Coherence

CHAPTER OUTLINE

- Introduction
- Wave Train
- Coherence Length and
- Coherence Time
- Bandwidth
- Relation Between Coherence Length and Bandwidth
- Coherence
- Determination of Coherence Length

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Introduction:

The light emitted by an ordinary light source is not an infinitely long, simple harmonic wave but is composed of a jumble of finite wave trains. We therefore call a real monochromatic source as a quasi-monochromatic source. The wave trains issuing out of a quasi-monochromatic source are as shown in figure

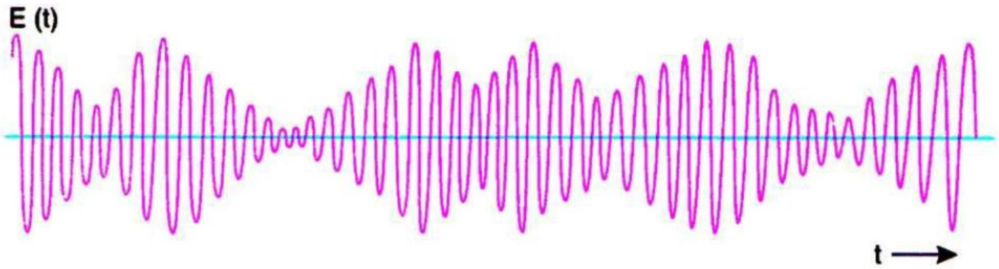

Fig.1

Waves train:

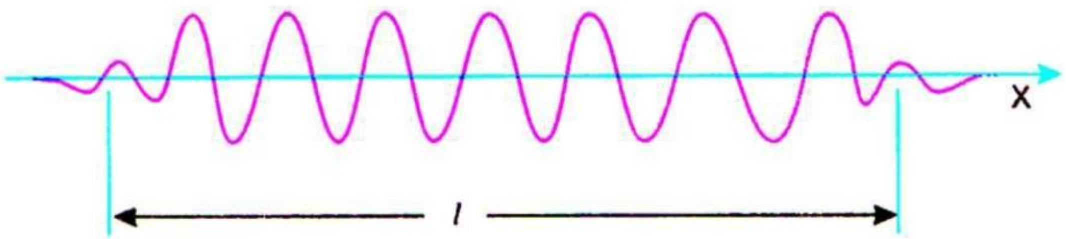

Fig.2: Shows a wave train generated by an atom.

If such a wave train lasts for a time interval Δt, then the length of the wave train in a vacuum is

$l = c \Delta t$ ………………….. (1)

Where c is the velocity of light in a vacuum. For example, if $\Delta t = 10^{-8}$s. and $c = 3 \times 10^{8}\ m/s$, then $l = (3 \times 10^{8}\ m/s)(10^{-8} s) = 3m$.

The number of oscillations present in the wave train is

$N = \frac{l}{\lambda}$ ………………….. (2)

Where λ is the wavelength. If we assume $\lambda = 5000A^{\circ} = 5 \times 10^{-7}m$, then $N = \frac{3m}{5\times10^{-7}m} = 6 \times 10^{6}$.

Thus, a wave train contains about a million wave oscillations in it.

Adding together the wave packets generated by all atoms in the light source, one finds a succession of wave trains, as shown in figure.

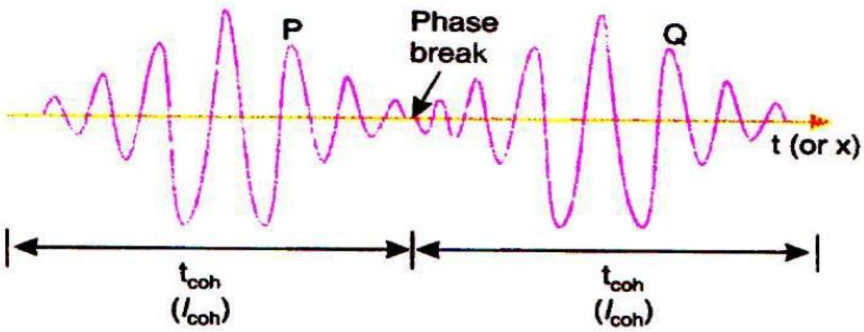

Fig.3

In passing from one wave train to the next, there is an abrupt change in the phase and also in plane of polarization. It is not possible to relate the phase at a point in wave train Q to a point in wave train P.

