

$$= \int \frac{2dz}{2-2z} = \int \frac{dz}{1-z} = -\int \frac{-dz}{1-z} = -\ln|1-z| + C$$

$$= -\ln|1 - \tan(\frac{x}{2})| + C$$

$$\textcircled{2} \int \frac{dx}{\sin x + \tan x} = \int \frac{\frac{2dz}{1+z^2}}{\frac{2z}{1+z^2} + \frac{2z}{1-z^2}} = \int \frac{\frac{2dz}{1+z^2}}{\frac{2z-2z^3+2z+2z^3}{(1+z^2)(1-z^2)}}$$

$$= \int \frac{\frac{2dz}{1+z^2}}{\frac{4z}{(1+z^2)(1-z^2)}} = \int \frac{2(1+z^2)(1-z^2)}{(1+z^2) \cdot 4z} dz = \int \frac{(1-z^2)}{2z} dz$$

$$= \frac{1}{2} \int \frac{1-z^2}{z} dz = \frac{1}{2} \int \frac{1}{z} dz - \frac{1}{2} \int z dz$$

$$= \frac{1}{2} \ln|z| - \frac{z^2}{4} + C = \frac{1}{2} \ln|\tan(\frac{x}{2})| - \frac{1}{4} \tan^2(\frac{x}{2}) + C$$

$$\textcircled{3} \int \frac{dx}{1-\cos x} = \int \frac{\frac{2dz}{1+z^2}}{1 - \frac{1-z^2}{1+z^2}} = \int \frac{\frac{2dz}{1+z^2}}{\frac{1+z^2-1+z^2}{1+z^2}} = \int \frac{2dz}{2z^2}$$

$$= \int \frac{dz}{z^2} = \int z^{-2} dz = -\frac{1}{z} + C = -\frac{1}{\tan(\frac{x}{2})} + C$$

Exercise: Find ①  $\int x \sqrt{x-2} dx$  ②  $\int \frac{\sqrt{x-4}}{x} dx$  ③  $\int x^5 \sqrt{x^3+1} dx$

④  $\int \frac{dx}{3+\sqrt{x}}$  ⑤  $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$  ⑥  $\int \frac{dx}{x(1-x^{\frac{1}{4}})}$  ⑦  $\int \frac{x}{\sqrt{x+1}} dx$  ⑧  $\int \frac{x^3}{\sqrt{1+x^2}} dx$

⑨  $\int \sin \sqrt{x} dx$  ⑩  $\int \frac{dx}{\sqrt{1+e^x}}$  ⑪  $\int \frac{1+\sqrt{x}}{1-\sqrt{x}} dx$  ⑫  $\int x^2 \sqrt{x+1} dx$  ⑬  $\int \frac{dx}{x-\sqrt[3]{x}}$

⑭  $\int \frac{\sqrt{x}}{1+\sqrt[4]{x}} dx$  ⑮  $\int \frac{dx}{2+\sin x}$  ⑯  $\int \frac{dx}{1+\sin x + \cos x}$  ⑰  $\int \frac{dx}{1+\sin x}$