$\therefore 2\int e^{x}\cos x\,dx = e^{x}\sin x + e^{x}\cos x$

$\therefore \int e^{x}\cos x\,dx = \frac{1}{2}\left[e^{x}\sin x + e^{x}\cos x\right] + c$

(7) Show that

(a) $\int \sin^{n} x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{(n-1)}{n}\int \sin^{n-2} x\,dx$ (ch.)

(b) $\int \cos^{n} x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{(n-1)}{n}\int \cos^{n-2} x\,dx$

$n \in \mathbb{Z}^{+}$

proof: (b) $\int \cos^{n} x\,dx = \int \cos^{n-1} x\cos x\,dx$

$u = \cos^{n-1} x \qquad dv = \cos x\,dx$

$du = (n-1)\cos^{n-2} x\cdot(-\sin x)\,dx \qquad v = \sin x$

$\therefore \int \cos^{n} x\,dx = \int u\,dv = uv - \int v\,du$

$= \cos^{n-1} x\sin x - \int \sin x\cdot(n-1)\cos^{n-2} x\,(-\sin x)\,dx$

$= \cos^{n-1} x\sin x + (n-1)\int \sin^{2} x\cos^{n-2} x\,dx$

$= \cos^{n-1} x\sin x + (n-1)\int (1-\cos^{2} x)\cos^{n-2} x\,dx$

$= \cos^{n-1} x\sin x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^{n} x\,dx$

$\therefore (n-1)\int \cos^{n} x\,dx + \int \cos^{n} x\,dx = \cos^{n-1} x\sin x + (n-1)\int \cos^{n-2} x\,dx$

$n\int \cos^{n} x\,dx = \cos^{n-1} x\sin x + (n-1)\int \cos^{n-2} x\,dx$

$\therefore \int \cos^{n} x\,dx = \frac{1}{n}\cos^{n-1} \cancel{x}\sin x + \frac{(n-1)}{n}\int \cos^{n-2} x\,dx$

ملاحظة: في فرع (b) عندما نعوض $n=4$ ينتج أن: