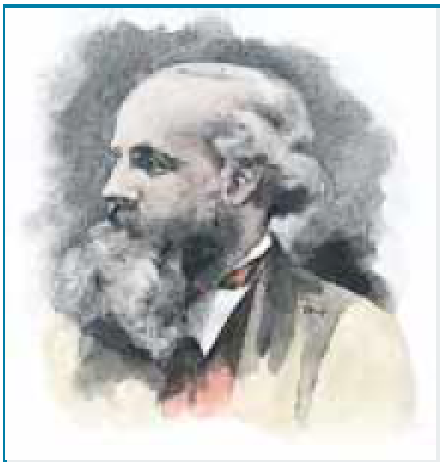


James Clerk Maxwell

Scottish Theoretical Physicist (1831–1879)

Maxwell developed the electromagnetic theory of light and the kinetic theory of gases, and explained the nature of Saturn's rings and color vision. Maxwell's successful interpretation of the electromagnetic field resulted in the field equations that bear his name. Formidable mathematical ability combined with great insight enabled him to lead the way in the study of electromagnetism and kinetic theory. He died of cancer before he was 50. (*North Wind Picture Archives*)

2.1 Vibrations and Waves

In our world of macroscopic bodies, water waves and sound waves are produced by moving masses of considerable size. Earthquakes produce waves as the result of sudden shifts in land masses. Water waves are produced by the wind or ships as they pass by. Sound waves are the result of quick movements of objects in the air. Any motion that repeats itself in equal intervals of time is periodic motion. The swinging of a clock pendulum, the vibrations of the prongs of a tuning fork, and a mass dancing from the lower end of a coiled spring are but three examples. These particular motions and many others like them that occur in nature are referred to as simple harmonic motion (SHM).

2.2 Waves

when disturbance passes through a medium, a series of point are affected. a local displacement fro equilibrium caused in one part of the medium is transmitted successively to next by interaction among particles, and such displacement together make up a wave. simple harmonic vibration of particles in the medium generates a simple harmonic wave. a **wave** ***is any disturbance , which travels through the medium due to the repeated periodic motion of the particles (of the medium) about their mean position.***

2.3 Transverse Waves

All light waves are classified as transverse waves. **Transverse waves** ***are those in which each small part of the wave vibrates along a line perpendicular to the direction of propagation and all parts are vibrating in the same plane.***

A wave machine for demonstrating transverse waves is shown in Fig. 1. When the handle His turned clockwise the small white balls at the top of the vertical rods move up and down with SHM. As each ball moves along a vertical line, the wave form ABCDEFG moves to the right. When the handle is turned counterclockwise, the wave form moves to the left. In either case each ball performs the exact same motion along its line of vibration, the difference being that each ball is slightly behind or ahead of its neighbor.

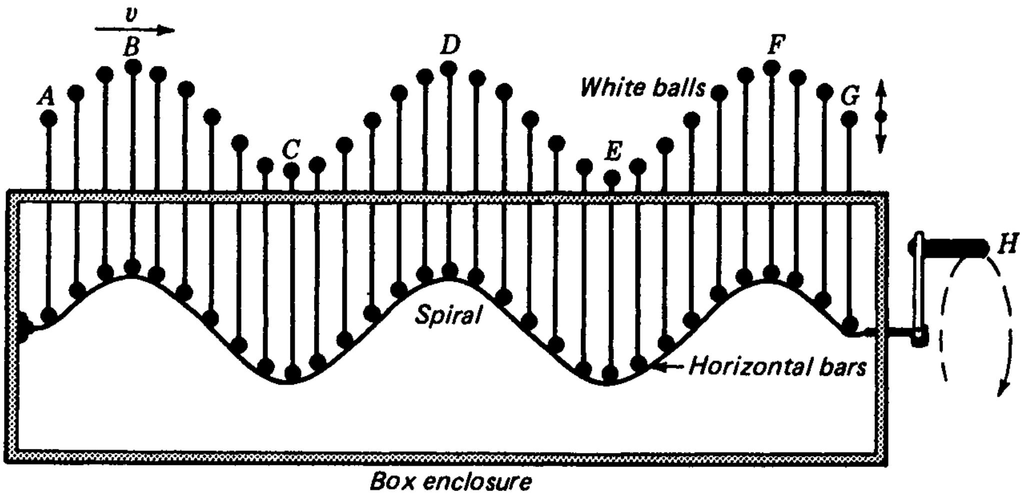
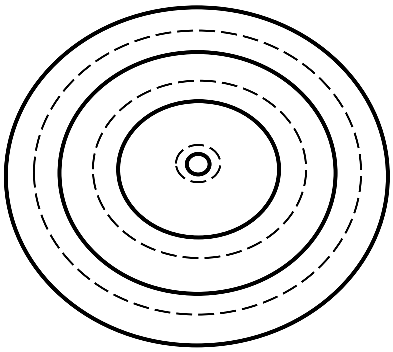


FIGURE 1: Machine for demonstrating transverse waves.

