

Production of Linearly Polarized

Linearly polarized light may be produced from polarized light using of the following five optical phenomena:

(i) Reflection, (ii) refraction, (iii) scattering, (iv) selective absorption (dichroism) and (v) Double refraction.

Brewster's Law

Sir David Brewster performed a series of experiments on the polarization of light by reflection at a number of surfaces. He found that the polarizing angle depends upon the refractive index of the medium. In 1892, Brewster proved that the *tangent of the angle at which polarization is obtained by reflection is numerically equal to the refractive index of the medium*. If θ_p is the angle and μ is the refractive index of the medium, then

$$\tan \theta_p = \mu \quad (1)$$

This is known as Brewster's law.

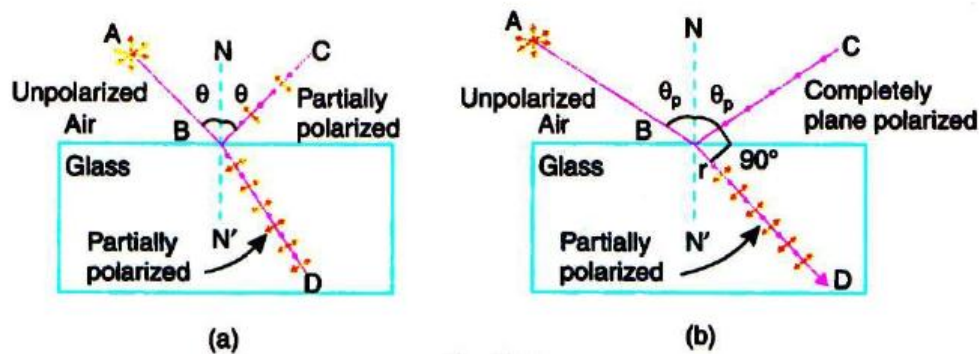


Figure (1)

If natural light is incident on a smooth surface at the polarizing angle, it is reflected along BC and refracted along BD, as shown in Fig. 1 (b). Brewster found that the maximum polarization of reflected ray occurs when it is at right angles to the refracted ray. It means that $\theta_p + r = 90^\circ$.

$$\therefore r = 90^\circ - \theta_p \quad (2)$$

According to Snell's law,

$$\frac{\sin \theta_p}{\sin r} = \frac{\mu_2}{\mu_1} \quad (3)$$



Where μ_2 is the absolute refractive index of reflecting surface and μ_1 is the refractive index of the surrounding medium. It follows from equation (2) and equation (3) that

$$\frac{\sin \theta_p}{\sin (90^\circ - \theta_p)} = \frac{\mu_2}{\mu_1}$$

or

$$\frac{\sin \theta_p}{\cos \theta_p} = \frac{\mu_2}{\mu_1}$$

$$\therefore \tan \theta_p = \frac{\mu_2}{\mu_1} \quad (4)$$

Equation (.4) shows that the polarizing angle depends on the refractive index of the reflecting surface. The polarizing angle θ_p is also known as **Brewster angle** and denoted as θ_B . Light reflected from any angle other than Brewster angle is partially polarized.

Application of Brewster's law:

- (i) Brewster's law can be used to determine the refractive indices of opaque materials.
- (ii) It helps us in calculating the polarizing angle necessary for total polarization of reflected light for any material if its refractive index is known. However, the law is not applicable for metallic surfaces.
- (iii) Two windows known as **Brewster windows** are used in gas lasers. They are arranged at Brewster angle at the two ends of the laser tube. Every time light passes through the windows on its way to the reflecting mirrors of the optical resonator, s-component is removed. At the end, light emerging from the laser consists of linearly polarized light of p-component.
- (iv) Another application utilizes the Brewster angle for transmitting a light beam into or out of an optical fiber without reflection losses.

The degree of polarization of the transmitted light is given by

$$P = \frac{I_p - I_s}{I_p + I_s} = \frac{m}{m + \left(\frac{2\mu}{1 - \mu^2} \right)^2} \quad (5)$$

Where m is the number of plates required and u is the refractive index of the material, if I_p and I_s are the intensities of the parallel and perpendicular components in the refracted light.

Effect of Analyzer on Plane Polarized Light—MALUS' LAW

When natural (unpolarized) light is incident on a polarizer, the transmitted light is linearly polarized. If this light further passes through an analyzer, the intensity varies with the angle between the transmission axes of the polarizer and analyzer. Malus studied the phenomenon in 1809 and formulated the law that bears his name. ***It states that the intensity of the polarized light transmitted through the analyzer is proportional to cosine square of the angle between the plane of transmission of the analyzer and the plane of transmission of the polarizer.*** This is known as Malus' law.

