



# **Chapter One**

## **Starting with MATLAB**

### **Outlines of this chapter:**

1. Describing the characteristics and purpose of the different windows in MATLAB.
2. How to use MATLAB for arithmetic operations with scalars.
3. The use of elementary math functions with scalars.
4. How to use the scalar variables in arithmetic calculations.
5. Introduces script files.

## LECTURE 1-1

### 1.1 STARTING MATLAB, MATLAB WINDOWS

1. The MATLAB desktop window with the default layout, Figure 1-1.
2. The layout has

- Tool-strip at the top,
- Current Folder Toolbar below it, and
- Four windows underneath.

At the top of the Tool-strip there are three tabs:

- HOME, MATLAB is used with the HOME tab selected.
- PLOTS, used to create plots, and
- APPS used for opening additional applications and Toolboxes of MATLAB.

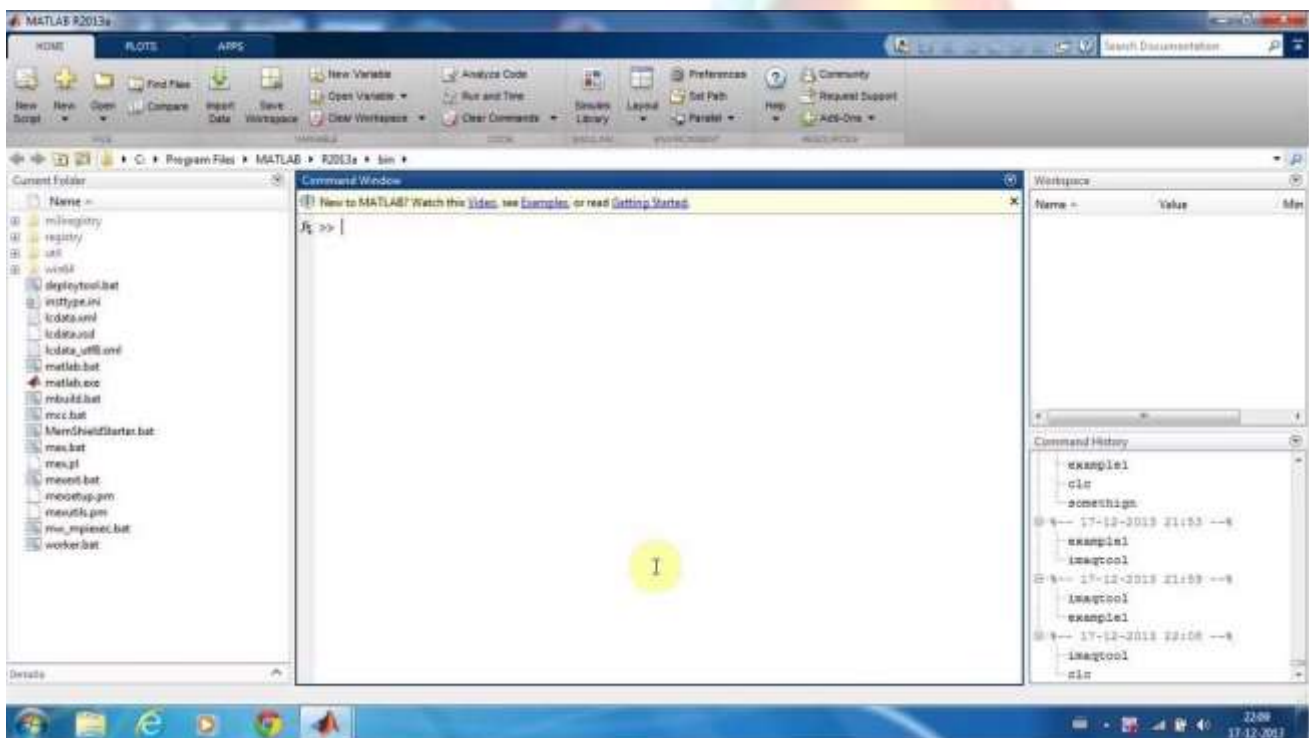


Figure (1-1) The default view of Matlab Desktop

Four windows that are displayed under the Tool strip:

- the Command Window (larger window at the center),
- the Current Folder Window (on the left) and
- the Workspace and
- Command History windows.

A list of several MATLAB windows and their purposes is given in Table 1-1.

**Table 1-1: MATLAB windows**

<b>Window</b>	<b>Purpose</b>
Command Window	Main window, enters variables, runs programs.
Figure Window	Contains output from graphic commands.
Editor Window	Creates and debugs script and function files.
Help Window	Provides help information.
Command History Window	Logs commands entered in the Command Window.