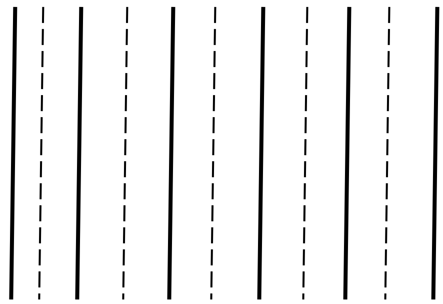
2.4 Wave Front and The Ray

wave front is defined as the locus of points, all of which are in the same phase. The electromagnetic waves radiated by a point

light source may be represented by spherical surface concentric with the source as in figure



At a sufficiently great distance from the source, where the radius of the spheres have become very large, the spherical surface can be considered plan and we have a train of plane wave as in figure.



a train of light waves may often be represented more simply by means of rays than by wave fronts.

2.5 Examples of Waves

Waves cane be classified according to the source that generates them:

1. **Mechanical waves:** mechanical waves or elastic waves are governed by Newton’s laws and require a material medium for their propagation. sound waves, seismic waves, water waves in bodies of water such as ocean, river, and ponds are examples of mechanical waves.
2. **Electromagnetic waves:** Visible light, radio waves, microwaves, x-rays and γ- rays belong to this category. Electromagnetic waves consist of oscillating electric and magnetic field and do not require a material medium for their propagation. They all travel in free space with same speed (c).

3. **Matter waves:** Atomic particles exhibit wave properties under certain condition. The laws of quantum mechanics govern such matter waves.
4. **Gravitational waves:** It is suggested that the cosmic bodies such as galaxies, stars produce gravitational waves and interact with each other through these waves. The gravitational waves are believed to with the velocity of light.

2.6 The Spectrum of Electromagnetic Waves

The various types of electromagnetic waves are listed in Figure 1, which shows the **electromagnetic spectrum**. Note the wide ranges of frequencies and wavelengths. No sharp dividing point exists between one type of wave and the next. Remember that **all forms of the various types of radiation are produced by the same phenomenon— accelerating charges**. The names given to the types of waves are simply for convenience in describing the region of the spectrum in which they lie.

