Solu

ⓐ $\int \sin^2 x \, dx = \int (\frac{1}{2} - \frac{1}{2}\cos 2x)\,dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x \, dx$

$= \frac{1}{2}x - \frac{1}{4}\int 2\cos 2x \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

$\therefore \int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

ⓓ $\int \cos^4 x \, dx = \int (\cos^2 x)^2 \, dx = \int (\frac{1}{2} + \frac{1}{2}\cos 2x)^2 \, dx$

$= \int (\frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x)\,dx$

$= \frac{1}{4}\int dx + \frac{1}{2}\int \cos 2x \, dx + \frac{1}{4}\int \cos^2 2x \, dx$

$= \frac{1}{4}x + \frac{1}{4}\int 2\cos 2x \, dx + \frac{1}{4}\int (\frac{1}{2} + \frac{1}{2}\cos 4x)\,dx$

$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}\int dx + \frac{1}{8}\int \cos 4x \, dx$

$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\int 4\cos 4x \, dx$

$\therefore \int \cos^4 x \, dx = \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

② ⓐ $\int \sin^2 x \cos^4 x \, dx$ ⓑ $\int \sin^4 x \cos^4 x \, dx$

Solu

ⓐ $\int \sin^2 x \cos^4 x \, dx = \int (\frac{1}{2} - \frac{1}{2}\cos 2x)(\frac{1}{2} + \frac{1}{2}\cos 2x)^2 \, dx$

$= \int (\frac{1}{2} - \frac{1}{2}\cos 2x)(\frac{1}{2} + \frac{1}{2}\cos 2x)(\frac{1}{2} + \frac{1}{2}\cos 2x)\,dx$

$= \int (\frac{1}{4} - \frac{1}{4}\cos^2 2x)(\frac{1}{2} + \frac{1}{2}\cos 2x)\,dx$

$= \int (\frac{1}{8} + \frac{1}{8}\cos 2x - \frac{1}{8}\cos^2 2x - \frac{1}{8}\cos^3 2x)\,dx$

$= \frac{1}{8}x + \frac{1}{16}\sin 2x - \frac{1}{8}\int \cos^2 2x \, dx - \frac{1}{8}\int \cos^3 2x \, dx$ (لم اكمل الحل)