**Table 1-1: MATLAB windows**

<b>Window</b>	<b>Purpose</b>
Workspace Window	Provides information about the variables that are stored.
Current Folder Window	Shows the files in the current folder.

## 1.2 Working in the Command Window

Notes for working in the Command Window:

### The semicolon ( ; ):

The use of: in the Command Window, the output is not displayed in the Command Window.

### Typing%:

When the symbol % (percent) is typed at the beginning of a line, the line is designated as a comment.

### The clc command:

The clc command clears the Command Window.

## 1.3 Arithmetic Operations with Scalars

1. The symbols of arithmetic operations with scalars and variables are given below

<i>Operation</i>	<i>Symbol</i>	<i>Example</i>
Addition	+	5+3
Subtraction	-	5-3
Multiplication	*	5*3
Right division	/	5/3
Left division	\	5\3=3/5 (with arrays)
Exponentiation	^	5^3 (means 5^3= 125)

### 1.3.1 Order of Precedence

1. MATLAB executes the calculations according to the order of precedence displayed below.

<u>Precedence</u>	<u>Mathematical Operation</u>
First	Parentheses. For nested parentheses, the innermost are executed first.
Second	Exponentiation.
Third	Multiplication, division (equal precedence).
Fourth	Addition and subtraction.

2. If two or more operations have the same precedence, the expression is executed from left to right.

### 1.3.2 Using MATLAB as a Calculator

- This is done in the Command Window by typing a mathematical expression and pressing the Enter key.
- This is demonstrated in Tutorial 1-1.

**Tutorial 1-1: Using MATLAB as a calculator.**

The screenshot shows the MATLAB Command Window with several calculations and their results. Annotations with arrows point to specific parts of the code to explain the order of operations.

```
>> 7+8/2
ans =
    11
>> (7+8)/2
ans =
    7.5000
>> 4+5/3+2
ans =
    7.6667
>> 5^3/2
ans =
    62.5000
>> 27^(1/3)+32^0.2
ans =
    5
>> 27^1/3+32^0.2
ans =
    11
>> 0.7854 - (0.7854)^3/(1*2*3)+0.785^5/(1*2*3*4*5)...
- (0.785)^7/(1*2*3*4*5*6*7)
ans =
    0.7071
>>
```

**Annotations:**

- For `7+8/2`: "Type and press Enter." and "8/2 is executed first."
- For `(7+8)/2`: "Type and press Enter." and "7+8 is executed first."
- For `4+5/3+2`: "5/3 is executed first."
- For `5^3/2`: "5^3 is executed first, /2 is executed next."
- For `27^(1/3)+32^0.2`: "1/3 is executed first, 27^(1/3) and 32^0.2 are executed next, and + is executed last."
- For `27^1/3+32^0.2`: "27^1 and 32^0.2 are executed first, /3 is executed next, and + is executed last."
- For the Taylor series expression: "Type three periods ... (and press Enter) to continue the expression on the next line." and "The last expression is the first four terms of the Taylor series for  $\sin(\pi/4)$ ."



## LECTURE 2-1

### 1.4 DISPLAY FORMATS

- ❖ The user can control the format in which MATLAB displays output on the screen.
- ❖ Default format for numerical values with four decimal digits (called short).
- ❖ Several of the available formats are listed and described in Table 1-2.

**Table 1-2: Display formats**

Command	Description	Example
<code>format short</code>	Fixed-point with 4 decimal digits for: $0.001 \leq \text{number} \leq 1000$ Otherwise display format short e.	<pre>&gt;&gt; 290/7 ans =     41.4286</pre>
<code>format long</code>	Fixed-point with 15 decimal digits for: $0.001 \leq \text{number} \leq 100$ Otherwise display format long e.	<pre>&gt;&gt; 290/7 ans =     41.428571428571431</pre>
<code>format short e</code>	Scientific notation with 4 decimal digits.	<pre>&gt;&gt; 290/7 ans =     4.1429e+001</pre>
<code>format long e</code>	Scientific notation with 15 decimal digits.	<pre>&gt;&gt; 290/7 ans =     4.142857142857143e+001</pre>
<code>format short g</code>	Best of 5-digit fixed or floating point.	<pre>&gt;&gt; 290/7 ans =     41.429</pre>
<code>format long g</code>	Best of 15-digit fixed or floating point.	<pre>&gt;&gt; 290/7 ans =     41.4285714285714</pre>
<code>format bank</code>	Two decimal digits.	<pre>&gt;&gt; 290/7 ans =     41.43</pre>
<code>format compact</code>	Eliminates empty lines to allow more lines with information displayed on the screen.	
<code>format loose</code>	Adds empty lines (opposite of compact).	

