amples: Find the integrals

$$\int \frac{x^3 + x^2 + x + 2}{x^4 + 3x^2 + 2} \, dx = \int \frac{x^3 + x^2 + x + 2}{(x^2+1)(x^2+2)} \, dx$$

$$\frac{x^3 + x^2 + x + 2}{(x^2+1)(x^2+2)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2}$$

$$\frac{x^3 + x^2 + x + 2}{(x^2+1)(x^2+2)} = \frac{(Ax+B)(x^2+2) + (Cx+D)(x^2+1)}{(x^2+1)(x^2+2)}$$

يضرب الطرفين بـ $(x^2+1)(x^2+2)$ ينتج أن :

$$x^3 + x^2 + x + 2 = (Ax+B)(x^2+2) + (Cx+D)(x^2+1)$$
$$x^3 + x^2 + x + 2 = \underline{Ax^3} + \underline{2Ax} + \underline{Bx^2} + 2B + \underline{Cx^3} + \underline{Cx} + \underline{Dx^2} + D$$

نساوي معاملات قوى x للطرفين ينتج أن :

$$\begin{aligned} A+C &= 1 \\ B+D &= 1 \\ 2A+C &= 1 \\ 2B+D &= 2 \end{aligned}$$

بالطرح $\longrightarrow A = 0 \Longrightarrow C = 1$

بالطرح $\longrightarrow B = 1 \Longrightarrow D = 0$

$$\therefore \frac{x^3 + x^2 + x + 2}{(x^2+1)(x^2+2)} = \frac{1}{x^2+1} + \frac{x}{x^2+2}$$

$$\therefore \int \frac{x^3 + x^2 + x + 2}{x^4 + 3x^2 + 2} \, dx = \int \frac{dx}{x^2+1} + \int \frac{x \, dx}{x^2+2} = \tan^{-1} x + \frac{1}{2} \int \frac{2x \, dx}{x^2+2}$$

$$= \tan^{-1} x + \frac{1}{2} \ln|x^2+2| + c$$

$$②\int \frac{x^2+x-2}{3x^3 - x^2 + 3x - 1} \, dx = \int \frac{x^2+x-2}{x^2(3x-1)+(3x-1)} \, dx = \int \frac{x^2+x-2}{(3x-1)(x^2+1)} \, dx$$