Consequently there is no correlation between the phase different wave trains. Each wave train has a sustained phase for only about 10^{-8}s, after which a new wave train is emitted with a totally random phase which also lasts only for about 10^{-8}s. The phase of the wave train from one atom will remain constant with respect to the phase of the wave train from another atom for utmost 10^{-8}s. It means that the wave trains can be coherent for a maximum 10^{-8}s only. If two light waves overlap, sustained interference is not observed since the phase relationship between the waves changes rapidly, nearly at the rate of 10^{-8} times per second.

Coherence length and coherence time:

$l_{coh} = c \, \Delta t$ ………………….. (3)

It is the time, Δt, during which the phase of the wave train does not become randomized but undergoes change in a regular systematic way. Coherence time is denoted by t_{coh}. We can therefor write.

$t_{coh} = \Delta t$ ………………….. (4)

$\therefore l_{coh} = c \, t_{coh}$ ………………….. (5)

A wave train consists of a group of waves, which have a continuous spread of wavelengths over a finite range $\Delta\lambda_\circ$ centered on a wavelength $\lambda_\circ$. According to Fourier analysis the frequency bandwidth $\Delta\nu$ is given by

$$\Delta\nu = \frac{1}{\Delta t}$$

Where Δt is the average lifetime of the excited state of the atom. However, Δt is time during which a wave train is radiated by atom and corresponds to the coherence time, t_{coh} , of the wave train.

$$\therefore \Delta\nu = \frac{1}{\Delta t} = \frac{1}{t_{coh}} \qquad \dots\dots\dots\dots\dots\dots\dots..(6)$$

Using the relation (5) in to equation (6), we get

$$\Delta\nu = \frac{c}{l_{coh}} \qquad \dots\dots\dots\dots\dots\dots\dots..(7)$$

Bandwidth:

A wave packet is not harmonic wave. Therefore, it cannot be represented mathematically by simple sin functions. The mathematical representation of a wave packet is done in terms of Fourier integral. If light emitted from a source is analyzed with help of a spectrograph, it is known to be made up of discrete spectral lines. Wave packets emitted by atoms form these spectral lines. Therefore, a spectral lines and wave packet are equivalent descriptions. The wavelength of a wave packet or a spectral line is not precisely defined. There is a continuous spread of wavelengths over a finite range, $\Delta\lambda$, centered on a wavelength $\lambda_\circ$. The maximum intensity of the wave packet occurs at $\lambda_\circ$ and the intensity falls off rapidly on either side of $\lambda_\circ$, as shown in figure.

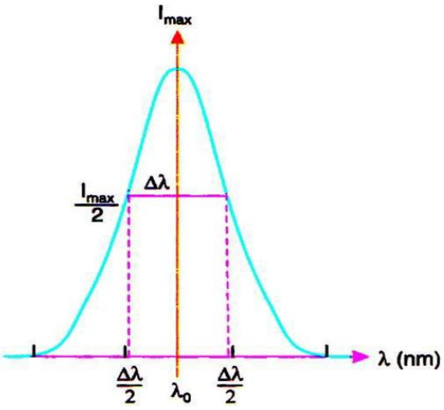

Fig.4

The spread of wavelengths is called the bandwidth. The bandwidth is the wavelength interval from$\lambda_\circ - \Delta\lambda/2$ to $\lambda_\circ + \Delta\lambda/2$ which contains the major portion of the energy of the wave packet. In practice a source, which is said to produce line spectrum, produce a number of sharp wavelength distributions.

Relation between coherence length and bandwidth:

The frequency and wavelength of a light wave are related through the equation

$$\nu = \frac{c}{\lambda} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(8)$$

Where $\lambda_\circ$ is the vacuum wavelength.

Differentiating equation (8) on both sides, we get

$$\Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(9)$$

Using the relation (7) into equation (9), we obtain

$$\therefore \frac{c}{l_{coh}} = -\frac{c}{\lambda^2}\Delta\lambda$$

Rearranging the terms, we get

$$l_{coh} = \frac{\lambda^2}{\Delta\lambda} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(10)$$

The minus sign has no significance and hence is ignored. Equation (10) means that the coherence length (the length of the wave packet) and the bandwidth of the wave packet are related to each other. The longer the wave packet, the narrower will be the bandwidth. In the limiting case, when the wave is infinitely long, we obtain monochromatic radiation of frequency $\nu_\circ$ (wavelength $\lambda_\circ$).