### 1.5 ELEMENTARY MATH BUILT-IN FUNCTIONS

- ✓ MATLAB has a very large library of built-in functions. A function has a name and an argument in parentheses.
- ✓ Some commonly used elementary MATLAB mathematical built-in functions are given in Tables 1-3 through 1-5.

Table 1-3: Elementary math functions

Function	Description	Example
<code>sqrt(x)</code>	Square root.	<pre>&gt;&gt; sqrt(81) ans =     9</pre>
<code>nthroot(x,n)</code>	Real $n$ th root of a real number $x$ . (If $x$ is negative $n$ must be an odd integer.)	<pre>&gt;&gt; nthroot(80,5) ans =     2.4022</pre>
<code>exp(x)</code>	Exponential ( $e^x$ ).	<pre>&gt;&gt; exp(5) ans =    148.4132</pre>

Table 1-3: Elementary math functions (Continued)

Function	Description	Example
<code>abs(x)</code>	Absolute value.	<pre>&gt;&gt; abs(-24) ans =     24</pre>
<code>log(x)</code>	Natural logarithm. Base $e$ logarithm ( $\ln$ ).	<pre>&gt;&gt; log(1000) ans =     6.9078</pre>
<code>log10(x)</code>	Base 10 logarithm.	<pre>&gt;&gt; log10(1000) ans =     3.0000</pre>
<code>factorial(x)</code>	The factorial function $x!$ ( $x$ must be a positive integer.)	<pre>&gt;&gt; factorial(5) ans =     120</pre>

Table 1-4: Trigonometric math functions

Function	Description	Example
<code>sin(x)</code> <code>sind(x)</code>	Sine of angle $x$ ( $x$ in radians). Sine of angle $x$ ( $x$ in degrees).	<pre>&gt;&gt; sin(pi/6) ans =     0.5000</pre>
<code>cos(x)</code> <code>cosd(x)</code>	Cosine of angle $x$ ( $x$ in radians). Cosine of angle $x$ ( $x$ in degrees).	<pre>&gt;&gt; cosd(30) ans =     0.8660</pre>
<code>tan(x)</code> <code>tand(x)</code>	Tangent of angle $x$ ( $x$ in radians). Tangent of angle $x$ ( $x$ in degrees).	<pre>&gt;&gt; tan(pi/6) ans =     0.5774</pre>
<code>cot(x)</code> <code>cotd(x)</code>	Cotangent of angle $x$ ( $x$ in radians). Cotangent of angle $x$ ( $x$ in degrees).	<pre>&gt;&gt; cotd(30) ans =     1.7321</pre>

Table 1-5: Rounding functions

Function	Description	Example
<code>round(x)</code>	Round to the nearest integer.	<pre>&gt;&gt; round(17/5) ans =      3</pre>
<code>fix(x)</code>	Round toward zero.	<pre>&gt;&gt; fix(13/5) ans =      2</pre>

Table 1-5: Rounding functions (Continued)

Function	Description	Example
<code>ceil(x)</code>	Round toward infinity.	<pre>&gt;&gt; ceil(11/5) ans =      3</pre>
<code>floor(x)</code>	Round toward minus infinity.	<pre>&gt;&gt; floor(-9/4) ans =     -3</pre>
<code>rem(x,y)</code>	Returns the remainder after $x$ is divided by $y$ .	<pre>&gt;&gt; rem(13,5) ans =      3</pre>
<code>sign(x)</code>	Signum function. Returns 1 if $x > 0$ , -1 if $x < 0$ , and 0 if $x = 0$ .	<pre>&gt;&gt; sign(5) ans =      1</pre>

Tutorial 1-2: Using the `sqrt` built-in function.

```
>> sqrt(64)
```

```
ans =
     8
```

Argument is a number.

```
>> sqrt(50+14*3)
```

```
ans =
  9.5917
```

Argument is an expression.

```
>> sqrt(54+9*sqrt(100))
```

```
ans =
    12
```

Argument includes a function.

```
>> (15+600/4)/sqrt(121)
```

```
ans =
    15
```

Function is included in an expression.

```
>>
```



## 1.6 DEFINING SCALAR VARIABLES

- Variable is a name made of a letter or a combination of several letters and digits that is assigned a numerical value.
- Once a variable is assigned a numerical value, it can be used in mathematical expressions, in functions, and in any MATLAB statements and commands.

### 1.6.1 The Assignment Operator

The assignment operator = assigns a value to a variable.