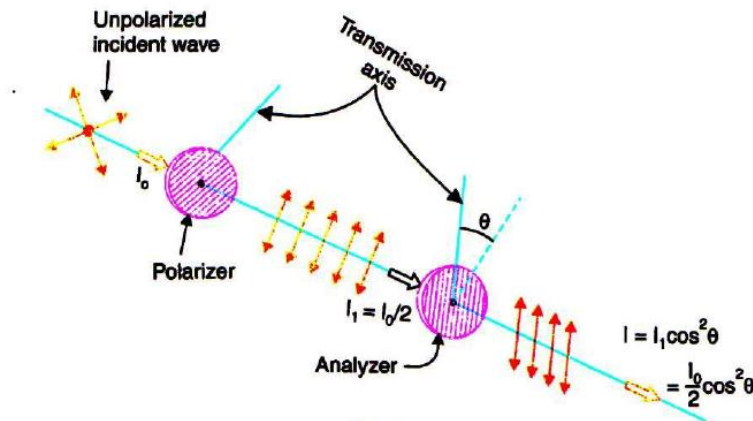


Figure (2)

If unpolarized light of intensity I_0 is incident on a polarizer, plane polarized light of intensity $I_0/2$ is transmitted by it. Let us denote $I_0/2$ by I_1 . This plane polarized light then passes through the analyzer. Let E be the amplitude of vibration and θ be the angle that this vibration makes with the axis of the analyzer. E can be resolved into two rectangular components: (i) E_y , parallel to the plane of transmission of the analyzer and (ii) E_x , perpendicular to the plane of analyzer. It is only the parallel component E_y that is transmitted by the analyzer.

$$E_y = E \cos \theta$$

The intensity corresponding to this component is

$$I = E^2 \cos^2 \theta = I_1 \cos^2 \theta = (I_0/2) \cos^2 \theta$$

Case (i): If $\theta = 0^\circ$	axes parallel	$I = I_1$
Case (ii): If $\theta = 90^\circ$	axes perpendicular	$I = 0$
Case (i): If $\theta = 180^\circ$	axes parallel	$I = I_1$
Case (i): If $\theta = 270^\circ$	axes perpendicular	$I = 0$

Thus, we obtain two positions of maximum intensity and two positions of zero intensity when we rotate the axis of the analyzer with respect to that of the polarizer.

Phase Difference Between e-Ray and o-Ray



We have seen that natural light incident on the surface of an anisotropic crystal undergoes double refraction and produces two plane polarized waves. *Even when a plane polarized light wave is incident on a birefringent crystal such that its electric vector makes an angle with the optic axis, then the polarized wave splits into two polarized waves namely e-ray and o-ray.* Let us consider the particular case of a slice of a positive crystal where the optic axis is parallel to refracting face of the crystal . The two waves travel along the same direction in the crystal but with different velocities. As a result, when the waves emerge from the rear face of the crystal, an optical path difference would have developed between them. The optical path difference can be calculated as follows:

Let d be the thickness of the crystal.

$$\begin{array}{l} \left. \begin{array}{l} \text{The optical path for o-ray} \\ \text{within the crystal} \end{array} \right\} = \mu_o d \\ \left. \begin{array}{l} \text{The optical path for e-ray} \\ \text{within the crystal} \end{array} \right\} = \mu_e d \\ \therefore \left. \begin{array}{l} \text{The optical path difference} \\ \text{between e-ray and o-ray} \end{array} \right\} \Delta = (\mu_e - \mu_o) d \end{array} \quad (6)$$

Consequently, a phase difference arises between the two waves. It is given by

$$\delta = \frac{2\pi}{\lambda} (\mu_e - \mu_o) d \quad (7)$$

As the two component waves are derived from the same incident wave, the two waves are in phase at the front face and have emerged from the crystal with a constant phase difference (see Fig. 3) and hence it may be expected that the waves are in a position to interfere with each other. However, as the planes of polarization of o-ray and e-ray are perpendicular to each other, interference cannot take place between e- rays and o-rays. The waves instead combine with each other to give an elliptically polarized wave.