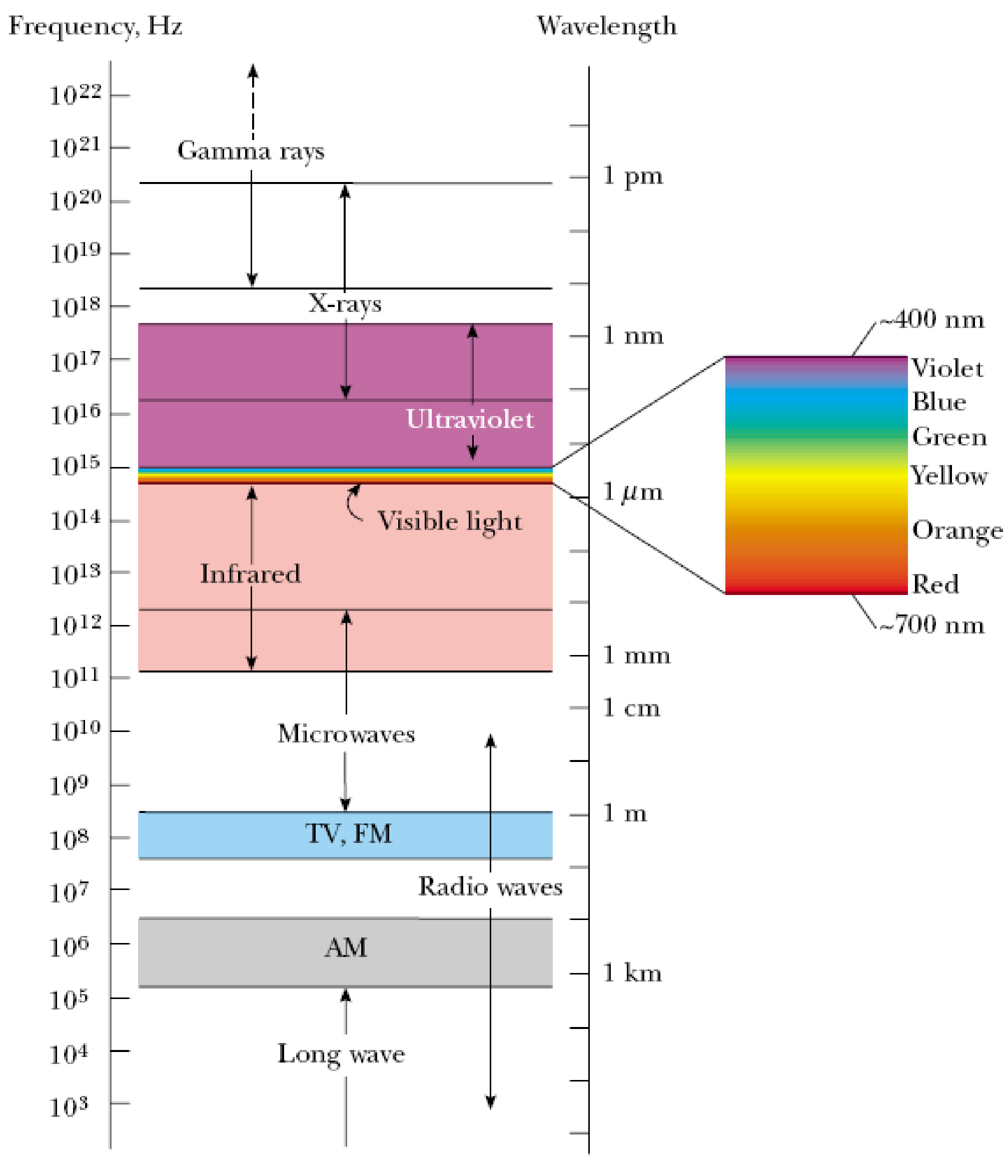


Figure 1: The electromagnetic spectrum. Note the overlap between adjacent wave types. The expanded view to the right shows details of the visible spectrum.

Radio waves, whose wavelengths range from more than 10^4 m to about 0.1 m, are the result of charges accelerating through conducting wires. They are generated by such electronic devices as *LC* oscillators and are used in radio and television communication systems.

Microwaves have wavelengths ranging from approximately 0.3 m to 10^{-4} m and are also generated by electronic devices. Because of their short wavelengths, they are well suited for radar systems and for studying the atomic and molecular properties of matter. Microwave ovens are an interesting domestic application of these waves. It has been suggested that solar energy could be harnessed by beaming microwaves to the Earth from a solar collector in space.

Infrared waves have wavelengths ranging from approximately 10^{-3} m to the longest wavelength of visible light, 7×10^{-7} m. These waves, produced by molecules and room-temperature objects, are readily absorbed by most materials. The infrared(IR) energy absorbed by a substance appears as internal energy because the energy agitates the atoms of the object, increasing their vibrational or translational motion, which results in a temperature increase. Infrared radiation has practical and scientific applications in many areas, including physical therapy, IR photography, and vibrational spectroscopy.

Visible light, the most familiar form of electromagnetic waves, is the part of the electromagnetic spectrum that the human eye can detect. Light is produced by the rearrangement of electrons in atoms and molecules. The various wavelengths of visible light, which correspond to different colors, range from red ($\lambda = 7 \times 10^{-7}$ m) to violet ($\lambda = 4 \times 10^{-7}$ m). The sensitivity of the human eye is a function of wavelength, being a maximum at a wavelength of about 5.5×10^{-7} m. With this in mind, why do you suppose tennis balls often have a yellow-green color?

Ultraviolet waves cover wavelengths ranging from approximately 4×10^{-7} m to 6×10^{-10} m. The Sun is an important source of ultraviolet (UV) light, which is the main cause of sunburn. Sunscreen lotions are transparent to visible light but absorb most UV light. The higher a sunscreen's solar protection factor (SPF), the greater the percentage of

UV light absorbed. Ultraviolet rays have also been implicated in the formation of cataracts, a clouding of the lens inside the eye.

Most of the UV light from the Sun is absorbed by ozone (O_3) molecules in the Earth's upper atmosphere, in a layer called the stratosphere. This ozone shield converts lethal high-energy UV radiation to infrared radiation, which in turn warms the stratosphere. Recently, a great deal of controversy has arisen concerning the possible depletion of the protective ozone layer as a result of the chemicals emitted from aerosol spray cans and used as refrigerants.