*Variable\_name = A numerical value, or a computable expression*

#### Examples

```
>> x=15
```

The number 15 is assigned to the variable x.

```
x =  
    15
```

MATLAB displays the variable name and its assigned value.

```
>> x=3*x-12
```

```
x =  
    33
```

A new value is assigned to x. The new value is 3 times the previous value of x minus 12.

```
>> a=12
```

Assign 12 to a.

```
a =  
    12
```

```
>> B=4
```

Assign 4 to B.

```
B =  
     4
```

```
>> C=(a-B)+40-a/B*10
```

```
C =  
    18
```

Assign the value of the expression on the right-hand side to the variable C.

```
>> a=12;
```

```
>> B=4;
```

```
>> C=(a-B)+40-a/B*10;
```

The variables a, B, and C are defined but are not displayed, since a semicolon is typed at the end of each statement.

```
>> C
```

```
C =  
    18
```

The value of the variable C is displayed by typing the name of the variable.

```
>> a=12, B=4; C=(a-B)+40-a/B*10
```

```
a =  
    12
```

```
C =  
    18
```

The variable B is not displayed because a semicolon is typed at the end of the assignment.



```
>> x=0.75;
>> E=sin(x)^2+cos(x)^2
E =
    1
>>
```

### 1.6.2 Rules about Variable Names

A variable can be named according to the following rules:

- ☒ Must begin with a letter.
- ☒ Can be up to 63 characters long.
- ☒ Can contain letters, digits, and the underscore character.
- ☒ Cannot contain punctuation characters (e.g., period, comma, and semicolon).
- ☒ MATLAB is case-sensitive: it distinguishes between uppercase and lowercase letters.
- ☒ No spaces are allowed between characters (use the underscore).
- ☒ Avoid using the name of a built-in function for a variable.

### 1.6.3 Predefined Variables and Keywords

1. There are 20 words, called keywords that are reserved by MATLAB for various purposes and cannot be used as variable names. These words are:

break case end for persistent catch classdef continue else function global if  
otherwise return spmd switch try while elseif parfor

2. Some of the predefined variables are:

Pre. Var.	Meaning
pi	The number $\pi$ .
ans	A variable that has the value of the last expression that was not assigned to a specific variable
eps	The smallest difference between two numbers which is approximately 2.2204e-016.
inf	Used for infinity.
i	as sqrt(-1), which is: 0 + 1.0000i. j Same as i.
NaN	Stands for Not-a-Number. Used when MATLAB cannot determine a valid numeric value. Example: 0/0

## LECTURE 3-1

### 1.7 USEFUL COMMANDS FOR MANAGING VARIABLES

1. The following are commands that can be used to eliminate variables or to obtain information about variables that have been created.

<u>Command</u>	<u>Outcome</u>
clear	Removes all variables from the memory
clear x y z	Removes only variables x, y, and z from the memory
Who	Displays a list of the variables currently in the memory
whos	Displays a list of the variables currently in the memory and their sizes together with information about their bytes and class.

### 1.8 SCRIPT FILES

The use of the script file instead of Command Window due to these reasons:

- ✚ The Command Window is difficult for execution more commands.
- ✚ The Commands in the Command Window cannot be saved and executed again.
- ✚ Only the last command is executed.
- ✚ The commands in the Command Window cannot be corrected or changed and the file cannot be saved and run again.

### 1.9 EXAMPLES OF MATLAB APPLICATIONS

Sample Problem 1-1: Trigonometric identity

A trigonometric identity is given by:

$$\cos^2 \frac{x}{2} = \frac{\tan x + \sin x}{2 \tan x}$$

Verify that the identity is correct by calculating each side of the equation, substituting  $x = \frac{\pi}{5}$ .

#### Solution

The problem is solved by typing the following commands in the Command Window.

```
>> x=pi/5;
>> LHS=cos(x/2)^2
LHS =
    0.9045
>> RHS=(tan(x)+sin(x))/(2*tan(x))
RHS =
    0.9045
```

Define x.

Calculate the left-hand side.

Calculate the right-hand side.

Sample Problem 1-2: Heat transfer

An object with an initial temperature of  $T_0$  that is placed at time  $t = 0$  inside a chamber that has a constant temperature of  $T_s$  will experience a temperature change according to the equation

$$T = T_s + (T_0 - T_s)e^{-kt}$$

where  $T$  is the temperature of the object at time  $t$ , and  $k$  is a constant. Determine, to the nearest degree, the temperature after three hours. Assume  $k = 0.45$ . First define all of the variables and then calculate the temperature using one MATLAB command.

Solution

```
>> Ts=38; T0=120; k=0.45; t=3;
```

```
>> T=round(Ts+(T0-Ts)*exp(-k*t))
```

```
T =  
    59
```

Round to the nearest integer.

**1.10 PROBLEMS**

The following problems can be solved by writing commands in the Command Window, or by writing a program in a script file and then executing the file.