Form equation (2), the coherence length may be defined as product of the number of wave oscillations N contained in the wave train and of the wavelength, λ. Thus,

$$l_{coh} = N\lambda \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(11)$$

Equation (10) and (11), we get

$$N = \frac{\lambda}{\Delta\lambda}$$

$$\therefore \frac{1}{N} = \frac{\Delta\ \lambda}{\lambda} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(12)$$

Equation (12) shows that the large the number of wave oscillations in a wave packet, the smaller is the bandwidth. In the limiting case, when N is infinitely large, that is when the wave packet is infinitely long; the wave will be monochromatic having a precisely defined wavelength. The dependence of bandwidth on the length of the wave packet is schematically shown in figure.

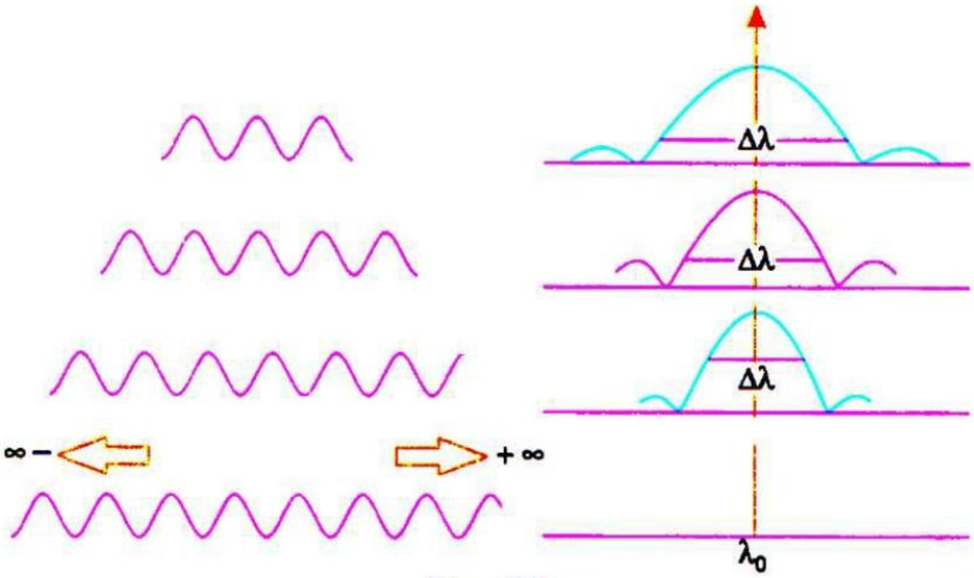

Fig.5

Coherence:

Coherence is an important property of light. It refers to the connection between the phase of light waves at one point and time, and the phase of the light waves at another point and time. Coherence effects are mainly divided into two categories: temporal and spatial. The temporal coherence is related directly to the finite bandwidth of the source, whereas the spatial coherence is related to the finite size of the source.

Temporal Coherence:

Temporal coherence is also known as longitudinal coherence. Let a point source of quasi monochromatic light S (Fig. 5) emit light in all directions. Let us consider light travelling along the line SP_1P_2. The phase relationship between the points P_1 and P_2 depends on the distance P_1P_2 and the coherence length of the light beam. The electric fields at P_1 and P_2 will be correlated in phase when a single wave train extends over greater length than the distance P_1P_2; that is if the distance P_1P_2 is less than the coherence length l_{coh}. Then, the waves are correlated in their rising and falling and they will preserve a constant phase difference. The points P_1 and P_2 would not have any phase relationship if the longitudinal distance P_1P_2 is greater than l_{coh}, since in such a case many wave trains would span the distance. It means different independent wave trains would be at P_1 and P_2 at any instant and therefore the phase at the two points would be independent of each other. The degree to which a correlation exists is known as the amount of longitudinal coherence.