X-rays have wavelengths in the range from approximately 10^{-8} m to 10^{-12} m. The most common source of x-rays is the stopping of high-energy electrons upon bombarding a metal target. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because x-rays damage or destroy living tissues and organisms, care must be taken to avoid unnecessary exposure or overexposure. X-rays are also used in the study of crystal structure because x-ray wavelengths are comparable to the atomic separation distances in solids (about 0.1 nm).

Gamma rays are electromagnetic waves emitted by radioactive nuclei (such as ^{60}Co and ^{137}Cs) and during certain nuclear reactions. High-energy gamma rays are a component of cosmic rays that enter the Earth's atmosphere from space. They have wavelengths ranging from approximately 10^{-10} m to less than 10^{-14} m. They are highly penetrating and produce serious damage when absorbed by living tissues. Consequently، those working near such dangerous radiation must be protected with heavily absorbing materials, such as thick layers of lead.

2.7 Characteristics of a Waves

Any waves is characterized by the following parameters:

1. **Time period *T*:** the period is the time interval required for two identical points (such as the crests) of adjacent waves to pass by a point.
2. **wavelength λ:** the wavelength is the minimum distance between any two identical points (such as the crests) on adjacent waves, as shown in Figure 2a.
3. **frequency ν:** the frequency of a periodic wave is the number of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval.

$$\nu = \frac{1}{\mathrm{T}}$$

4. **Amplitude A:** The maximum displacement in a waveform is known as the amplitude.
5. **Velocity *υ***: Each time the source (of disturbance) vibrates one, the wave moves forward at distance (λ) . If there are (ν) vibrations in one second, the waves moves forward at a distance of. These distance that the wave moves in one second is the velocity of (ν λ)the wave. Thus,

$$\upsilon = \nu\lambda$$

$$\upsilon = \frac{\lambda}{\mathrm{T}}$$

6. **Phase angle Φ :** The extent of displacement of particles in the medium and direction of their displacement change from point to point along the wave. The quantity, which represents the

displacement , is called phase of the vibration, Φ . The phase may be expressed in terms of degrees or radians; or as the ratio of time t to the time period T. or as the ratio of the distance x to the wavelength, . The ratio t/T and x/λ are fractional numbers and have a maximum value of 1. When expressed in terms of radians (or degrees), the maximum value that phase can take is 2π radians (or 360).

7. **Intensity, I :** The energy transferred on an average by a wave in unit time, through a unit area perpendicular to is propagation direction, is known as the intensity of the wave. It is established that the intensity of a wave is directly proportional to the square of the amplitude of wave . Thus,

$$I \propto |A|^2$$

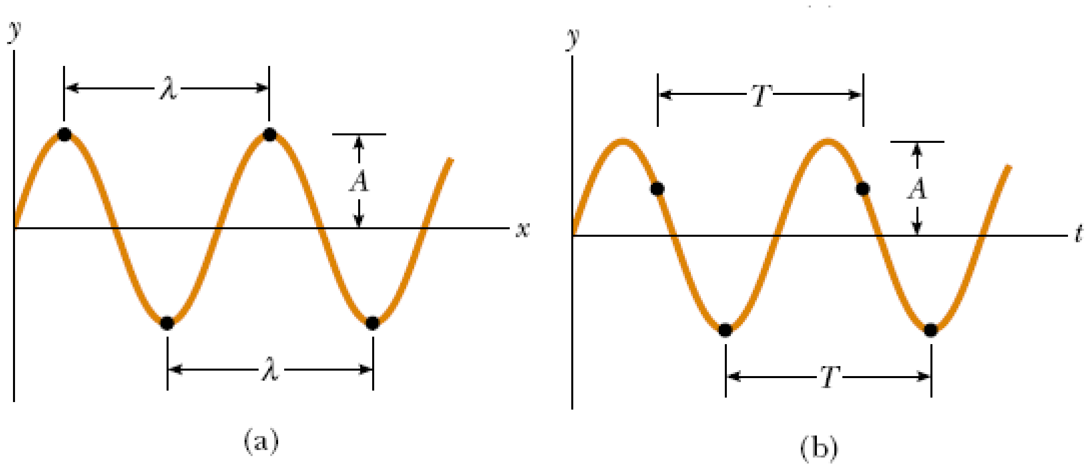


Figure 2: (a) The wavelength λ of a wave is the distance between adjacent crests or adjacent troughs. (b) The period *T* of a wave is the time interval required for the wave to travel one wavelength.