1. Calculate:

$$(a) \frac{22 + 5.1^2}{50 - 6.3^2}$$

$$(b) \frac{44}{7} + \frac{8^2}{5} - \frac{99}{3.9^2}$$

2. Calculate:

$$(a) \frac{\sqrt{41^2 - 5.2^2}}{e^5 - 100.53}$$

$$(b) \sqrt[3]{132} + \frac{\ln(500)}{8}$$

3. Calculate:

$$(a) \frac{14.8^3 - 6.3^2}{(\sqrt{13} + 5)^2}$$

$$(b) 45\left(\frac{288}{9.3} - 4.6^2\right) - 1065e^{-1.5}$$

4. Calculate:

$$(a) \frac{24.5 + 64/3.5^2 + 8.3 \cdot 12.5^3}{\sqrt{76.4} - 28/15}$$

$$(b) (5.9^2 - 2.4^2)/3 + \left(\frac{\log_{10} 12890}{e^{0.3}}\right)^2$$

5. Calculate:

$$(a) \cos\left(\frac{7\pi}{9}\right) + \tan\left(\frac{7}{15}\pi\right) \sin(15^\circ)$$

$$(b) \sin^2 80^\circ - \frac{(\cos 14^\circ \sin 80^\circ)^2}{\sqrt[3]{0.18}}$$

6. Define the variable  $x$  as  $x = 6.7$ , then evaluate:

$$(a) 0.01x^5 - 1.4x^3 + 80x + 16.7$$

$$(b) \sqrt{x^3 + e^x - 51/x}$$

7. Define the variable  $t$  as  $t = 3.2$ , then evaluate:

$$(a) 56t - 9.81\frac{t^2}{2}$$

$$(b) 14e^{-0.1t} \sin(2\pi t)$$



8. Define the variables  $x$  and  $y$  as  $x = 5.1$  and  $y = 4.2$ , then evaluate:

$$(a) \frac{3}{4}xy - \frac{7x}{y^2} + \sqrt{xy}$$

$$(b) (xy)^2 - \frac{x+y}{(x-y)^2} + \sqrt{\frac{x+y}{2x-y}}$$

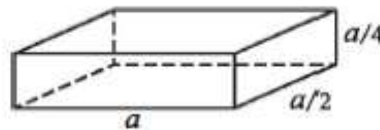
9. Define the variables  $a$ ,  $b$ ,  $c$ , and  $d$  as:

$$a = 12, b = 5.6, c = \frac{3a}{b^2}, \text{ and } d = \frac{(a-b)^c}{c}, \text{ then evaluate:}$$

$$(a) \frac{a}{b} + \frac{d-c}{d+c} - (d-b)^2$$

$$(b) e^{\frac{d-c}{a-2b}} + \ln\left(c - d + \frac{b}{a}\right)$$

10. A sphere has a radius of 24 cm. A rectangular prism has sides of  $a$ ,  $a/2$ , and  $a/4$ .

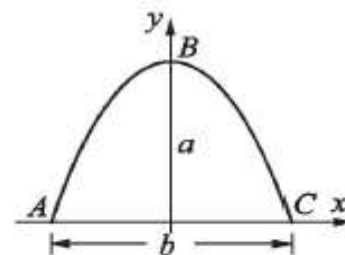


- (a) Determine  $a$  of a prism that has the same volume as the sphere.  
 (b) Determine  $a$  of a prism that has the same surface area as the sphere.

11. The arc length of a segment of a parabola  $ABC$  of an ellipse with semi-minor axes  $a$  and  $b$  is given approximately by:

$$L_{ABC} = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a} \ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right).$$

- (a) Determine  $L_{ABC}$  if  $a = 11$  in. and  $b = 9$  in.



12. Two trigonometric identities are given by:

$$(a) \sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x \quad (b) \sin^2 x \cos^2 x = \frac{1}{8} \cos 4x$$

For each part, verify that the identity is correct by calculating the values of the left and right sides of the equation, substituting  $x = \frac{\pi}{12}$ .

13. Two trigonometric identities are given by:

$$(a) \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \quad (b) \cos 4x = 8(\cos^4 x - \cos^2 x) + 1$$

For each part, verify that the identity is correct by calculating the values of the left and right sides of the equation, substituting  $x = 24^\circ$ .

14. Define two variables:  $\alpha = \pi/6$ , and  $\beta = 3\pi/8$ . Using these variables, show that the following trigonometric identity is correct by calculating the values of the left and right sides of the equation.

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

15. Given:  $\int x \sin ax dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$ . Use MATLAB to calculate the following definite integral:  $\int_{\frac{\pi}{3}}^{\frac{3\pi}{2}} x \sin(0.6x) dx$ .