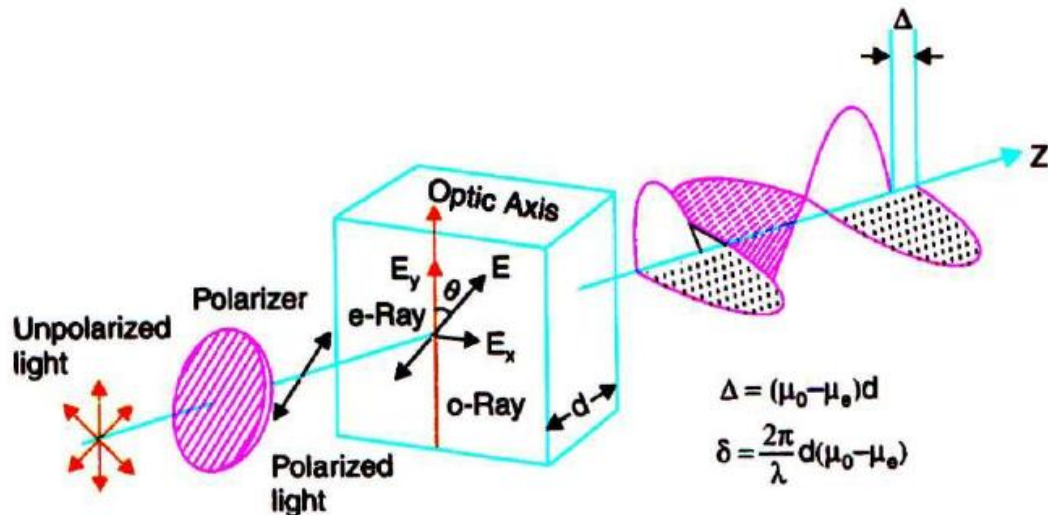


Figure (3)

(H.W) SUPERPOSITION OF WAVES LINEARLY POLARISED AT RIGHT ANGLES

SUPERPOSITION OF e-RAY AND o-RAY

Now we are in a position to understand what happens when e-ray and o- ray overlap on each other after emerging from an anisotropic crystal plate. It is obvious that they cannot produce interference fringes as in a double slit experiment. On the other hand, they combine to produce different states of polarization depending upon their optical path difference.

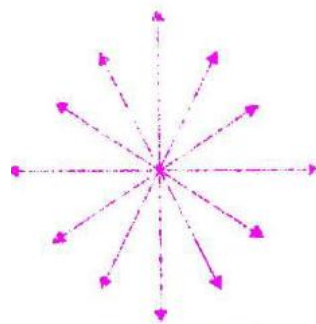
1. When the optical path difference is 0 or an even or odd multiple of $\lambda/2$, the resultant light wave is linearly polarised.
2. When the optical path difference is $\lambda/4$, the resultant light wave is **elliptically polarised**.
3. In the particular instance when the wave amplitudes are equal and the optical path difference is $\lambda/4$, the resultant light wave is **circularly polarised**.

TYPES OF POLARISED LIGHT

We can now sum up the various types of polarised light as follows.

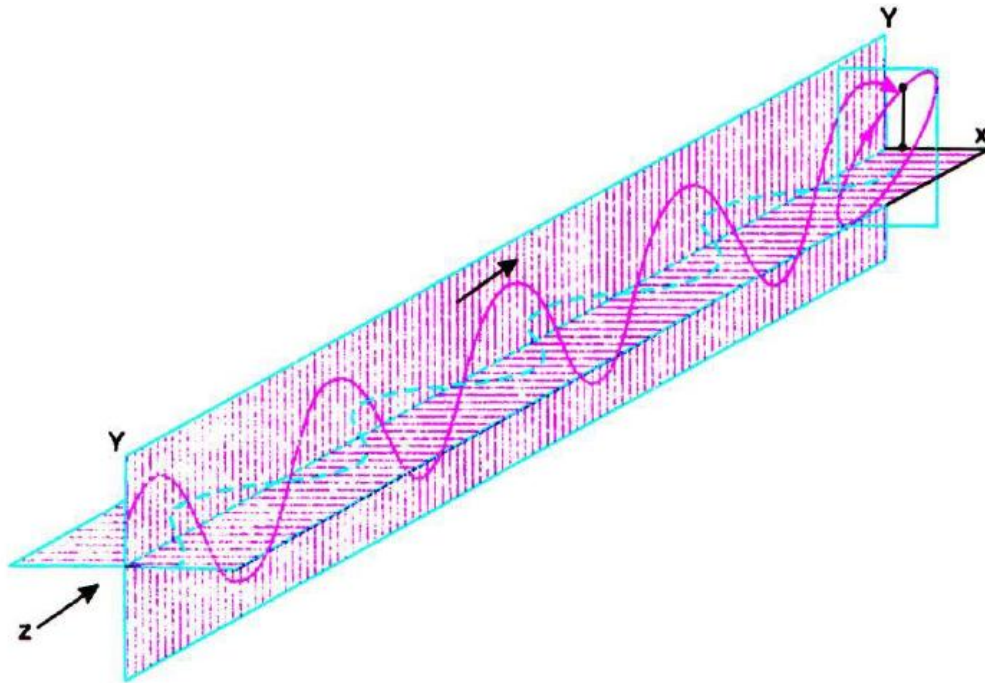
- i) unpolarized light, which consists of sequence of wave trains, all oriented at random. It is considered as the resultant of two optical vector components, which are incoherent.
- ii) Linearly polarised light, which can be regarded as a resultant of two coherent linearly polarised waves.

