Introduction:

When waves encounter obstacles (or openings), they bend round the edges of the obstacles, if the dimensions of the obstacles are comparable to the wavelength of the waves. The bending of waves around the edges of an obstacle is called diffraction.

Fig. 1 illustrates the passage of waves through an opening. When the opening is large compared to the wavelength, the waves do not bend round the edges. When the opening is small, the bending round the edges is noticeable. When the opening is very small, the waves spread over the entire surface behind the opening. The opening acts an independent source of waves, which propagate in all directions. The diffraction effect is observable quite close to the opening when the size of the opening is very small. When the opening is large, diffraction effect is observed at greater distances from the opening. In general diffraction of waves becomes noticeable only when the size of the obstacle is comparable to a wavelength.
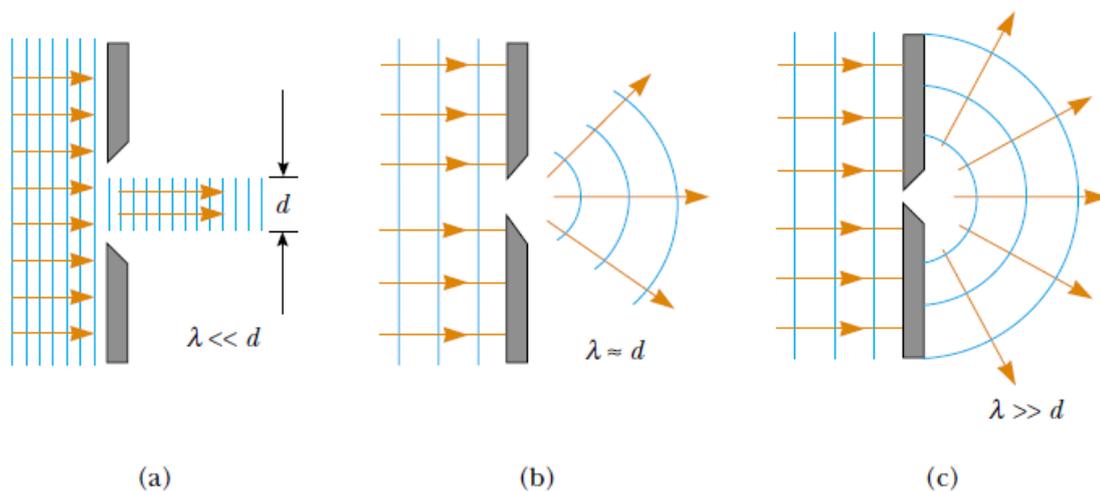

Figure 1: A plane wave of wavelength λ is incident on a barrier in which there is an opening of diameter *d*. (a) When λ << *d*, the rays continue in a straight-line path and the ray approximation remain valid. (b) When λ ≈ *d*, the rays spread out after passing through the opening. (c) When λ >> *d*, the opening behaves as a point source emitting spherical waves.

However, diffraction phenomenon is not readily apparent in case of light waves. It becomes significant when the aperture size is of the order of one wavelength wide. Diffraction and interference are basically equivalent.

Huygens-Fresnel Theory:

According to Huygen's wave theory of light, each progressive wave produces secondary waves, the envelope of which forms the secondary wave front. In figure 2(a). S is a source of monochromatic light and MN is a small aperture. XY is the

screen placed in the path of light. AB is the illuminated portion of the screen and above A and below B is the region of the geometrical shadow. Considering MN as the primary wave front. According to Huygen's construction, if secondary wave fronts are drawn, one would expect encroachment of light in the geometrical shadow. Thus, the shadows formed by small obstacles are not sharp. This bending of light round the edges of an obstacle or the encroachment of light within the geometrical shadow is known as diffraction. Similarly, if an opaque obstacle MN is placed in the path of light [Figure 2 (b)], there should be illumination in the geometrical shadow region AB also. But the illumination in the geometrical shadow of an obstacle is not commonly observed because the light sources are not point sources and secondly the obstacles used are of very large size compared to the wavelength of light. If a shadow of an obstacle is cast by an extended source, say a frosted electric bulb, light from every point on the surface of the bulb forms its own diffraction pattern (bright and dark diffraction bands) and these overlap such that no single pattern can be identified. The term diffraction is referred to such problems in which one considers the resultant effect produced by a limited portion of a wave front.
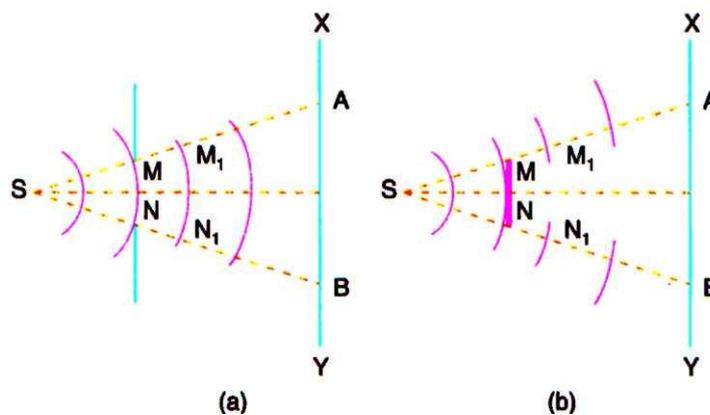

Figure 2

Diffraction phenomena are part of our common experience. The luminous border that surrounds the profile of a mountain just before the sun rises behind it, the light streaks that one sees while looking at a strong source of light with half shut eyes and the colored spectra (arranged in the form of a cross) that one sees while viewing a distant source of light through a fine piece of cloth are all examples of diffraction effects.

Augustine Jean Fresnel in 1815, combined in a striking manner Huygens' wavelets with the principle of interference and could satisfactorily explain the bending of light round obstacles and also the rectilinear propagation of light.

Fresnel's Assumptions:

According to Fresnel, the resultant effect at an external point due to a wave front will depend on the factors discussed below:

In figure 3, S is a point source of monochromatic light and MN is a small aperture. XY is the screen and SO is perpendicular to XY. MCN is the incident spherical wave front due to the point source S. To obtain the resultant effect at a point P on the screen, Fresnel assumed the following:
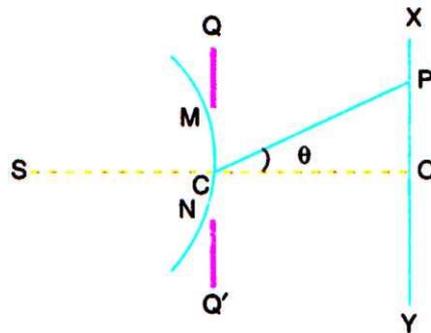

Figure 3

(1) A wave front can be divided into a large number of strips or zones called Fresnel's zones of small area and the resultant effect at any point will depend on the combined effect of all the secondary waves emanating from the various zones;

(2) The effect at a point due to any particular zone will depend on the distance of the point from the zone;

(3) The effect at P will also depend on the obliquity of the point with reference to the zone under consideration, e.g. due to the part of the wave front at C. the effect will be a maximum at O and decreases with increasing obliquity. It is a maximum in a direction radially outwards from C and it decreases in the opposite direction. The effect at a point due to the obliquity factor is proportional to $(1+ \cos\theta)$ where $\angle PCO = \theta$. considering an elementary wave front at C, the effect is maximum at O because $\theta = 0$ and $\cos\theta = 1$. Similarly, in a direction tangential to the primary wave front at C (along CQ) the resultant effect is one half of that along CO because $\theta = 90°$ and $\cos 90° = 0$. In the direction CS, the resultant effect is zero since $\theta = 180°$ and $\cos180° = -1$ and $I + \cos180° = 1-1 = 0$. This property of the secondary waves eliminates one of the difficulties experienced with the simpler form of Huygens principle viz... That if the secondary waves

spread out in all directions from each point on the primary wave front. They should give a wave traveling forward as well as backward. Now, as the amplitude at the rear of the wave is zero there will evidently be no back wave.