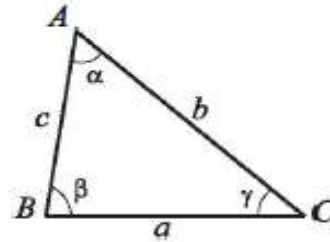
16. In the triangle shown  $a = 5.3$  in.,  $\gamma = 42^\circ$ , and  $b = 6$  in. Define  $a$ ,  $\gamma$ , and  $b$  as variables, and then:

(a) Calculate the length  $b$  by using the Law of Cosines.

$$(\text{Law of Cosines: } c^2 = a^2 + b^2 - 2ab \cos \gamma)$$

(b) Calculate the angles  $\beta$  and  $\gamma$  (in degrees) using the Law of Cosines.

(c) Check that the sum of the angles is  $180^\circ$ .



17. In the triangle shown  $a = 5$  in.,  $b = 7$  in., and  $\gamma = 25^\circ$ . Define  $a$ ,  $b$ , and  $\gamma$  as variables, and then:

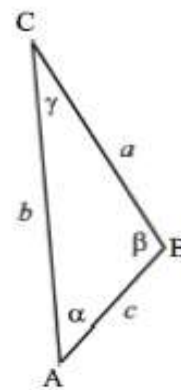
(a) Calculate the length of  $c$  by substituting the variables in the Law of Cosines.

$$(\text{Law of Cosines: } c^2 = a^2 + b^2 - 2ab \cos \gamma)$$

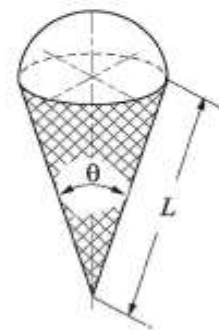
(b) Calculate the angles  $\alpha$  and  $\beta$  (in degrees) using the Law of Sines.

(c) Verify the Law of Tangents by substituting the results from part (b) into the right and left sides of the equation.

$$\text{Law of Tangents: } \frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\left[\frac{1}{2}(\alpha+\beta)\right]}$$



18. In the ice cream cone shown,  $L = 4$  in. and  $\theta = 35^\circ$ . The cone is filled with ice cream such that the portion above the cone is a hemisphere. Determine the volume of the ice cream.



19. For the triangle shown,  $a = 48$  mm,  $b = 34$  mm, and  $\gamma = 83^\circ$ . Define  $a$ ,  $b$ , and  $\gamma$  as variables, and then:

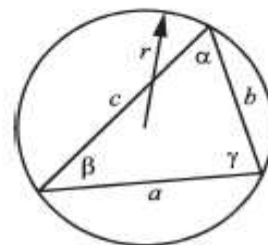
(a) Calculate  $c$  by substituting the variables in the Law of Cosines.

$$(\text{Law of Cosines: } c^2 = a^2 + b^2 - 2ab \cos \gamma)$$

(b) Calculate the radius  $r$  of the circle circumscribing the triangle using the formula:

$$r = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

$$\text{where } s = (a+b+c)/2.$$





20. The parametric equations of a line in space are:

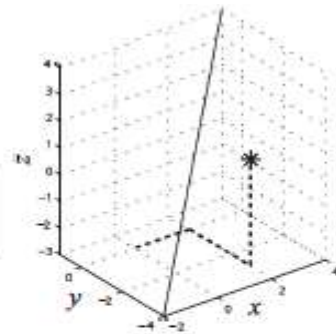
$x = x_0 + at$ ,  $y = y_0 + bt$ , and  $z = z_0 + ct$ . The distance  $d$  from a point  $A(x_A, y_A, z_A)$  to the line can be calculated by:

$$d = d_{A0} \sin \left[ \arccos \left( \frac{(x_A - x_0)a + (y_A - y_0)b + (z_A - z_0)c}{d_{A0} \sqrt{a^2 + b^2 + c^2}} \right) \right]$$

where  $d_{A0} = \sqrt{(x_A - x_0)^2 + (y_A - y_0)^2 + (z_A - z_0)^2}$ .

Determine the distance of the point  $A(2, -3, 1)$

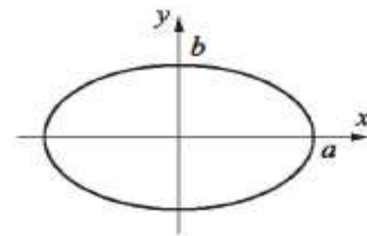
from the line  $x = -4 + 0.6t$ ,  $y = -2 + 0.5t$ , and  $z = -3 + 0.7t$ . First define the variables  $x_0$ ,  $y_0$ ,  $z_0$ ,  $a$ ,  $b$ , and  $c$ , then use the variable (and the coordinates of point  $A$ ) to calculate the variable  $d_{A0}$ , and finally calculate  $d$ .