2.8 Mathematical Representation of Travelling Waves

Consider a pulse traveling to the right on a long string, as shown in Figure 3. Figure 3a represents the shape and position of the pulse at time $t = 0$. At this time, the shape of the pulse, whatever it may be, can be represented by some mathematical function which we will write as $y(x, 0) = f(x)$. This function describes the transverse position y of the element of the string located at each value of x at time $t = 0$. Because the speed of the pulse is v, the pulse has traveled to the right a distance vt at the time t(Fig. 3b). We assume that the shape of the pulse does not change with time. Thus, at time t, the shape of the pulse is the same as it was at time $t = 0$, as in Figure 3a.

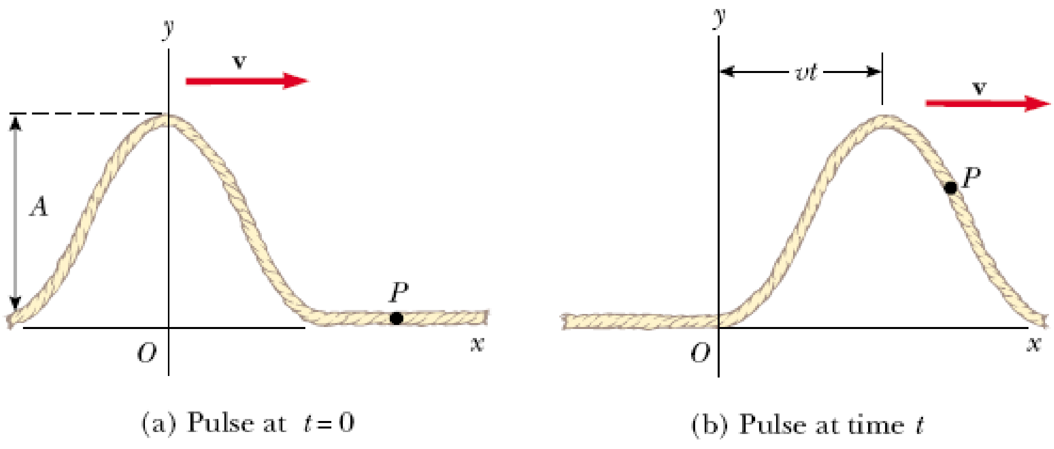


Figure 3 A one-dimensional pulse traveling to the right with a speed v. (a) At $t = 0$, the shape of the pulse is given by $y = f(x)$. (b) At some later time t, the shape remains unchanged and the vertical position of an element of the medium any point P is given by $y = f(x - vt)$.

Consequently, an element of the string at x at this time has the same y position as an element located at $x - vt$ had at time $t = 0$:

$$y(x, t) = y(x - vt, 0)$$

In general, then, we can represent the transverse position y for all positions and times, measured in a stationary frame with the origin at O, as

$$y(x, t) = y(x - vt) \qquad (2.1)$$

Similarly, if the pulse travels to the left, the transverse positions of elements of the string are described by

$$y(x, t) = y(x + vt) \qquad (2.2)$$

The function y, sometimes called the **wave function**, depends on the two variables x and t. For this reason, it is often written $y(x, t)$, which is read "y as a function of x and t."

It is important to understand the meaning of y. Consider an element of the string at point P, identified by a particular value of its x coordinate. As the pulse passes through P, the y coordinate of this element increases, reaches a maximum, and then decreases to zero. **The wave function y(x, t) represents the y coordinate—the transverse position—of any element located at position x at any time t**. Furthermore, if t is fixed (as, for example, in the case of taking a snapshot of the pulse), then the wave function $y(x)$, sometimes called the **waveform**, defines a curve representing the actual geometric shape of the pulse at that time.

Consider the sinusoidal wave in Figure 2a, which shows the position of the wave at $t = 0$. Because the wave is sinusoidal, we expect the wave function at this instant to be expressed as $y(x, 0) = A \sin ax$, where A is the amplitude and a is a constant to be determined. At $x = 0$, we see that $y(0, 0) = A \sin a(0) = 0$, consistent with Figure2a. The next value of x for which y is zero is $x = \lambda/2$. Thus,

$$y(\lambda/2, 0) = A \sin a(\lambda/2) = 0$$

For this to be true, we must have $a(\lambda/2) = \pi$, or $a = 2\pi / \lambda$. Thus, the function describing the positions of the elements of the medium through which the sinusoidal wave is traveling can be written

$$y(x, 0) = A \sin (2\pi / \lambda\, x) \qquad (2.3)$$

where the constant A represents the wave amplitude and the constant λ is the wavelength. We see that the vertical position of an element of the medium is the same whenever x is increased by an integral multiple of λ. If the wave moves to the right with a speed v, then the wave function at some later time t is

$$y(x,t) = A \sin\left[\frac{2\pi}{\lambda} \quad (x - vt)\right] \qquad (2.4)$$

If the wave were traveling to the left, the quantity $x - vt$ would be replaced by $x + vt$, as we learned when we developed Equations (2.1) and (2.2).