Rectilinear Propagation of Light:

ABCD is a plane wave front perpendicular to the plane of the paper [Figure 4 (a)] and P is an external point at a distance b perpendicular to ABCD. To find the resultant intensity at P due to the wave front ABCD, Fresnel's method consists in dividing the wave front into a number of half period elements or zones called Fresnel's zones and to find the effect of all the zones at the point P.

If spheres are constructed with P as centre and radii equal to $b+\lambda/2$. $b+2\lambda/2$. $b+3\lambda/2$etc.. they will cut out circular areas of radii OM_1, $OM2,OM_3$, etc.. on the wave front. These circular zones are called half period zones or half period elements. Each zone differs from its neighbour by a phase difference of π or path difference of $\lambda/2$. Thus the secondary waves starting from the point O and M_1 and reaching P will have a phase difference of π or a path difference $\lambda/2$. A Fresnel half period zone with respect to an actual point P is a thin annular zone of the primary wave front in which the secondary waves from any two corresponding points of neighboring zones differ in path by $\lambda/2$.
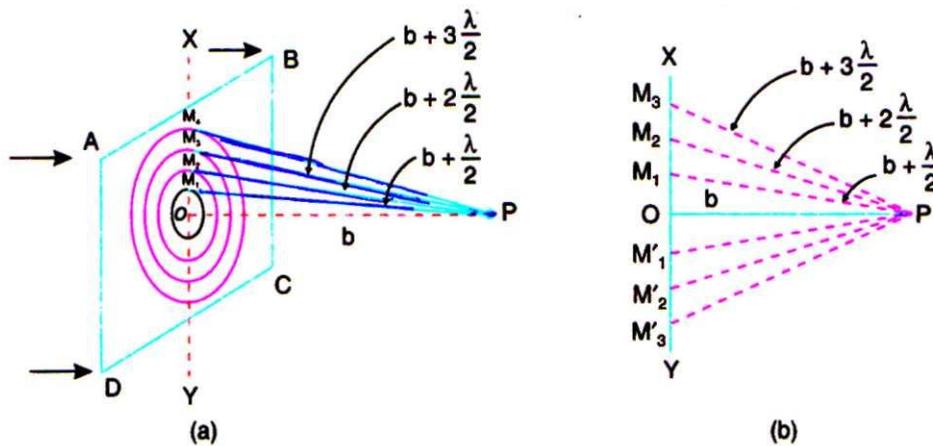

Figure (4)

In Fig. 4 (b) O is the pole of the wave front XY with reference to the external point P. OP is perpendicular to XY. In Fig. 4 (c) 1.2.3 etc are the half period zones constructed on the primary wave front XY. OM_1 is the radius of the first zone. OM_2 is the radius of the second zone and so on. P is the point at which the resultant intensity has to be calculated.
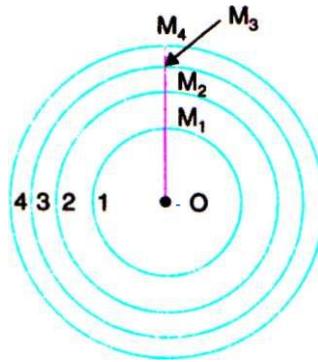
Figure (4)

$OP = b, OM_1 = r_1, OM_2 = r_2, OM_3 = r_3$, etc.

And

$$M_1P = b + \frac{\lambda}{2}, M_2P = b + \frac{2\lambda}{2}, M_3P = b + \frac{3\lambda}{2} \text{ etc.}$$

The area of the first half period zone is

$$\pi OM_1{}^2 = \pi[M_1P^2 - OP^2] = \pi\left[\left[b + \frac{\lambda}{2}\right]^2 - b^2\right] = \pi\left[b\lambda + \frac{\lambda^2}{4}\right] = \pi b\lambda \qquad (1)$$

As λ is small, λ^2 term is neglected.

The radius of the first half period zone is, $r_1 = OM_1 = \sqrt{b\lambda}$

The radius of the second half period zone is, $OM_2 = [M_2P^2 - OP^2]^{1/2}$

$$= [(b + \lambda)^2 - b^2]^{1/2}$$

$$= \sqrt{2b\lambda}$$

The area of the second half period zone, $= \pi[OM_2 - OM_1{}^2]$

$$= \pi[2b\lambda - b\lambda]$$

$$= \pi b\lambda \qquad (2)$$

Thus, the area of each half period zone is equal to $\pi b\lambda$. Also the radii of the 1st, 2nd, 3td, etc. half period zones are $\sqrt{1b\lambda}$, $\sqrt{2b\lambda}$, $\sqrt{3b\lambda}$ etc. Therefore, the radii are proportional to the square roots of the natural numbers. However, it should be remembered that the areas of the zones are not constant but are dependent on - (i) λ, the wavelength of light and (ii) b, the distance of the point from the wave front.

The area of the zone increases with increase in the wavelength of light and with increase in the distance of the point P from the wave front.

As discussed in Fresnel's Assumptions the effect at a point will depend on (i) the distance of P from the wave front, (ii) the area of the zone and (iii) the obliquity factor. Here, the area of each zone is the same. The secondary waves reaching the point P are continuously out of phase and in phase with reference to the central or the first half period zone. Let m_1, m_2, m_3, etc. represent the amplitudes of vibration of the ether particles at P due to secondary waves from the 1st, 2nd, 3^{td}, etc. half period zones (see Figure 5). As we consider the zones outwards from O, the obliquity increases and hence the quantities m_1, m_2, m_3, etc. are of continuously decreasing order. Thus, m_1 is slightly greater than m_2; m_2 is slightly greater than m_3 and so on. Due to the phase difference of π between any two consecutive zones, if the displacements of the ether particles due to odd numbered zones are in the positive direction, then due to the even numbered zones the displacement will be in the negative direction at the same instant. As the amplitudes are of gradually decreasing magnitude, the amplitude of vibration at P due to any zone can be approximately taken as the mean of the amplitudes due to the zones preceding and succeeding it.

$$\text{e. g.} \quad m_2 = \frac{m_1 + m_3}{2}$$

The resultant amplitude at P, at any instant is given by,

$A = m_1 - m_2 + m_3 - m_4 \dots. + m_n$ if n is odd.