21. The circumference of an ellipse can be approximated by:

$$C = \pi[3(a+b) - \sqrt{(3a+b)(a+3b)}]$$

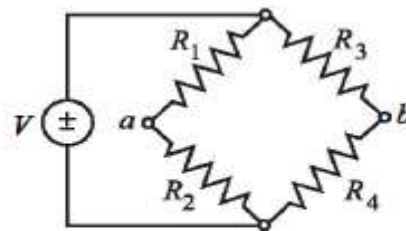
Calculate the circumference of an ellipse with  $a = 16$  in. and  $b = 11$  in.



22. 315 people have to be transported using buses that have 37 seats. By typing one line (command) in the Command Window, calculate how many seats will remain empty if enough buses will be ordered to transport all the people. (Hint: use MATLAB built-in function `ceil`.)
23. 739 apples are to be packed and shipped such that 54 are placed in a box. By typing one line (command) in the Command Window, calculate how many apples will remain unpacked if only full boxes can be shipped. (Hint: use MATLAB built-in function `fix`.)
24. Assign the number 316,501.673 to a variable, and then calculate the following by typing one command:
- (a) Round the number to the nearest hundredth.
  - (b) Round the number to the nearest thousand.
25. The voltage difference  $V_{ab}$  between points  $a$  and  $b$  in the Wheatstone bridge circuit is:

$$V_{ab} = V \left( \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} \right)$$

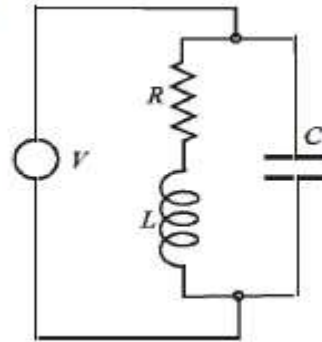
Calculate the voltage difference when  $V = 14$  volts,  $R_1 = 120.6$  ohms,  $R_2 = 119.3$  ohms,  $R_3 = 121.2$  ohms, and  $R_4 = 118.8$  ohms.



26. The resonant frequency  $f$  (in Hz) for the circuit shown is given by:

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

Calculate the resonant frequency when  $L = 0.15$  henrys,  $R = 14$  ohms, and  $C = 2.6 \times 10^{-6}$  farads.



27. The number of combinations  $C_{n,r}$  of taking  $r$  objects out of  $n$  objects is given by:

$$C_{n,r} = \frac{n!}{r!(n-r)!}$$

- (a) Determine how many combinations are possible in a lottery game for selecting 6 numbers that are drawn out of 49.  
 (b) Using the following formula, determine the probability of guessing two out of the six drawn numbers.

$$\frac{C_{6,2} C_{43,4}}{C_{49,6}}$$

(Use the built-in function `factorial`.)

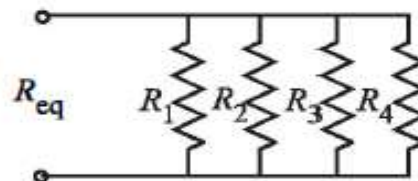
28. The formula for changing the base of a logarithm is:

$$\log_a N = \frac{\log_b N}{\log_b a}$$

- (a) Use MATLAB's function `log(x)` to calculate  $\log_4 0.085$ .  
 (b) Use MATLAB's function `log10(x)` to calculate  $\log_6 1500$ .

29. The equivalent resistance,  $R_{eq}$ , of four resistors,  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$ , that are connected in parallel is given by:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

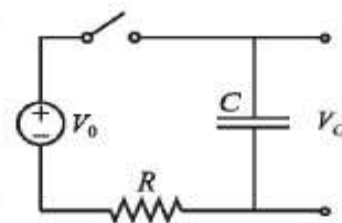


Calculate  $R_{eq}$  if  $R_1 = 120\Omega$ ,  $R_2 = 220\Omega$ ,  $R_3 = 75\Omega$ , and  $R_4 = 130\Omega$ .

30. The voltage  $V_C$   $t$  seconds after closing the switch in the circuit shown is:

$$V_C = V_0(1 - e^{-t/(RC)})$$

Given  $V_C = 36$  V,  $R = 2500\Omega$ , and  $C = 1600\mu\text{F}$ , calculate the current 8 seconds after the switch is closed.