By definition, the wave travels a distance of one wavelength in one period T. Therefore, the wave speed, wavelength, and period are related by the expression

$$\upsilon = \frac{\lambda}{T}$$

Substituting this expression for v into Equation 4, we find that

$$y = A \sin\left[2\pi \quad \left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \qquad (2.5)$$

This form of the wave function shows the *periodic* nature of y. (We will often use y rather than $y(x, t)$ as a shorthand notation.) At any given time t, y has the *same* value at the positions x, $x + \lambda$, $x + 2\lambda$, and so on.

Furthermore, at any given position *x*, the value of *y* is the same at times *t*, *t* + *T*, *t* + 2*T*, and so on.

We can express the wave function in a convenient form by defining two other quantities, the angular **wave number** *k* (usually called simply the wave number) and the **angular frequency** *w*:

$$k = \frac{2\,\pi}{\lambda} \tag{2.6}$$

$$w = \frac{2\,\pi}{T} \tag{2.7}$$

Using these definitions, we see that Equation(2.5)can be written in the more compact form

$$y \;=\; A\sin(k\,x \;\; - w\,t) \tag{2.8}$$

Equation (2.8) represents a **progressive** or **travelling wave.**

1. The wave is said to be **monochromatic** because it has a single frequency, ν.
2. It is an undamped wave since its **amplitude A** is constant along the direction of propagation. It is a **plane wave,** since the amplitude is constant everywhere.
3. It represents a continuous train of wave stretching from *x* = -∞ to *x* = +∞. The disturbance is sinusoidal and continues forever.
4. It is a mathematically **idealized wave.** Such ideal waves **do not occur** in nature.

For many purposes the light disturbance at any point can be represented by the single scalar quantity "*y*". It is assumed that the variation of *y* are propagated in the form of a wave motion, and equation (2.8) represents the light wave.

2.9 General Wave Equation

To know how the displacement *y* varies as a function of space, *x* and time, *t* we have to do partial differentiation of *y* with respect to *x* and *t*.

$$\frac{\partial y}{\partial x} = \frac{2\pi}{\lambda}\cos\left[\frac{2\pi}{\lambda}(x - vt)\right] \qquad (2.9)$$

$$\frac{\partial y}{\partial t} = -\frac{2\pi v}{\lambda}\cos\left[\frac{2\pi}{\lambda}(x - vt)\right] \qquad (2.10)$$

Combining both these equations and eliminating equal factors, we get

$$\frac{\partial y}{\partial x} = -\frac{1}{v}\frac{\partial y}{\partial t} \qquad (2.11)$$

If we take second derivatives, it will hold for any sinusoidal wave, independent of the direction of travel, either –*x* or +*x*.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \text{ or } \frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2} \qquad (2.12)$$

We replace *y* by the more general term ξ, which stands for any disturbance.

$$\frac{\partial^2 \xi}{\partial t^2} = v^2\frac{\partial^2 \xi}{\partial x^2} \qquad (2.13)$$

This is one-dimensional wave equation. It connects the variations of ξ in space and time to the velocity of propagation of the wave.

If we are to include wave propagating in any direction, we need to extend the right hand term to the *y* and *z* axes, and replace it by

$$\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2}$$

Using the Laplacian operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, we can write the equation as

$$\frac{\partial^2 \xi}{\partial t^2} = v^2 \nabla^2 \xi \qquad (2.14)$$

This is general three-dimensional wave equation.

2.10 Phase Velocity

The wave function given by Equation (2.8) assumes that the vertical position y of an element of the medium is zero at $x = 0$ and $t = 0$. This need not be the case. If it is not, we generally express the wave function in the form

$$y = A \sin[(k\,x - w\,t) + \Phi] \qquad (2.15)$$

where Φ is the initial phase of the wave which is determined by our choice the beginning of counting x and t. Let us fix a value of the phase by assuming that

$$[(k\,x - w\,t) + \Phi] = \text{Constant} \qquad (2.16)$$

This expression determines the relation between the time t and the place where the phase has a fixed value. The value $\frac{dx}{\partial t}$ calculated from (2.16) gives the velocity with which the given value of the phase propagates.

$$k\frac{dx}{\partial t} - w = 0$$

$$\frac{dx}{\partial t} = \frac{w}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda = v \quad (2.17)$$

Thus, the velocity of wave of wave propagation " v" is the velocity of phase propagation and it is therefore called the **phase velocity.**

The **phase velocity** " v" of a wave is the velocity with which the wave front moves forward. It is the same as velocity of propagation of wave. When the waves are travelling through a non-dispersing medium, the common velocity of waves is the phase velocity.