(If n is even, the last quantity is $-m_n$).

$$\therefore A = \frac{m_1}{2} + \left[\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right] + \left[\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right] + \cdots$$

But $m_2 = \dfrac{m_1}{2} + \dfrac{m_3}{2}$ and $m_4 = \dfrac{m_3}{2} + \dfrac{m_5}{2}$

$$A = \frac{m_1}{2} + \frac{m_n}{2} \dots \dots \dots \dots \text{if n is odd.}$$

$$A = \frac{m_1}{2} + \frac{m_{n-1}}{2} - m_n \dots \dots \dots \dots \text{if n is even.}$$

If the whole wave front ABCD is unobstructed, the number of half period zones that can be constructed with reference to the point P is infinite i.e. $n \rightarrow \infty$. As the amplitudes are of gradually diminishing order, m_n and m_{n-1}, tend to be zero.
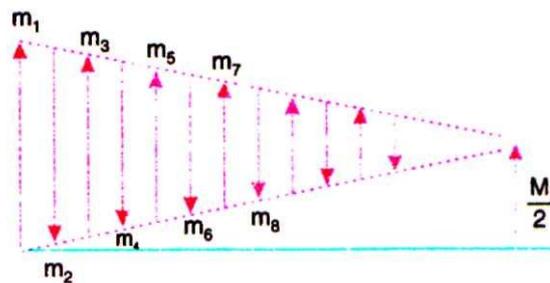
Therefore, the resultant amplitude at P due to the whole wave front $= A = \dfrac{m_1}{2}$ (3)

The intensity at a point is proportional to the square of the amplitude.

$$\therefore I \propto \frac{m_1^2}{4} \tag{4}$$

Thus, the intensity at P is only one-fourth of that due to the first half period zone alone. Here, only half the area of the first half period zone is effective in producing the illumination at the point P. A small obstacle of the size of half the area of the first half period zone placed at will screen the effect of the whole wave front and the intensity at P due to the rest of the wave front will be zero. While

considering the rectilinear propagation of light, the size of the obstacle used is far greater than the area of the first half period zone and hence the bending effect of light round corners (diffraction effects) cannot be noticed. In the case of sound waves, the wavelengths are far greater than the wavelength of light, and hence the area of the first half period zone for a plane wave front of sound is very large. If the effect of sound at a point beyond an obstacle is to be shadowed, an obstacle of very large size has to be used to get no sound effect. if the size of the obstacles placed in the path of light is comparable to the wavelength of light, then it is possible to observe illumination in the region of the geometrical shadow also. Thus, rectilinear propagation of light is only approximately true.

Figure 5

Zone Plate:

A zone plate is a specially constructed screen such that light is obstructed from every alternate zone. It can be designed so as to cut off light due to the even numbered zones or that due to the odd numbered zones. The correctness of Fresnel's method in dividing a wave front into half period zones can be verified with its help.

To construct a zone plate, concentric circles are drawn on white paper such that the radii are proportional to the square roots of the natural numbers [as shown in (Rectilinear Propagation of Light) the radii are proportional to the square roots of the natural numbers]. The odd numbered zones (i.e. 1st, 3^{td}, 5th etc.) are covered with black ink and a reduced photograph is taken. The drawing appears as shown in   figure 6 (b). The negative of the photograph will be as shown in figure 6 (a). In the developed negative, the odd zones are transparent to incident light and the even zones will cut off light.

(a)   Zone Plate   (b)
Figure 6

If such a plate is held perpendicular to an incident beam of light and a screen is moved on the other side to get the image, it will be observed that maximum brightness is possible at some position of the screen say b cm from the zone plate (Figure 7) XO is the upper half of the incident plane wave front. P is the point at which the light intensity is to be considered. The distance of the point P from the wave front is b. $OM_1 = r_1, OM_2 = r_2,$ etc. are the radii of the zones.

$r_1 = \sqrt{b\lambda}$ and $r_2 = \sqrt{2b\lambda}$ where λ is the wavelength of light.

$$r_n = \sqrt{nb\lambda} \quad \text{or} \quad b = \frac{r_n^2}{n\lambda} \tag{5}$$

If the source is at a large distance from the zone plate, a bright spot will be obtained at P. As the distance of the source is large, the incident wave front can be taken as a plane one with respect to the small area of the zone plate. The even numbered zones cut off the light and hence resultant amplitude at P = A = m₁ + m₂ + m₃ + etc. In this case the focal length of the zone plate f_n is given by

$$f_n = b = \frac{r_n^2}{n\lambda} \tag{6}$$

Thus, a zone plate has different foci for different wavelengths. The radius of the nth zone increases with increasing value of λ. It is very interesting to note that as the even numbered zones are opaque, the intensity at P is much greater than that when the whole wave front is exposed to the point P.

In the first case the resultant amplitude is given by

$A = m_1 + m_3 \ldots + m_n$ (n is odd)

When the whole wave front is unobstructed, the amplitude is given by
$A = m_1 - m_2 + m_3 - m_4 \ldots + m_n$
$= \frac{m_1}{2}$ (if n is very large and n odd).

If a parallel beam of white light is incident on the zone plate, different colours come to focus at different points along the line OP. Thus, the function of a zone plate is similar to that of a convex (converging) lens and a formula connecting the distance of the object and image points can be obtained for a zone plate also.
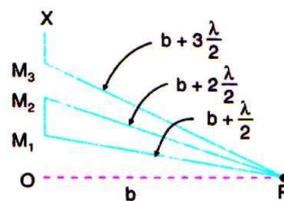

Figure 7

Action of a Zone Plate for an Incident Spherical Wave Front:

Let XY represent the section of the zone plate perpendicular to the plane of the paper. S is a point source of light. P is the position of the screen for a bright image, 'a' is the distance of the source from the zone plate and b is the distance of the screen from the plate. $OM_1, OM_2, OM_3, (r_1, r_2, r_3)$ etc. are the radii of 1st, 2nd, 3^{td}, etc. half period zones. The position of the screen is such that from one zone to the next there is an increasing path difference of $\frac{\lambda}{2}$.

Thus,

$$SO + OP = a + b$$
$$SM_1 + M_1P = a + b + \frac{\lambda}{2} \qquad (7)$$
$$SM_2 + M_2P = a + b + \frac{2\lambda}{2} \quad \text{and so on.}$$
From $\Delta\, SM_1O$
$$SM_1 = \left(SO^2 + OM_1{}^2\right)^{1/2}$$
$$= (a^2 + r_1{}^2)^{1/2}$$
Similarly from $\Delta\, OM_1P$
$$M_1P = \left(OP^2 + OM_1{}^2\right)^{1/2}$$
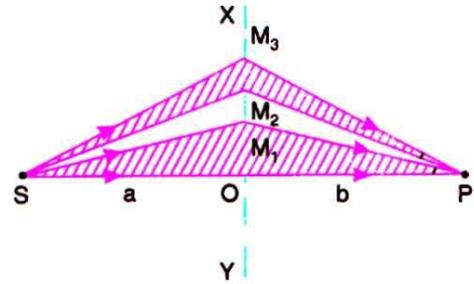$$= (b^2 + r_1{}^2)^{1/2}$$

Figure 8

Substituting the values of SM_1 and M_1P in equation (7), we get

$$(a^2 + r_1{}^2)^{1/2} + (b^2 + r_1{}^2)^{1/2} = a + b + \frac{\lambda}{2}$$

$$a\left[1 + \frac{r_1{}^2}{a^2}\right]^{1/2} + b\left[1 + \frac{r_1{}^2}{b^2}\right]^{1/2} = a + b + \frac{\lambda}{2}$$

$$a + \frac{r_1{}^2}{2a} + b + \frac{r_1{}^2}{2b} = a + b + \frac{\lambda}{2}$$

$$\frac{r_1{}^2}{2}\left[\frac{1}{a} + \frac{1}{b}\right] = \frac{\lambda}{2}$$

Similarly for r_n i.e. the radius of the nth zone, the relation can be written as

$$r_n{}^2\left[\frac{1}{a} + \frac{1}{b}\right] = n\lambda \qquad (8)$$

Applying the sign convention,

$$\frac{1}{a} - \frac{1}{b} = \frac{n\lambda}{r_n{}^2} = \frac{1}{f_n} \qquad (9)$$

or $\quad f_n = \dfrac{r_n{}^2}{n\lambda}$

Equation (9) is similar to the equation $\left[\dfrac{1}{v} - \dfrac{1}{u}\right] = \dfrac{1}{f}$ in the case of lenses with a and b as the object and image distances and f_n the focal length. Thus, a zone plate acts as a converging lens. A zone plate has a number of foci which depend on the number of zones used as well as the wavelength of light employed.

