

~~حيث ان A و B عدلان حقيقيان ينبغي ايجابهما وكما هو موضح في الامثلة الالآتية :-~~

amples: Find the integrals

$$\int \frac{x^3 + x^2 + x + 2}{x^4 + 3x^2 + 2} dx = \int \frac{x^3 + x^2 + x + 2}{(x^2 + 1)(x^2 + 2)} dx$$

$$\frac{x^3 + x^2 + x + 2}{(x^2 + 1)(x^2 + 2)} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{x^2 + 2}$$

$$\frac{x^3 + x^2 + x + 2}{(x^2 + 1)(x^2 + 2)} = \frac{(Ax + B)(x^2 + 2) + (Cx + D)(x^2 + 1)}{(x^2 + 1)(x^2 + 2)}$$

بفرد الطرفين بـ $(x^2 + 1)(x^2 + 2)$ ينتج أن :-

$$\begin{aligned} x^3 + x^2 + x + 2 &= (Ax + B)(x^2 + 2) + (Cx + D)(x^2 + 1) \\ x^3 + x^2 + x + 2 &= \underline{Ax^3} + \underline{2Ax} + \underline{Bx^2} + \underline{2B} + \underline{Cx^3} + \underline{Cx} + \underline{Dx^2} + \underline{D} \end{aligned}$$

بشأوي معاملات قوى x للطرفين ينتج أن :-

$$\begin{aligned} A + C &= 1 \\ B + D &= 1 \\ 2A + C &= 1 \\ 2B + D &= 2 \end{aligned}$$

بالطرف $A = 0 \rightarrow C = 1$
 بالطرف $B = 1 \rightarrow D = 0$

$$\therefore \frac{x^3 + x^2 + x + 2}{(x^2 + 1)(x^2 + 2)} = \frac{1}{x^2 + 1} + \frac{x}{x^2 + 2}$$

$$\begin{aligned} \therefore \int \frac{x^3 + x^2 + x + 2}{x^4 + 3x^2 + 2} dx &= \int \frac{dx}{x^2 + 1} + \int \frac{x dx}{x^2 + 2} = \tan^{-1} x + \frac{1}{2} \int \frac{2x dx}{x^2 + 2} \\ &= \tan^{-1} x + \frac{1}{2} \ln|x^2 + 2| + C \end{aligned}$$

$$2) \int \frac{x^2 + x - 2}{3x^3 - x^2 + 3x - 1} dx = \int \frac{x^2 + x - 2}{x^2(3x - 1) + (3x - 1)} dx = \int \frac{x^2 + x - 2}{(3x - 1)(x^2 + 1)} dx$$