③ $\int \sqrt{1-4x^2}\,dx = \int \sqrt{1-(2x)^2}\,dx \rightarrow$ Let $2x = \sin\theta$

$\rightarrow 2\,dx = \cos\theta\,d\theta \rightarrow dx = \frac{\cos\theta}{2}\,d\theta$ & $\theta = \sin^{-1}(2x)$

(ثم عوضنا واكمل الحل)

④ $\int \frac{dx}{x^2+16} = \frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right) + c$ (بموجب قانون اخذ سابقاً)

or Let $x = 4\tan\theta \rightarrow dx = 4\sec^2\theta\,d\theta$ & $\theta = \tan^{-1}\left(\frac{x}{4}\right)$

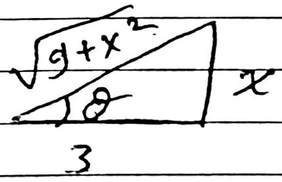
$\therefore \int \frac{dx}{x^2+16} = \int \frac{4\sec^2\theta\,d\theta}{16\tan^2\theta+16} = \int \frac{4\sec^2\theta\,d\theta}{16(\tan^2\theta+1)} = \int \frac{\sec^2\theta\,d\theta}{4\sec^2\theta}$

$= \frac{1}{4}\int d\theta = \frac{1}{4}\theta + c = \frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right) + c$

⑤ $\int \frac{dx}{\sqrt{9+x^2}} \rightarrow x = 3\tan\theta \rightarrow dx = 3\sec^2\theta\,d\theta$ & $\tan\theta = \frac{x}{3}$

$\therefore \int \frac{dx}{\sqrt{9+x^2}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9+9\tan^2\theta}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9(1+\tan^2\theta)}} = \int \frac{3\sec^2\theta\,d\theta}{3\sec\theta}$

$= \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + c$



$\therefore \int \frac{dx}{\sqrt{9+x^2}} = \ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right| + c$

⑥ $\int \frac{x^3\,dx}{(3+x^2)^{\frac{5}{2}}} \rightarrow x = \sqrt{3}\tan\theta \rightarrow dx = \sqrt{3}\sec^2\theta\,d\theta$

(ثم عوضنا واكمل الحل)

⑦ $\int \frac{dx}{x\sqrt{x^2-5}} = \frac{1}{\sqrt{5}}\sec^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$ (بموجب قانون أخذ سابقاً)