Difference Between a Zone Plate and a Convex Lens:

For a given wavelength of light, a convex lens has only one focal length given by

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Where f is the focal length of the lens, μ is the refractive index of the material of the lens and R_1 and R_2 are the radii of curvature. In a convex lens, the violet rays of light come to focus nearer the lens than the red rays of light because for a given material the refractive index for violet rays of light is more than for red rays of light.

In the case of a zone plate, there are a number of foci between the point O and P (Figure 8). Each focus corresponds to the position where, with reference to P an odd number of half period elements can be constructed on each zone. As the screen is moved nearer the zone plate, the area of the half period elements decreases and more half period elements can be present on each zone. If P_m is the position on the image when (2m - 1) half period elements can be present on each zone. f_m the focal length of the zone plate is given by

$$f_m = \frac{r_n{}^2}{(2m-1)n\lambda} \tag{10}$$

Putting m = 1, 2. 3... etc., the different positions of the screen for a bright image can be obtained. In equation (10), r_n is the radius of the nth zone of the wave front. λ is the wavelength of light and (2m - 1) is the number of odd half period elements present on each zone. For example, if the position of the screen is such that with reference to the point P. three half period elements can be constructed on each zone, then the focal length of the zone plate f_3 is given by

$$f_3 = \frac{r_n{}^2}{5n\lambda}$$

With the decrease in the focal length of the zone plate, the brightness of the image decreases. Let the first zone contain only one period element. Then, the amplitude at P due to this zone is my If the first zone contains three half period

elements for a particular position of the screen, then the amplitude at P due to the first zone

$$= m_1 - m_2 + m_3 = \frac{m_1}{2} + \left[\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right] + \frac{m_3}{2} = \frac{m_1}{2} + \frac{m_3}{2}$$

But, $\frac{m_1 + m_3}{2}$ is less than m_1, because $m_1 > m_3$. Further, in a zone plate (for the same number of odd half period elements contained in each zone) the focal length for violet light is more than for red light, which is reverse in the case of a convex lens.

Comparison between a Zone Plate and a Convex Lens:

1. Both the zone plate and convex lens form a real image of the object and the equations connecting the conjugate distances are similar.
2. The local lengths of both depend on the wavelength, λ and hence suffer from chromatic aberration. The chromatic aberration in a zone plate is much more severe than in a convex lens.
3. A zone plate acts simultaneously as a convex lens and as a concave lens. In addition to a real image, a virtual image is also formed simultaneously. A convex lens forms only a real image.
4. In case of zone plate the image is formed by the diffraction phenomenon. In case of a convex lens the image is formed due to refraction of light.
5. The zone plate has got multiple foci on either side of the plate. Hence, the intensity of the image formed will be much less. Convex lens has only one focus. As all the light is focused at one point, the intensity of the image will be more.
6. In a zone plate, waves reaching the image point through any two alternate zones differ in path by λ and in phase by 2π. In case of a convex lens all the rays reaching the image point have zero path or phase difference.
7. A zone plate can be used over a wide range of wavelengths from microwaves to x-rays. Glass lens cannot be used beyond the visible region.

Distinction between Interference and Diffraction:

The main differences between interference and diffraction are as follows:

INTERFERENCE	DIFFRACTION
1. Interference is the result of interaction of light coming from different wave fronts originating from the source.	1. Diffraction is the result of interaction of light coming from different parts of the same wave front.
2. Interference fringes may or may not be of the same width.	**2.** Diffraction fringes are not of the same width.
3. Regions of minimum intensity are perfectly dark.	3. Regions of minimum intensity are not perfectly dark.
4. All bright bands are of same intensity.	4. The different maxima are of varying intensities with maximum intensity for central maximum.

Fresnel and Fraunhoflfer types of Diffraction:

   The diffraction phenomena are broadly classified into two types: Fresnel diffraction and Fraunhoffer diffraction.

   1. *Fresnel diffraction:* In this type of diffraction, the source of light and the screen are effectively at finite distances from the obstacle (Figure 9a). Observation of Fresnel diffraction phenomenon does not require any lenses. The incident wave front is not planar. As a result, the phase of secondary wavelets is not the same at all points in the plane of the obstacle. The resultant amplitude at any point of the screen is obtained by the mutual interference of secondary wavelets from different elements of unblocked portions of wave front. It is experimentally simple but the analysis proves to be very complex.
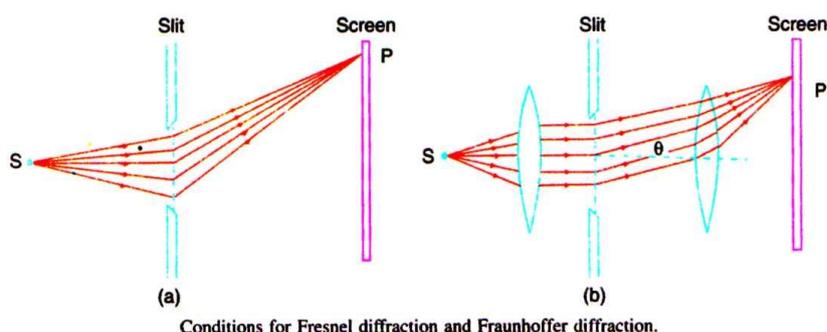

Conditions for Fresnel diffraction and Fraunhoffer diffraction.

Figure 9

2. Fraunhoffer diffraction: In this type of diffraction, the source of light and the screen are effectively at infinite distances from the obstacle. Fraunhoffer diffraction pattern can be easily observed in practice. The conditions required for Fraunhoffer diffraction are achieved using two convex lenses, one to make the light from the source parallel and the other to focus the light after diffraction on to the screen (Figure 9b). The diffraction is thus produced by the interference between parallel rays. The incident wave front as such is plane and the secondary wavelets, which originate from the unblocked portions of the wave front, are in the same phase at every point in the plane of the obstacle. This problem is simple to handle mathematically because the rays are parallel. The incoming light is rendered parallel with a lens and diffracted beam is focused on the screen with another lens.

Fresnel class of diffraction phenomenon is treated in this chapter.

Diffraction at a Circular Aperture

Let AB be a small aperture (say a pin hole) and S is a point source of monochromatic light. X Y is a screen perpendicular to the plane of the paper and P is a point on the screen. SP is perpendicular to the screen O is the center of the aperture and r is the radius of the aperture. Let the distance of a source from the aperture be a (SO = a) and the distance of the screen from me aperture be b (OP=b). QOQ is the incident spherical wave front and with reference to the point P, O is the pole of the wave front (Fig. 10). To consider the intensity at P, half period zones can be constructed with P as center and radii $b+\frac{\lambda}{2}, b+\frac{2\lambda}{2}$ etc. on the exposed wave front AOB. Depending on the distance of P from the aperture (i.e. the distance b) the number of half period zones that can be constructed may be odd or even. If the distance a is such that only one half period zone can be constructed, then the intensity at P will be proportional to m_1^2 (where m_1 is the amplitude due to the first zone at P). On the other hand, if the whole of wave front is exposed to the point P. On the other hand, if the whole of the wave front is exposed to the point P. the resultant amplitude is $\frac{m_1}{2}$ or the intensity at P will be proportional to $\frac{m_1^2}{4}$ the position of the screen can be altered so as to construct 2, 3 or more half period zones for the same area of the aperture. If only two zones are exposed, the resultant amplitude at P = $m_1 - m_2$ (minimum) and if 3 zones are exposed, the amplitude = $m_1 - m_2 + m_3$ (maximum) and so on. Thus by continuously altering the value of b, the point P becomes alternately bright and dark depending on whether odd or even number of zones are exposed by the aperture.
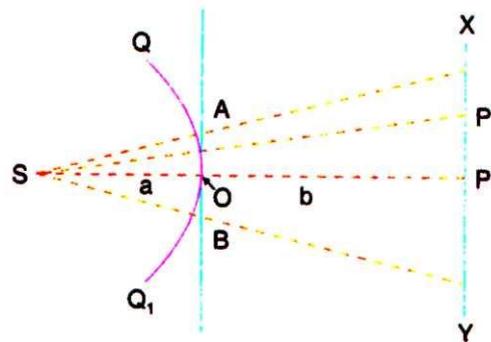
Figure 10