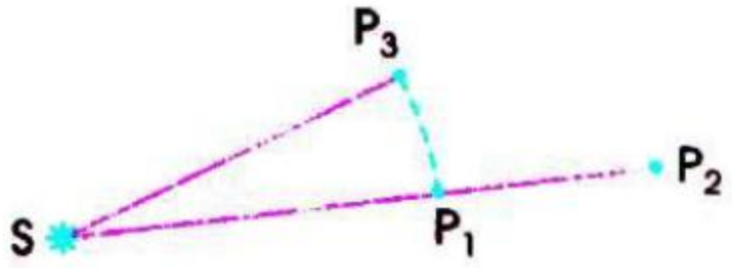

Fig.6

Monochromaticity:

Form equation (11) and Fig. 5 we conclude that temporal coherence is indicative of mono chromaticity of the source. An ideally monochromatic source is an absolutely coherent source. The degree of mono chromaticity of a source is given by

$$\xi = \frac{\Delta\nu}{\nu_\circ} \qquad (13)$$

When the ratio $\frac{\Delta\nu}{\nu_\circ} = 0$, the light wave is ideally monochromatic.

Purity of spectral line:

The width of a spectral line is given by $\Delta\ \lambda$. (See Fig. 4). It is seen from equation (11) that it is related to the temporal coherence. Thus,

$$\Delta\ \lambda = \frac{\lambda^2}{l_{coh}}$$

Spatial Coherence:

Spatial coherence refers to the continuity and uniformity of a wave in a direction perpendicular to the direction of propagation. ***If the phase difference for any two fixed points in a plane normal to the wave propagation does not vary with time, then the wave is said to exhibit spatial coherence.*** It is also known as lateral coherence. Again looking at the point source S (Fig. 6), $SP_1 = SP_3$ and therefore, the fields at points P_1 and P_3 would have the same phase. Thus, an ideal point source exhibits spatial coherence, as the waves produced by it are likely to have the same phase at points in space, which are equidistant from the source. On the other hand, an extended source is bound to exhibit lesser lateral spatial coherence. Two points on the source separated by a lateral distance greater than one wavelength will behave quite independently. Therefore, correlation is absent between the phases of the waves emitted by them. The degree of contrast of the interference fringes produced by a source is a measure of the degree of the spatial coherence of its waves. The higher the contrast, the better is the spatial coherence.

Determination of Coherence Length:

The coherence length can be measured by means of Michelson interferometer. In a Michelson interferometer, a light beam from the source S is incident on a semi-silvered glass plate G (see Fig. 7) and gets divided into two components: one component is reflected, 1 .and the other,2, is transmitted. These two beams, 1 and 2, are reflected back at mirrors M_1 and M_2 respectively and are received by the telescope where interference fringes are produced. It is obvious that the beams produce stationary interference only if they are coherent.

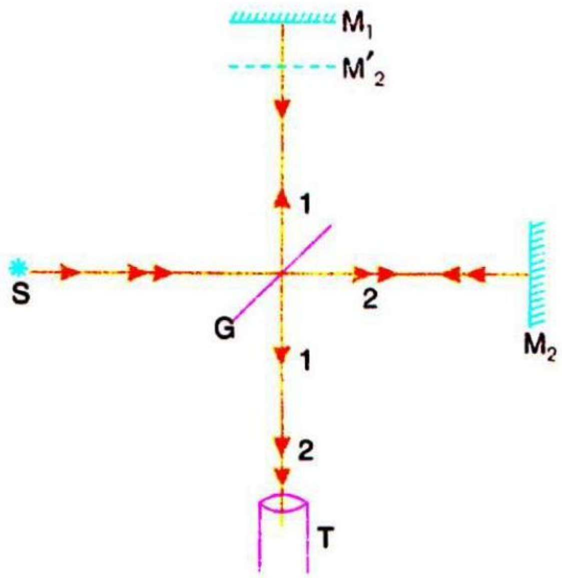

Fig.7

Let M_2' be the image of M_2 formed by G. If the reflecting surfaces M_1 and M_2' (the image of M_2) are separated by a distance d, then 2d will be the path difference between the interfering waves. The condition of fixed phase relationship between the two waves, 1 and 2, will be satisfied if

$$2\text{d} \ll l_{coh}$$

In such a case distinct interference fringes will be seen. If, however,

$$2\text{d} \gg l_{coh}$$

then the phases of the two waves are not correlated and interference fringes will not be seen. To determine the coherence length of waves emitted by a light source, the distance d between the mirrors M_1 and M_2' (the image of M_2) is varied by moving one of the mirrors. As the distance varies, the contrast of the fringes decreases and ultimately they disappear. The path difference 2d at the particular stage where the fringes disappear gives us the coherence length.

