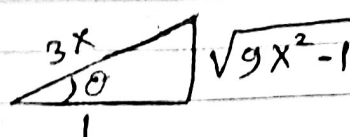


Let $x = \sqrt{5} \sec \theta \rightarrow dx = \sqrt{5} \sec \theta \tan \theta d\theta$ and $\theta = \sec^{-1}\left(\frac{x}{\sqrt{5}}\right)$

∴ $\int \frac{dx}{x \sqrt{x^2 - 5}} = \int \frac{\sqrt{5} \sec \theta \tan \theta d\theta}{\sqrt{5} \sec \theta \sqrt{5 \sec^2 \theta - 5}} = \int \frac{\tan \theta d\theta}{\sqrt{5} \tan \theta}$

$= \frac{1}{\sqrt{5}} \int d\theta = \frac{1}{\sqrt{5}} \theta + C = \frac{1}{\sqrt{5}} \sec^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$

8) $\int \frac{\sqrt{9x^2 - 1}}{x} dx = \int \frac{\sqrt{(3x)^2 - 1}}{x} dx$



∴ Let $3x = \sec \theta \rightarrow 3 dx = \sec \theta \tan \theta d\theta$

∴ $\int \frac{\sqrt{9x^2 - 1}}{x} dx = \int \frac{\sqrt{\sec^2 \theta - 1}}{\frac{\sec \theta}{3}} \cdot \frac{\sec \theta \tan \theta d\theta}{3}$

$= \int \tan \theta \cdot \tan \theta d\theta = \int \tan^2 \theta d\theta = \int (\sec^2 \theta - 1) d\theta$

$= \tan \theta - \theta + C = \sqrt{9x^2 - 1} - \sec^{-1}(3x) + C$

9) $\int \frac{dx}{x^2 \sqrt{4x^2 - 9}}$ $\rightarrow 2x = 3 \sec \theta \rightarrow 2 dx = 3 \sec \theta \tan \theta d\theta$

∴ $\int \frac{dx}{x^2 \sqrt{4x^2 - 9}} = \int \frac{\frac{3}{2} \sec \theta \tan \theta d\theta}{\frac{9}{4} \sec^2 \theta \sqrt{9 \sec^2 \theta - 9}}$ (تم بسط واكسر الكل)

Exercise: Find ① $\int \sqrt{4-x^2} dx$ ② $\int \frac{dx}{x^2 \sqrt{16-x^2}}$ ③ $\int x^3 \sqrt{16-x^2} dx$

④ $\int \frac{x^3 dx}{\sqrt{2-x^2}}$ ⑤ $\int \frac{dx}{(1-x^2)^{3/2}}$ ⑥ $\int \frac{dx}{(4+x^2)^2}$ ⑦ $\int \frac{\sqrt{1+x^2}}{x} dx$ ⑧ $\int \frac{dx}{x^2 \sqrt{x^2+25}}$

⑨ $\int \frac{dx}{(9x^2-1)^{3/2}}$ ⑩ $\int \frac{\sqrt{x^2-9}}{x} dx$ ⑪ $\int \frac{dx}{x^2 \sqrt{x^2-16}}$ ⑫ $\int \frac{dx}{\sqrt{x^2-1}}$ ⑬ $\int \frac{\sqrt{2x^2-4}}{x} dx$