Now let us consider a point P' on the screen XY (Fig. 11). Let S to P' be joined. The line SP' meets the wave front at O' .O' is the pole of the wave front with reference to the point P'. Now let us construct half period zones with the point O' as the pole of the wave front. The upper half of the wave front is cut off by the obstacle. If the first two zones are cut off by the obstacle between the points O' and A X and if only the 3rd, 4th and 5th zones are exposed by the aperture AOB, then intensity at P' will be maximum. Thus if odd number of half period zones are exposed, point P' will be of maximum intensity and if even number of zones are exposed, point P' will be of minimum intensity. As the distance of P' from P increases the intensity of maxima and minima gradually decreases. It is because with the point P' far removed from P, the most effective central half period zones are cut off by the obstacle between the points O' and A. With the outer zones the obliquity increases with reference to the point P' and hence the intensity of maxima and minima also will be less. If the point P' happens to be of maximum intensity, then all the points lying on a circle of radius PP' on the screen also will be of maximum intensity. Thus with a circular aperture, the diffraction pattern will be concentric bright and dark rings with the center P bright or dark depending on the distance b. The width of the rings continuously decreases.
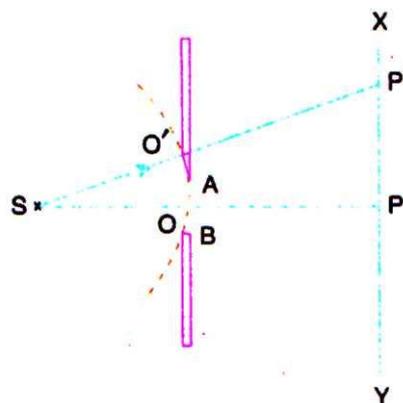

Figure 11

Mathematical Treatment of Diffraction at a Circular Aperture:

In Fig. 12 S is a point source of a monochromatic light, AB is the circular aperture and P is point on the screen. O is the center of the circular aperture. The line SOP is perpendicular to the circular aperture AB and the screen at P. The screen is perpendicular to the plane of the paper.
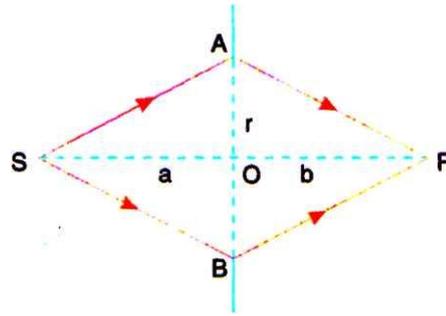

Figure 12

Let δ be the path difference for the wave reaching P along the paths SAP and SOP.

$$SO = a \; ; \; OP = b; \; OA = r$$
$$\delta = SA + AP - SOP$$

$$= \left(a^2 + r^2\right)^{1/2} + \left(b^2 + r^2\right)^{1/2} - (a+b) \qquad = a\left(1 + \frac{r^2}{a^2}\right)^{1/2} + b\left(1 + \frac{r^2}{b^2}\right)^{1/2} - (a+b)$$

$$= a\left(1 + \frac{r^2}{2a^2}\right) + b\left(1 + \frac{r^2}{2b^2}\right) - (a+b) \qquad = \frac{r^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$\therefore \qquad \frac{1}{a} + \frac{1}{b} = \frac{2\delta}{r^2} \tag{11a}$$

If the position of the screen is such that n full number of half period zones can be constructed on the aperture, then the path difference $\delta = \frac{n\lambda}{2}$ or $2\delta = n\lambda$, Substituting the value of 2δ in (11 a), we get,

$$\frac{1}{a} + \frac{1}{b} = \frac{n\lambda}{r^2} \tag{11b}$$

The point P will be of maximum or minimum intensity depending on whether n is odd or even. If the S source is at infinite distance (for an incident plane wave front), then $a = \infty$ and

$$\frac{1}{b} = \frac{1}{f} = \frac{n\lambda}{r^2} \tag{12}$$

If n is odd, P will be a bright point. The idea of focus at P does not mean that it is always a bright point.

Intensity at a Point Away From the Centre:

In Fig. 13 AB is a circular aperture and P and P' are two points on the screen. PP' = x and OP = b. OP is perpendicular to the screen.
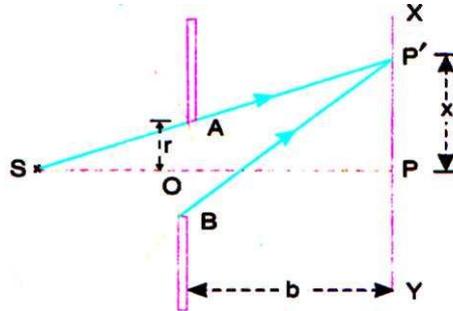

Figure 13

Let r be the radius of the aperture. The path difference between the secondary waves from A and B and reaching P' can be given by,

$$\delta = BP' - AP'$$
$$= \sqrt{b^2 + (x+r)^2} - \sqrt{b^2 + (x-r)^2} \qquad = b\left(1 + \frac{(x+r)^2}{2b^2}\right) - b\left(1 + \frac{(x-r)^2}{2b^2}\right)$$
$$= \left(b + \frac{(x+r)^2}{2b}\right) - \left(b - \frac{(x-r)^2}{2b}\right) \qquad = \frac{1}{2b}\left[(x+r)^2 - (x-r)^2\right]$$
$$\therefore \qquad \delta = \frac{1}{2b}(4xr) = \frac{2rx}{b} \tag{13}$$

The point P' will be dark if the path difference,

$$\delta = 2n\frac{\lambda}{2} \quad (2n \text{ means even number of zones}).$$
$$2n\frac{\lambda}{2} = \frac{2rx_n}{b}$$

or
$$x_n = \frac{nb\lambda}{2r} \tag{14}$$

where x_n gives the radius of nth dark ring.

Similarly, if $\qquad \delta = \frac{(2n+1)\lambda}{2},$

then $\qquad \dfrac{(2n+1)\lambda}{2} = \dfrac{2rx_n}{b}$

or $\qquad x_n = \dfrac{(2n+1)\,b\,\lambda}{4r}$

(15)

where x_n gives the radius of the nth bright ring.

The objective of a telescope consists of an achromatic convex lens and a circular aperture is fixed in front of the lens. Let the diameter of the aperture be D (= 2r). While viewing distant objects, the incident wave front is plane and the

diffraction pattern consists of a bright centre surrounded by dark and bright rings of gradually decreasing intensity. The radii of the dark rings are given by

$$x_n = \frac{nb\lambda}{2r} = \frac{nb\lambda}{D} \qquad (16)$$

The radius of the first dark ring is, $x_1 = \frac{b\lambda}{D}$

For an incident plane wavefront, $b = f$ the focal length of the objective.

$$\therefore \qquad x_1 = \frac{f\lambda}{D}$$

The value of x_n measures the distance of the first secondary minimum from the central bright maximum. However, according to Airy's theory, the radius of the first dark ring is given by

$$x_1 = \frac{1.22 f\lambda}{D} \qquad (17)$$

It is interesting to note that the size of the central image depends on λ, the wavelength of light *f*, the focal length of the lens and D, diameter of the lens aperture.