- iii) Partially polarised light, which is a mixture of linearly polarised light and unpolarised light. Partially polarized light is represented as shown in Figure (4)
- iv) Elliptically polarised light, which is the resultant of two coherent waves having different amplitudes and a constant phase difference of 90° (see Fig. 5). In elliptically polarized light, the magnitude of electric vector E changes with time and the vector E rotates about the direction of propagation. If we imagine that we are looking at the light wave advancing towards us, we would observe that the tip of the E vector traces an ellipse. If we look from sides, we would find that the tip of E sweeps a flattened helix in space. When we are looking back towards the source, if the rotation of E vector occurs clockwise, it is said to be a **right-elliptically polarised** wave. If it rotates anticlockwise, as we look back toward the source it is said to be a **left-elliptically polarised** wave.
- v) Circularly polarised light, which is the resultant of two coherent waves having same amplitudes and a constant phase difference of 90° . A light wave is said to be **circularly polarised**, if the magnitude of the electric vector E stays constant but the vector rotates about the direction of propagation such that it goes on sweeping a circular helix in space. If we imagine that the wave is advancing toward our eyes, we would find that the tip of the E vector of the wave traces a circle. If the rotation of the tip of E is clockwise, as seen by an observer looking back towards the source, then the wave is said to be right-circularly polarised. If the tip of E rotates anticlockwise, as seen by an observer looking back toward the source, the wave is said to be left-circularly polarised.



Schematic representation of partially polarized light.

Figure (4)



Elliptically polarised light is produced when two orthogonal coherent waves having different amplitudes and a phase difference of 90° , superpose on each other.

Figure (5)

EFFECT OF POLARIZER ON TRANSMISSION OF POLARISED LIGHT

(i) If unpolarized light is incident on a polarizer, the transmitted light will be linearly polarised light. The intensity of the transmitted polarised light will be half the intensity of unpolarized light incident on the polarizer. The intensity of the transmitted light does not change on rotation of the polarizer.

(ii) If partially polarised light is incident on a polarizer, the intensity of the transmitted light will be dependent on the direction of the transmission axis of the polarizer. The intensity of the transmitted light will vary from a maximum value I_{\max} to a minimum value I_{\min} one full rotation of the polarizer. Two positions of I_{\max} and two positions of I_{\min} one complete rotation.

(iii) If plane polarised light is incident on the polarizer, the intensity of transmitted light varies from zero to a maximum value. Two positions of zero intensity and two positions of full intensity I occur in one complete rotation of the polarizer.

(iv) When circularly polarised light is incident on a polarizer, the intensity of the transmitted light stays constant in any position of the polarizer. The circular vibrations may be resolved into two mutually perpendicular linear vibrations of equal amplitude. When the circularly polarised light is incident on the polarizer, the vibrations parallel to its transmission axis pass through the polarizer while the perpendicular component is obstructed. When the polarizer is rotated, there is always a component of constant



intensity parallel to the axis of the polarizer, which is freely transmitted. Hence, the intensity of the transmitted light is the same for all positions of the polarizer.

(v) In case of elliptically polarised light, the intensity of the light transmitted through the polarizer varies with the rotation of the polarizer from I_{\max} to I_{\min} . I_{\max} is found when the polarizer axis coincides with the semi-major axis of the ellipse and I_{\min} occurs when the polarizer axis coincides with the semi-minor axis of the ellipse.

WORKED OUT PROBLEMS

Example 20.1: Unpolarized light falls on two polarizing sheets placed one on top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of the transmitted light is one third intensity of the incident beam?

