$$\therefore \int \frac{dx}{x\sqrt{5x^2-3}} = \int \frac{\frac{du}{\sqrt{5}}}{\frac{u}{\sqrt{5}}\sqrt{u^2-3}} = \frac{\sqrt{5}}{\sqrt{5}}\int \frac{du}{u\sqrt{u^2-3}} = \int \frac{du}{u\sqrt{u^2-3}}$$

$$= \frac{1}{\sqrt{3}} \sec^{-1}\left(\frac{u}{\sqrt{3}}\right) + c = \frac{1}{\sqrt{3}} \sec^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{3}}\right) + c$$

(12) $\int \frac{dx}{\sqrt{1+9x^2}} = \int \frac{dx}{\sqrt{1+(3x)^2}}$ $\rightarrow$ Let $u = 3x \rightarrow du = 3\,dx \rightarrow \frac{du}{3} = dx$

$$\therefore \int \frac{dx}{\sqrt{1+9x^2}} = \int \frac{\frac{du}{3}}{\sqrt{1+u^2}} = \frac{1}{3}\int \frac{du}{\sqrt{1+u^2}} = \frac{1}{3}\sinh^{-1} u + c = \frac{1}{3}\sinh^{-1}(3x) + c$$

(13) $\int \frac{dx}{\sqrt{x^2-2}} = \cosh^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$

(14) $\int \frac{e^x\,dx}{1-e^{2x}} = \int \frac{e^x\,dx}{1-(e^x)^2}$ $\rightarrow$ Let $u = e^x \rightarrow du = e^x\,dx$

$$\therefore \int \frac{e^x\,dx}{1-e^{2x}} = \int \frac{du}{1-u^2} = \tanh^{-1} u + c = \tanh^{-1}(e^x) + c$$

Exercise

Find the integrals

(1) $\int \frac{dx}{\sqrt{2-x^2}}$ (2) $\int \frac{x\,dx}{\sqrt{1-x^4}}$ (3) $\int \frac{dx}{\sqrt{9-4x^2}}$ (4) $\int \frac{\sec^2 x\,dx}{\sqrt{1-\tan^2 x}}$

(5) $\int \frac{e^{-x}\,dx}{\sqrt{1-e^{-2x}}}$ (6) $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$ (7) $\int \frac{e^x\,dx}{1+e^{2x}}$ (8) $\int \frac{x\,dx}{1+x^4}$

(9) $\int \frac{dx}{\sqrt{x}\,(1+x)}$ (10) $\int \frac{x\,dx}{3+x^4}$ (11) $\int \frac{dx}{4+9x^2}$ (12) $\int \frac{e^{x+1}\,dx}{4+e^{2x+2}}$

(13) $\int \frac{dx}{1+16x^2}$ (14) $\int \frac{dx}{x\sqrt{9x^2-1}}$ (15) $\int \frac{\cos x\,dx}{\sin x\sqrt{\sin^2 x-1}}$ (16) $\int \frac{dx}{\sqrt{4x^2-9}}$

(17) $\int \frac{dx}{\sqrt{1-e^{2x}}}$ (18) $\int \frac{\sin x\,dx}{\sqrt{1+\cos^2 x}}$ (19) $\int \frac{dx}{x\sqrt{1+4x^2}}$ (20) $\int \frac{dx}{\sqrt{9x^2-25}}$

(21) $\int \frac{dx}{x\sqrt{1+x^6}}$ (22) $\int \frac{dx}{x\sqrt{(\ln x)^2-1}}$ (23) $\int \frac{x^2\,dx}{\sqrt{1+x^6}}$ (24) $\int \frac{dx}{e^x\sqrt{1-e^{-2x}}}$