Diffraction at an Opaque Circular Disc:

S is a point source of monochromatic light. CD is an opaque disc and MN is the screen. P is a point on the screen such that SAP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. XY is the incident spherical wave front. EF is the geometrical shadow. With reference to the point P, the wave front can be divided into half period zones taking the center of the disc (A) as the pole (Fig. 14). If one half period zone can be constructed on the surface of the disc,
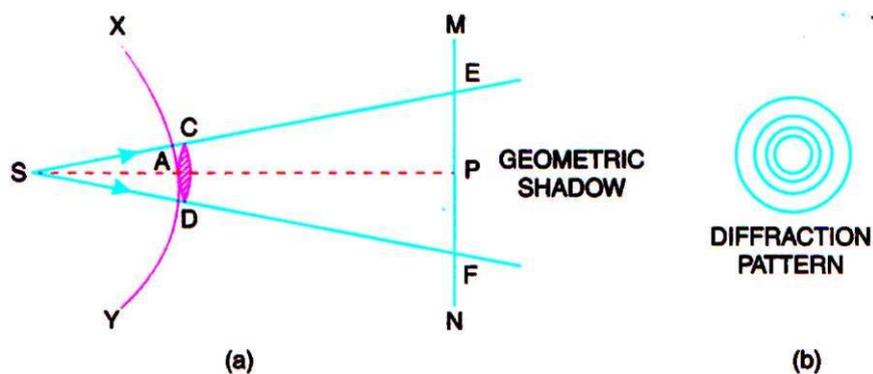

Figure 14

the rest of the zones are exposed to the point P and the resultant amplitude at $p = \frac{m_2}{2}$ approximately, where m_2 is the amplitude due to the second zone $\left[m_2 - m_3 + m_4 - \cdots \frac{m_2}{2} \right.$ approximately$\left. \right]$. Similarly, if two half period zones can be constructed on the surface of the disc, the resultant amplitude at P due to the

exposed zones will be $\frac{m_3}{2}$ and so on. Thus, the point P will always be bright but the intensity at P decreases with increase in the diameter of the disc. That is. with a large diameter of the disc, the most effective central zones will be cut off by the disc and the exposed outer zones are more oblique with reference to the point P. Thus (at P) the center of the geometrical shadow will be bright as i f the disc were absent. The diffraction pattern consists of a central bright spot surrounded by alternate bright and dark rings, as shown in Fig. 14 (b).

Diffraction Pattern Due to a Straight Edge:

Let S be narrow slit illuminated by a source of monochromatic light of wavelength. X. The length of the slit is perpendicular to the plane of the paper. AD is the straight edge and the length of the edge is parallel to the length of slit (Fig. 15). XY is the incident cylindrical wave front. P is a
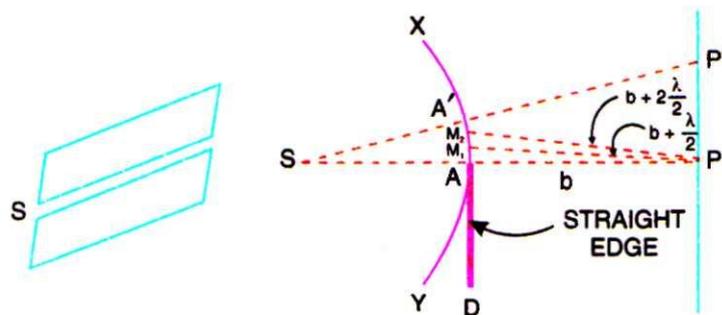

Figure 15

point on the screen and SAP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. Below the point P is the geometrical shadow and above P is the illuminated portion. Let the distance AP be b. With reference to the point P, the wave front can be divided into a number of half period strips, as shown in Fig. 16. XY is the wave front, A is the pole of the wave front and AM_1, M_1M_2, M_2M_3 etc measure of the thickness of the 1st, 2nd, 3nd etc. half period strips. With the increase in the order of the strip, the area of the strip decreases (Fig. 16).
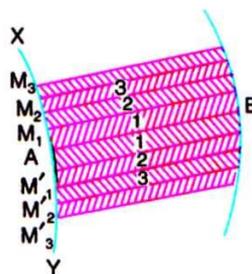

Figure 16

In Fig. 15, AP = b,

$$PM_1 = b + \frac{\lambda}{2} \quad \text{and} \quad PM_2 = b + \frac{2\lambda}{2} \text{ etc.}$$

Let P' be a point on the screen in the illuminated portion (Fig. 17). To calculate the resultant effect at P' due to the wave front XY, let us join S to P'. This line meets the wave front at B. B is the pole of the wave front with reference to the point P' and the intensity at P' will depend mainly on the number of half period strips enclosed between the points A and B. The effect at P 'due to the wave front above B is same at all points on the screen whereas it is different at different points due to the wave front between B and A. The point P' will be of maximum intensity, if the number of half period strips enclosed between B and A is odd and the intensity at P' will be minimum i f the number of half period strips enclosed between B and A is even.

Positions of Maximum and Minimum Intensity:

Let the distance between the slit and the straight edge be a and the distance between the straight edge and the screen be b (Fig. 17). Let PP' be x.

The path difference, $\delta = AP' - BP'$

$$= (b^2 + x^2)^{1/2} - [SP' - SB]$$

$$= (b^2 + x^2)^{1/2} - \left(\sqrt{(a+b)^2 + x^2} - a \right)$$

$$= b\left[1 + \frac{x^2}{2b^2} \right] - (a+b)\left[1 + \frac{x^2}{2(a+b)^2} \right] + a$$

$$= \frac{x^2}{2}\left(\frac{1}{b} - \frac{1}{a+b} \right) = \frac{x^2}{2}\left(\frac{a+b-b}{b(a+b)} \right)$$

$$\therefore \quad \delta = \frac{x^2}{2} \cdot \frac{a}{b(a+b)} \tag{18}$$

The point P' will be of maximum intensity if $\delta = (2n+1)\dfrac{\lambda}{2}$

$$\therefore \quad (2n+1)\frac{\lambda}{2} = \frac{ax_n^2}{2b(a+b)}$$

$$x_n^2 = \frac{(2n+1)(a+b)b\lambda}{a}$$

or $$x_n = \sqrt{\frac{(2n+1)(a+b)b\lambda}{a}}$$

where x_n is the distance of the nth bright band from P. Similarly, P' will be of minimum intensity if $\delta = 2n\dfrac{\lambda}{2}$

$$\therefore \qquad 2n\frac{\lambda}{2} = \frac{ax_n^2}{2b(a+b)} \quad \text{or} \quad x_n = \frac{\sqrt{2n(a+b)b\lambda}}{a} \tag{19}$$

where x_n is the distance of the nth dark band from P. Thus, diffraction bands of varying intensity (roughly corresponding to maxima and minima) are observed above the geometrical shadow i.e. above P and the bands disappear and uniform illumination occurs if P' is far away from P.