The light from a sodium lamp has coherence length of the order of 1 mm, that of green mercury line is about 1 cm, neon red line 3 cm, red cadmium line 30 cm, orange krypton line 80 cm and that of a commercial He-Ne laser is about 15m. The coherence length of light from some of the lasers goes up to a few km.

WORKED OUT PROBLEMS

Example 1: A sodium atom radiates for 4×10^{-12}s. What is the coherence length of light from a sodium lamp?

Solution: it is given that coherence time $\text{t}_{coh} = 4 \times 10^{-12}\text{s}$.

Coherence length

$$l_{coh} = \text{c}\,\text{t}_{coh} = (3 \times 10^8\ \text{m/s})(4 \times 10^{-12}\text{s}) = 12 \times 10^{-4}\text{m} = 1.2\text{mm}.$$

Example 2: Calculate the coherence length for CO_2 laser whose line width is 1×10^{-5}nm at IR emission wavelength of 10.6 µm.

Solution: Coherence length $l_{coh} = \frac{\lambda^2}{\Delta\lambda} = \frac{(10.6\times10^{-6})^2\text{m}^2}{1\times10^{-5}\times10^{-9}\text{m}} = 11.2\text{km}.$

Example 3: Compute the coherence length of yellow light with 5893A° in 10^{-12} second pulse duration. Find also the bandwidth.

Solution: Coherence length $l_{coh} = \text{c}\,\text{t}_{coh} = (3 \times 10^8\ \text{m/s})(10^{-12}\text{s}) = 0.33\text{mm}$

Bandwidth is given by $\Delta\ \lambda = \frac{\lambda^2}{l_{coh}} = \frac{(5893\times10^{-10}\text{m})^2}{0.33\times10^{-3}\text{m}} = 11.6\text{A}^\circ.$

Home work:

1. Calculate the frequency bandwidth for white light (frequency range 4×10^{14}Hz to 7.5×10^{14}Hz). Also find (i) coherence time and (ii) coherence length of white light.
2. A quasi-monochromatic source emits radiations of mean wavelength λ=5461A° and has bandwidth $\Delta\nu = 10^9$Hz. Calculate coherence time, coherence length and frequency stability.
3. An optical filter has a line width of 1.5 nm and mean wavelength 550 nm. With white light incident on the filter, calculate (i) coherence length and (ii) the number of wavelengths in the wave train.
4. The spectral purity of a source can be appreciated via the quantity $\Delta\nu/\nu$, the frequency stability. For example, a Hg^{198} low-pressure isotope lamp (λ_{air} = 546.078 nm) has a bandwidth of $\Delta\nu = 1000$ MHz. Compute the coherence length and coherence time of the light, as well as the frequency stability.
5. A Michelson interferometer is illuminated by red cadmium light with a mean wavelength of 643.847 nm and a line width of 0.0013 nm. The initial setting is for zero O.P.D., i.e. d = O. One mirror is then slowly moved until the fringes disappear by how much must it be shifted? How many wavelengths does this correspond to?
6. Suppose the experiment described in Problem 5 were repeated with light (λ = 682.8 nm) from a He-Ne laser having a frequency stability of 2 parts per 10^{10} .What mirror displacement would now be needed to cause the fringes to vanish?
7. In 1963 Jaseija, Javan and Townes attained a short-term frequency stability of roughly 8 parts per 10^{14} with a He-Ne gas laser at λ == 1153 nm. Compute the coherence time and coherence length.
8. Roughly what is the line width of a hypothetical source if it has an uninterrupted transition time of 10^{-8} s (i.e. assume Δt == 10^{-8} s)? Compute the coherence length as well. The vacuum wavelength equals 650 nm.
9. Red light (λ == 650 nm) emerging from an ordinary filter is comprised of wave trains about 50λ in length. What is the line width, $\Delta\lambda$, passed by the filter? Determine the maximum range over which the mirror in a Michelson interferometer can be moved before the fringes in this case become unobservable.