2.11 Complex Representation of a Plane Wave

An expression similar to equation (2.8) can be written in terms of cosine as

$$y = A \cos(k x - w t) \quad (2.18)$$

We can express equation (2.8) and (2.18) in a single equation, using Euler's formula.

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$y = A e^{i(k x - w t)} \text{ or } y = A e^{-i(w t - k x)} \quad (2.19)$$

The advantage of the above complex representation as follows. The complex quantity used to represent the wave may be split into its space and time parts to give

$$y = A e^{-iw t} e^{ik x}$$

It is seen to consist complex amplitude $\psi = A e^{ik x}$ and a harmonic factor $e^{-iw t}$.

2.12 Wave Packets

A group of waves of finite length, such as that illustrated in Figure 4, is produced. The mathematical representation of a wave packet of this type is rather more complex. Since wave packets are of frequent occurrence, however, some features of their behavior should be mentioned here. In the first place, the wavelength is not well defined. If the packet is sent through any device for measuring wavelengths, e.g., light through a diffraction grating, it will be found to yield a continuous spread over a certain range $\Delta\lambda$. The maximum intensity will occur at the value of $\lambda_\circ$ indicated in Figure 4, but energy will appear in other wavelengths, the intensity dying off more or less rapidly on either side of $\lambda_\circ$. The larger the number N of waves in the group, the smaller the spread $\Delta\lambda$, and in fact theory shows that $\Delta\lambda / \lambda_\circ$ is approximately equal to l/N. Hence only when N is very large may we consider the wave to have an accurately defined wavelength.

$$\frac{1}{N} \approx \frac{\Delta\lambda}{\lambda_\circ} \qquad (2.20)$$

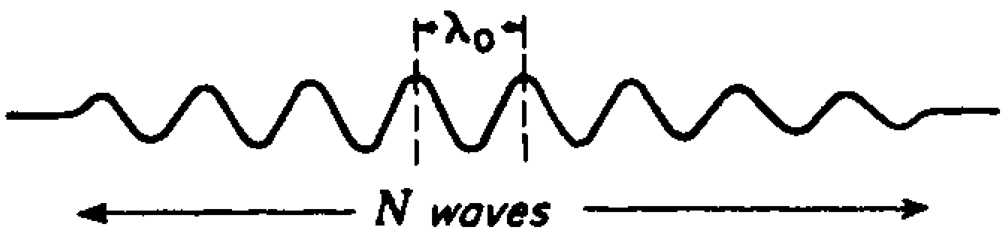


Figure 4: Example of a wave packet.

If the medium through which the packet travels is such that the velocity depends on frequency, two further phenomena will be observed. The individual wave crests will travel with a velocity different from that of the packet as a whole, and the packet will spread out as it progresses.

We then have two velocities, the wave (or phase) velocity and the group velocity. The relation between these will be derived in this Chapter.

In light sources, the radiating atoms emit wave trains of finite length. Usually, because of collisions or damping arising from other causes, these packets are very short. According to the theorem mentioned above, the consequence is that the spectrum lines will not be very narrow but will have an appreciable width $\Delta\lambda$. A measurement of this width will yield the effective "lifetime" of the electromagnetic oscillators in the atoms and the average length of the wave packets.

2.13 Group Velocity

The wave packet generally has the maximum amplitude at a particular value of *x* and **the velocity of this maximum amplitude point is called the group velocity** see figure 5. Thus, the velocity at which a wave packet(or a pulse) travels is the group velocity of the wave packet. This velocity also represents the velocity with which energy of the wave packet is transmitted.