Intensity at a Point Inside the Geometrical Shadow (Straight Edge):

If P' is a point below P (Fig. 18) and B is the new pole of the wave front with reference to the point P', then the half period strips below B are cut off by the obstacle and only the uncovered half period strips above B will be effective in producing the illumination at P'. As P' moves farther from P. more number of half period strips above B is also cut off and the intensity gradually falls. Thus within the geometrical shadow, the intensity gradually falls off depending on the position of P' with respect to P. The intensity distribution on the screen due to a straight edge is shown in Fig. 19. S is the source, AD is the straight edge and MN is the screen. In the illuminated portion PM, alternate bright and dark bands of gradually diminishing intensity will be observed and the intensity falls off gradually in the region of the geometrical shadow. Thus according to the wave theory, the shadows cast by obstacles in the path of light are not sharp and hence rectilinear propagation of light is only approximately true. In general, there is gradual fading of intensity in the region of the geometrical shadow and with monochromatic light bright and dark bands (diffraction bands) are observed in the illuminated portion of the screen. However, with white light coloured bands will be observed and the bands of shorter wavelength are nearer the point P.
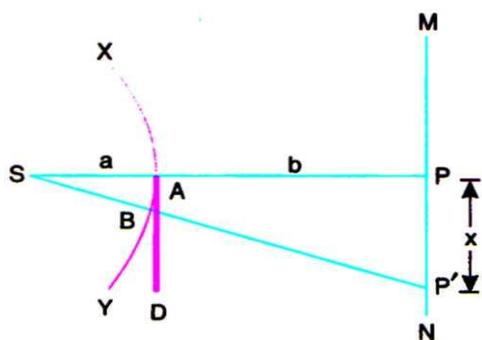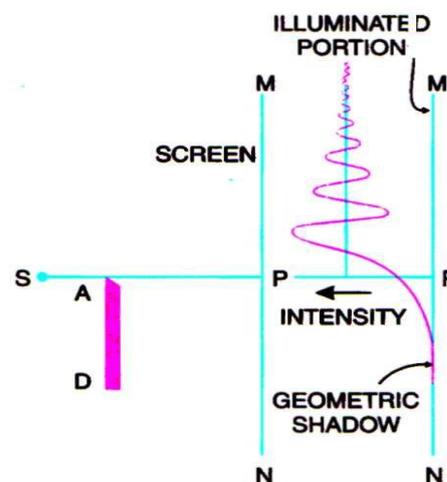

Fig. 18

Fig. 19

Diffraction Pattern Due to a Narrow Slit:

S is a narrow slit illuminated by monochromatic light. The length of the slit is perpendicular to the plane of the paper. AB is a rectangular aperture parallel to the slit, MN is the screen and P is a point on the screen such that SOP is perpendicular to the plane of the paper; XY is the incident cylindrical wave front (Fig. 20). On the screen, EF is the illuminated portion and above E and below F is the region of the geometrical shadow.
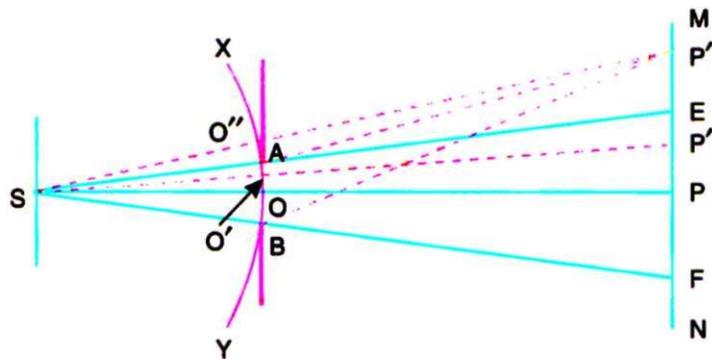

Fig. 20

If the slit AB is wide, then with reference to the point P, the cylindrical wave front can be divided into a large number of half period strips and the resultant amplitude at P will be $\frac{m_1}{2}$ where hi, is the amplitude due to the first half period strip. Thus, the point P will be illuminated. Even points very near to P will be equally illuminated. If the wave front is divided with reference to points nearer P, the number of half period strips above and below the new pole in the exposed portion of the wave front will be quite large and hence this results in uniform illumination.

Now, let us consider a point P' nearer to the edge of the geometrical shadow (see Fig. 20). Let us join S to P'. Here O' is the pole of the wave front with reference to the point P'. If the wave front is divided into half period strips, the number of half period strips between O' and B will be quite large and the illumination at P 'due to the lower portion of the wave front will be the same at all points near the edge of the geometrical shadow. But the intensity at P due to the exposed portion of the wave front between A and O' will depend on the number of half period strips present. If the number of half period elements is odd, the point P' will be of maximum intensity and if it is even the point will be of minimum intensity.

Let P" be a point in the region of the geometrical shadow. Let us join S to P". Here O" is the pole of the wave front with reference to the point P". If the wave front is divided into half period elements, then the upper half of the wave front

between X and O" is cut off by the obstacle and only a portion between A and B is exposed to the point P". If the number of half period elements exposed by AB is odd, then P" will be of maximum intensity and if it is even, it will be of minimum intensity. But as the most effective central half period strips between O" and A are cut off, the intensity falls off rapidly in the region of the geometrical shadow and maxima and minima cannot be distinguished. The intensity distribution due to a wide aperture is shown in Fig. 21 (b).
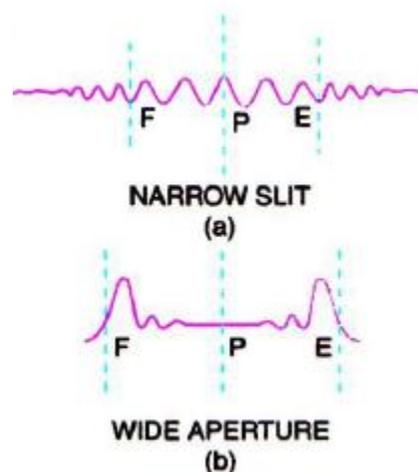

F   P   E

NARROW SLIT
(a)

F   P   E

WIDE APERTURE
(b)

Fig. 21

On the other hand if the slit is narrow, the intensity at the point P will depend on the number of half period strips that can be constructed on the exposed wave front between A and B. If the number of half period strips is odd, the intensity at P will be maximum and if it is even, the intensity at P will be minimum (Fig. 17.20). Thus, the point P can be bright or dark. If we consider a point P' in the illuminated portion EF of the screen, the intensity at P will depend on number of half period strips that can be constructed between A and O' where O' is the pole of the wave front with reference to the point P'. I f the number of half periods strips between A and O' is odd, P' will be a point of maximum intensity. Thus, between E and F alternate bright and dark bands will be observed and the point P may be bright or dark. Now consider a point P" in the region of the geometrical shadow (Fig. 20). O" is the pole of the wave front with reference to the point P" and the intensity at P" will depend on the number of half period strips exposed by the slit AB. The upper half of the wave front above O" is obstructed by the obstacle and even the most effective central half period strips between O" and A are cut off by the obstacle. Thus, at P "which is far away from E, the maxima and minima become indistinguishable. There is no marked transition between the diffraction bands observed in the geometrical shadow and the illuminated portion. In the intensity distribution on the screen due to a narrow slit (say less than the wavelength of light), a broad central maximum will be observed in the illuminated portion and

the intensity variation cannot be distinguished. The intensity gradually falls off in the region of geometrical shadow.

