



على حدة: لا يسهل كل من

$$① \int \sin^m x \cos^n x dx \quad ② \int \sin^m x \sin^n x dx \quad ③ \int \cos^m x \cos^n x dx$$

حيث إن m و n أعداد حقيقية و $m \neq n$ تستخدم التطبيقات الآتية

$$\sin x \cos y = \frac{1}{2} \sin(x-y) + \frac{1}{2} \sin(x+y)$$

$$\cos x \cos y = \frac{1}{2} \cos(x-y) + \frac{1}{2} \cos(x+y)$$

$$\sin x \sin y = \frac{1}{2} \cos(x-y) - \frac{1}{2} \cos(x+y)$$

Examples

$$① \int \sin 7x \cos 3x dx = \int \left(\frac{1}{2} \sin(7-3)x + \frac{1}{2} \sin(7+3)x \right) dx$$

$$= \frac{1}{2} \int \sin 4x dx + \frac{1}{2} \int \sin 10x dx$$

$$= \frac{1}{8} \int 4 \sin 4x dx + \frac{1}{20} \int 10 \sin 10x dx = \frac{-1}{8} \cos 4x - \frac{1}{20} \cos 10x + C$$

$$\therefore \int \sin 7x \cos 3x dx = \frac{-1}{8} \cos 4x - \frac{1}{20} \cos 10x + C$$

$$② \int \sin 2x \sin 4x dx = \int \left[\frac{1}{2} \cos(2-4)x - \frac{1}{2} \cos(2+4)x \right] dx$$

$$= \frac{1}{2} \int \cos(-2x) dx - \frac{1}{2} \int \cos 6x dx = \frac{1}{2} \int \cos 2x dx - \frac{1}{2} \int \cos 6x dx$$

$$= \frac{1}{4} \int 2 \cos 2x dx - \frac{1}{12} \int 6 \cos 6x dx$$

$$\therefore \int \sin 2x \sin 4x dx = \frac{1}{4} \sin 2x - \frac{1}{12} \sin 6x + C$$

$$③ \int \cos 5x \cos 3x dx \quad (Ch)$$

Exercise: Find ① $\int \cos^3 x \sin x dx$ ② $\int \sin^2(5x) dx$ ③ $\int \cos^2(3x) dx$

$$④ \int \sin^2(2x) \cos^3(2x) dx \quad ⑤ \int \sin^3(2x) \cos^2(2x) dx \quad ⑥ \int \sin^2 x \cos^2 x dx$$

$$⑦ \int \sin 4x \cos 4x dx \quad ⑧ \int \sin x \cos 2x dx \quad ⑨ \int \cos 3x \cos 2x dx$$