31. Radioactive decay of carbon-14 is used for estimating the age of organic material. The decay is modeled with the exponential function  $f(t) = f(0)e^{kt}$ , where  $t$  is time,  $f(0)$  is the amount of material at  $t = 0$ ,  $f(t)$  is the amount of material at time  $t$ , and  $k$  is a constant. Carbon-14 has a half-life of approximately 5,730 years. A sample taken from the ancient footprints of Acahualinca in Nicaragua shows that 77.45% of the initial ( $t = 0$ ) carbon-14 is present. Determine the estimated age of the footprint. Solve the problem by writing a program in a script file. The program first determines the constant  $k$ , then calculates  $t$  for  $f(t) = 0.7745f(0)$ , and finally rounds the answer to the nearest year.

32. The greatest common divisor is the largest positive integer that divides the numbers without a remainder. For example, the GCD of 8 and 12 is 4. Use the MATLAB Help Window to find a MATLAB built-in function that determines the greatest common divisor of two numbers. Then use the function to show that the greatest common divisor of:

- (a) 91 and 147 is 7.  
(b) 555 and 962 is 37.

33. The Moment Magnitude Scale (MMS), denoted  $M_w$ , which measures the total energy released by an earthquake, is given by:

$$M_w = \frac{2}{3} \log_{10} M_0 - 10.7$$

where  $M_0$  is the magnitude of the seismic moment in dyne-cm (measure of the energy released during an earthquake). Determine how many times more energy was released from the largest earthquake in the world, in Chile ( $M_w = 9.5$ ), 1960, than the earthquake in Rat Island, Alaska ( $M_w = 8.7$ ), in 1965.

34. According to special relativity, a rod of length  $L$  moving at velocity  $v$  will shorten by an amount  $\delta$ , given by:

$$\delta = L \left( 1 - \sqrt{1 - \frac{v^2}{c^2}} \right)$$

where  $c$  is the speed of light (about  $300 \times 10^6$  m/s). Calculate how much a rod 2 m long will contract when traveling at 5,000 m/s.

35. The value  $B$  of a principal  $P$  that is deposited in a saving account with a fixed annual interest rate  $r$  after  $n$  years can be calculated by the formula:

$$B = P \left( 1 + \frac{r}{m} \right)^{nm}$$

where  $m$  is the number of times that the interest is compounded annually. Consider a \$80,000 deposit for 5 years. Determine how much more money will be earned if the interest is compounded daily instead of yearly.

36. Newton's law of cooling gives the temperature  $T(t)$  of an object at time  $t$  in terms of  $T_0$ , its temperature at  $t = 0$ , and  $T_s$ , the temperature of the surroundings.

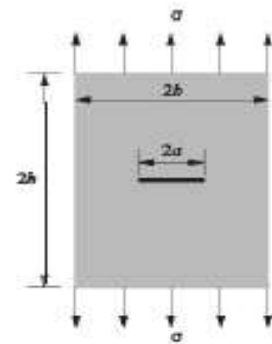
$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

A police officer arrives at a crime scene in a hotel room at 9:18 PM, where he finds a dead body. He immediately measures the body's temperature and find it to be  $79.5^{\circ}\text{F}$ . Exactly one hour later he measures the temperature again, and find it to be  $78.0^{\circ}\text{F}$ . Determine the time of death, assuming that victim body temperature was normal ( $98.6^{\circ}\text{F}$ ) prior to death, and that the room temperature was constant at  $69^{\circ}\text{F}$ .

37. The stress intensity factor  $K$  predicts the stress state (stress intensity) near a crack tip. For a plate with a crack and loading shown in the figure,  $K$  is given by:

$$K = \sigma \sqrt{\pi a} \left[ \frac{1 - \frac{a}{2b} + 0.326 \left( \frac{a}{b} \right)^2}{\sqrt{1 - \frac{a}{b}}} \right]$$

Determine  $K$  for the case where  $\sigma = 12000 \text{ psi}$ ,  $h = 5 \text{ in.}$ , and  $b = 4 \text{ in.}$ , and  $a = 1.5 \text{ in.}$



38. The spread of a computer virus through a computer network can be modeled by:

$$N(t) = 20e^{0.15t}$$

where  $N(t)$  is the number of computers infected and  $t$  time in minutes.

- Determine how long it takes for the number of infected computers to double.
- Determine how long it takes for 1,000,000 computers to be infected.

39. Use the Help Window to find a display format that displays the output as a ratio of integers. For example, the number 3.125 will be displayed as 25/8. Change the display to this format and execute the following operations:

- (a)  $5/8 + 16/6$  (b)  $1/3 - 11/13 + 2.7^2$

40. Stirling's approximation for large factorials is given by:

$$n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

Use the formula for calculating 20!. Compare the result with the true value obtained with MATLAB's built-in function `factorial` by calculating the error ( $Error = (TrueVal - ApproxVal) / TrueVal$ ).