Diffraction Due to a Narrow Wire:

In Fig. 22, S is a narrow slit illuminated by monochromatic light, AB is the diameter of the narrow wire and MN is the screen. The length of the wire is parallel to the illuminated slit and perpendicular to the plane of the paper. The screen is also perpendicular to the plane of the paper. XY is the incident cylindrical wave front. P is a point on the screen such that SOP is perpendicular to the screen. EF is the region of the geometrical shadow and above E and below F, the screen is illuminated.
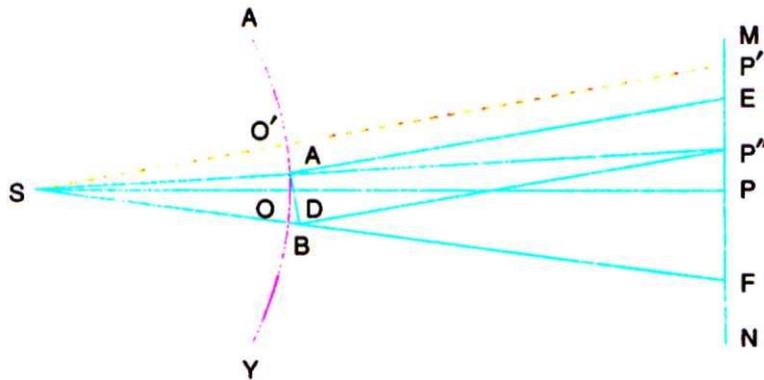

Fig. 22

Now, let us consider a point P' on the screen in the illuminated portion. Let us join S to O'. a point on the wave front. O' is the pole of the wave front with reference to P'. The intensity P' due to the wave front above O' is the same at all points and the effect due to the wave front BY is negligible. The intensity at P' will be a maximum or a minimum depending on whether the number of half period strips between O 'and A is odd or even. Thus, in the illuminated portion of the screen, diffraction bands of gradually diminishing intensity will be observed. The distinction between maxima and minima will become less if P' is far away from the edge E of the geometrical shadow. Maxima and minima cannot be distinguished if the wire is very narrow, because in that case the portion BY of the wave front also produces illumination at P.

Next let us consider a point P" in the region of the geometrical shadow. Interference bands of equal width will be observed in this region due to the fact that the points A and B, of the incident wave front, are similar to two coherent sources. The point P" will be of maximum or minimum intensity, depending on whether the path difference (BP" - AP") is equal to even or odd multiples of $\frac{\lambda}{2}$ .The fringe width β is given by

$$\beta = \frac{D\lambda}{d}$$

where D is the distance between the wire and the screen, X is the wave length of light and d is the distance between the two coherent sources. In this case d=2r where 2r is diameter of the wire (AB=2r).

$$\therefore \qquad \beta = \frac{D\lambda}{2r} \qquad\qquad (19)$$

$$\therefore \qquad r = \frac{D\lambda}{2\beta} \qquad\qquad (20)$$

$$\text{or} \qquad \lambda = \frac{2r\beta}{D} \qquad\qquad (21)$$

Here, β the fringe width corresponds to the distance between any two consecutive maxima. Thus, from equations (20) and 21), knowing the values of r or λ ; λ or r can be determined. In Fig. 23 the bands marked V represent the interference bands in the region of the geometrical shadow and the bands marked *"b"* and "c "represent the diffraction bands in the illuminated portion. The intensity distribution due to a narrow wire is shown in Fig. 24 *(a)*. The center of the geometrical shadow is bright.
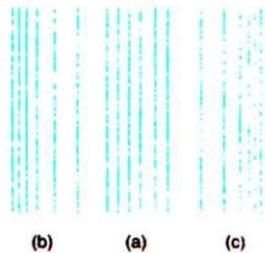

(b)   (a)   (c)

Fig. 23

On the other hand, if the wire is very thick, the interference bands cannot be noticed. From equation (19), $\beta = \frac{D\lambda}{2r}$; where β is the fringe width. As the diameter of the wire increases the fringe width decreases and if the wire is sufficiently thick, the width of the interference fringes decreases considerably and they cannot be distinguished. The intensity falls off rapidly in the geometrical shadow. The diffraction pattern in the illuminated portion will be similar to that of a thin wire Fig. 24(b). Coloured fringes will be observed with white light.
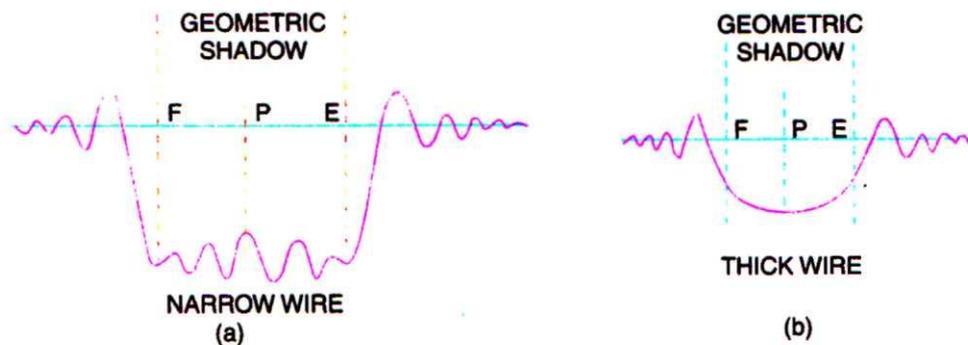
Fig. 24

WORKED OUT EXAMPLES

Example 17.1: What is the radius of the first half period zone in a zone plate behaving like a convex lens of focal length 60 cm for light of wavelength $6000\,\text{Å}$?

Solution: Given that $f = 60$ cm $= 0.6$ m, $n = 1$, and $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m

$$f = \frac{r_n^2}{n\lambda} \quad \therefore r_n^2 = fn\lambda$$

$\therefore \qquad r_1^2 = 0.6\,\text{m} \times 1 \times 6 \times 10^{-7}\,\text{m} = 36 \times 10^{-8}\,\text{m}^2$

or $\qquad r_1 = 6 \times 10^{-4}\,\text{m} = \textbf{0.6 mm.}$

Example 17.2: A circular aperture of 1.2 mm diameter is illuminated by plane waves of monochromatic light. The diffracted light is received on a distant screen which is gradually moved towards the aperture. The centre of the circular patch of light first becomes dark when the screen is 30 cm from the aperture. Calculate the wavelength of light.

Solution: Given that diameter $= 1.2$ mm $= 0.12$ cm

$$\text{Radius} = r = 0.06 \text{ cm}$$
$$b = 30 \text{ cm}$$

Here

$$\left(b^2 + r^2\right) = (b + \lambda)^2$$

$$30^2 + (0.06)^2 = (30 + \lambda)^2$$

$$\lambda = \frac{(0.06)^2\,\text{cm}^2}{2 \times 30\,\text{cm}} \quad (\lambda^2 \text{ neglected})$$

$$= 0.00006 \text{ cm} = \textbf{6000 Å}$$

PROBLEMS FOR PRACTICE

1. The diameter of the first ring of a zone plate is 1.1 mm. If plane waves $(\lambda = 6000 \ \mathring{A})$ fall on the plate, where should the screen be placed so that light is focused to a bright spot?

2. Find the radii of the first three transparent zones of a zone plate whose first focal length is 1 m for $\lambda = 5893\mathring{A}.$

3. The diameter of the central zone of a zone plate is 3 mm. If a point source of light $(\lambda = 6000 \ \mathring{A})$ is placed at a distance of 5 m from it. calculate the position of the first image.

4. A parallel beam of monochromatic light of $\lambda = 5000 \ \mathring{A}$ is incident normally on a plate having a circular hole of diameter 1mm. The screen is at the farthest position for which the axial point is almost black. The screen is moved towards the plate so that the axial point is again seen black. How far is the screen moved from the first position to the second?

5. A narrow slit illuminated by light of wave length $4500^\circ \ A$ is placed at a distance of 20 cm from a straight edge. Calculate the distance between the second and third diffraction maxima formed on a screen placed at a distance of 60 cm from straight edge.