Solution:

Intensity of the light transmitted through the first polarizer $I_1 = I_0 / 2$,

where I_0 is the intensity of the incident unpolarized light.

Intensity of the light transmitted through the second polarizer is $I_2 = I_1 \cos^2 \theta$

where θ is the angle between the characteristic directions of the polarizer sheets.

But $I_2 = I_0 / 3$ (given)

$$\therefore I_2 = I_1 \cos^2 \theta = I_0 \cos^2 \theta / 2 = I_0 / 3$$

$$\therefore \cos^2 \theta = 2 / 3$$

$$\therefore \cos \theta = 0.8165$$

$$\therefore \theta = 35.3^\circ$$

Example 20.2: If the plane of vibration of the incident beam makes an angle of 30° with the optic axis, compare the intensities of extraordinary and ordinary light.

Solution: Intensity of the extraordinary ray $I_e = E^2 \cos^2 \theta$

Intensity of the ordinary ray $I_o = E^2 \sin^2 \theta$

$$\frac{I_e}{I_o} = \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{\cos^2 30^\circ}{\sin^2 30^\circ} = 3$$



Example 20.3: A plane-polarized light is incident perpendicularly on a quartz plate cut with faces parallel to optic axis. Find the thickness of quartz plate, which introduces phase difference of 60° between e- and o-rays.

Solution:

The path difference between the waves is given by $\Delta = (\mu_e - \mu_o)d$

The phase difference between the waves is given by $\delta = \frac{2\pi}{\lambda} \Delta$

$$\Delta = \frac{60^\circ}{360^\circ} \cdot \lambda = \frac{\lambda}{6}$$

$$\therefore d = \frac{\lambda}{6(\mu_e - \mu_o)}$$

$$d = \frac{5400 \text{ \AA}}{6(1.553 - 1.544)} = \frac{0.54}{0.054} \mu\text{m} = 10 \mu\text{m}$$

Example 20.4: Calculate the thickness of double refracting plate capable of producing a path difference of $\lambda / 4$ between extraordinary and ordinary waves. Given : $\lambda = 5890 \text{ \AA}$,

$$\mu_o = 1.53, \mu_e = 1.54.$$

Solution:

$$d = \frac{\lambda}{4(\mu_e - \mu_o)} = \frac{5890 \text{ \AA}}{4(1.54 - 1.53)} = \frac{0.589}{0.04} \mu\text{m} = 14.7 \mu\text{m}$$

Example 20.5: Calculate the least thickness of a calcite plate which would convert plane polarized light into circularly polarized light. Given $\mu_o = 1.658$, $\mu_e = 1.486$ and wavelength of light is 5890 \AA .

Solution:

Plane polarized light gets converted into circularly polarized light by a suitably oriented quarter wave plate. Its thickness is given by

$$\therefore d = \frac{\lambda}{4(\mu_o - \mu_e)} = \frac{5890}{4(1.658 - 1.486)}$$

or

$$d = 0.856 \mu\text{m}.$$

Example 20.6: Plane polarized light passes through a calcite plate with its optic axis parallel to the faces. Calculate the least thickness of the plate for which the emergent beam will be plane polarized. Given $\mu_o = 1.6584$, $\mu_e = 1.4864$ and wavelength of light is 5000 \AA .

Solution:

When a plane-polarized beam is incident on a half wave plate, the emergent beam will also be plane polarized but the plane of polarization undergoes a rotation through an angle 2θ .

$$d = \frac{\lambda}{2(\mu_o - \mu_e)} = \frac{5 \times 10^{-7} \text{ m}}{2(1.6584 - 1.4864)} = 1.45 \mu\text{m}$$

Example 20.7: The rotation in the plane of polarization in a certain substance is $10^\circ/\text{cm}$. Calculate the difference between the refractive indices for right and left circularly polarized light in the substance. Given $\lambda = 5893 \text{ \AA}$.

$$\text{Solution: } \delta = \frac{2\pi}{\lambda} [\mu_R - \mu_L] d \quad \therefore [\mu_R - \mu_L] = \frac{\delta \lambda}{2\pi d}$$



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It is given that $\frac{\delta}{d} = 10^\circ = \frac{10 \times 2\pi}{360^\circ} = \frac{\pi}{18}$ radian/cm and $\lambda = 5893 \text{ \AA} = 5893 \times 10^{-8} \text{ cm}$.

$$\mu_R - \mu_L = \frac{\pi}{18} \cdot \frac{5893 \times 10^{-8} \text{ cm}}{2\pi} = 1.6 \times 10^{-6}.$$