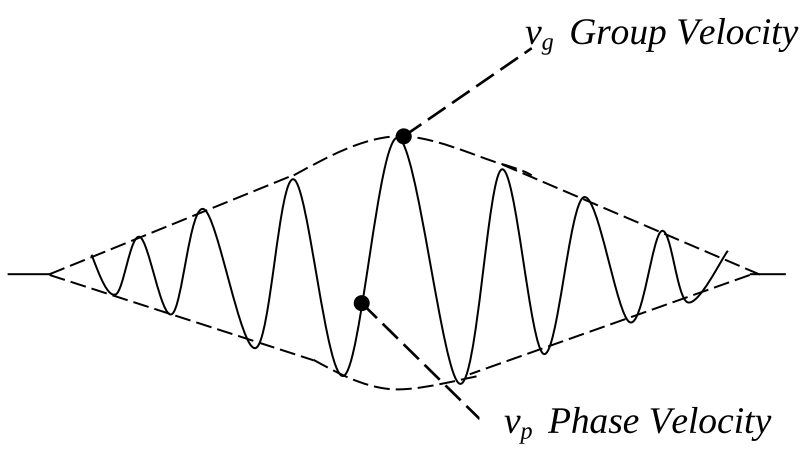


Figure 5: Group velocity

let each component wave in the wave packet has its own phase velocity, ($\upsilon=\nu\lambda$). The wave packet has amplitude that is large in a small region and very small outside it. The amplitude of the wave packet varies with x and t. Such variation of amplitude is called modulation of the wave. The velocity of propagation of the modulation is known as the group velocity, υ_g. It is given by

$$\upsilon_g = \frac{dw}{dk} \tag{2.21}$$

$$\upsilon_g = \frac{d(\upsilon k)}{dk} = \upsilon + k\frac{d\upsilon}{dk}$$

We further write

$$\frac{d\upsilon}{dk} = \frac{d\upsilon}{d\lambda}\frac{d\lambda}{dk}$$

$$\lambda = \frac{2\pi}{k}$$

Differentiating the above expression, we get

$$\frac{d\lambda}{dk} = -\frac{2\pi}{k^2} = -\frac{\lambda}{k}$$

$$k\frac{d\upsilon}{dk} = -\lambda\frac{d\upsilon}{d\lambda}$$

$$\upsilon_g = \upsilon - \lambda\frac{d\upsilon}{d\lambda} \tag{2.22}$$

This is the relation that connects phase velocity and group velocity.

2.14 Real Light Waves

It is now very easy to see why natural light behaves in different way from radio waves or other electromagnetic waves though it belongs to the same family of waves. We have been accustomed to regard light waves as ideal harmonic waves of infinite extension. Now we have to modify this visualization in view of discreteness in the emission process of light. We compare here the features of real light waves with those of ideal waves.

a) Real light waves are of limited extension:

Ideal waves are of infinite extension in both space and time are of constant amplitude.

Light emitted from common sources is in the form of wave train (or wave packet). The amplitude varies from one end of the wave packet to the other end. A jumble of such wave packets constitutes the real light wave.

b) Real light waves are not monochromatic:

Ideal waves are harmonic and posses a single frequency. Hence, they are strictly monochromatic.

In contrast, the wave train emitted by a light source are not harmonic but are pulses of short duration. Such non-harmonic waves may be regarded as arising due to the superposition of a series of harmonic waves having a range of frequencies $\Delta\nu$ centred about a central frequency $\nu_\circ$. The degree of monochromaticity of source is given by

$$\xi = \frac{\Delta\nu}{\nu_\circ}$$

Where $\Delta\nu$ is the band width. When $\Delta\nu/\nu_\circ = 0$ the radiation is ideally monochromatic. If $\Delta\nu/\nu_\circ \ll 1$, the radiation is **quasi-monochromatic.**

c) Real light waves are non-directional:

In case of real light waves, there is no definite direction of propagation as light is emitted randomly and in all possible directions. Therefore the light is divergent and is intensity diminishes at large distance from the source.

d) Real light waves are incoherent:

Coherence means the coordinated motion of several waves. When two or more waves are coherent, they will maintain a fixed and predictable phase relationship with each other. Monochromatic plane waves are ideally coherent.

As the emission acts occur without any coordination in the source, the resulting wave trains will not have any correlation in their phase of wave trains vary at random from one wave train to another wave train and fluctuate irregularly at a rate of about 10^8 times per second.

Consequently, the real light waves are incoherent.

e) Real light waves are unpolarized:

Light waves belong to the category of transverse waves. In ideal transverse waves, the vibrations are perpendicular to the direction of propagation and are confined to a plane perpendicular to it. Therefore. The waves are polarized.

In case of real light waves, each wave train taken alone is polarized. However, owing to the haphazardness in the acts of emission of wave trains by atoms, the different wave trains posses different orientations of planes of polarization. The radiation consists of wave trains with planes of vibration distributed in all possible directions about the direction of propagation. Therefore. The real light is highly disordered and unpolarized.