

تكاميل الدوال الزائرية العكسية

$$\textcircled{1} \int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1} x + c \quad \text{and} \quad \int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + c$$

$$\textcircled{2} \int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x + c \quad \text{and} \quad \int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + c$$

$$\textcircled{3} \int \frac{dx}{1-x^2} = \tanh^{-1} x + c \quad \text{and} \quad \int \frac{dx}{a^2-x^2} = \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) + c$$

$$\textcircled{4} \int \frac{dx}{1-x^2} = \coth^{-1} x + c \quad \text{and} \quad \int \frac{dx}{a^2-x^2} = \frac{1}{a} \coth^{-1}\left(\frac{x}{a}\right) + c$$

$$\textcircled{5} \int \frac{dx}{x\sqrt{1-x^2}} = -\operatorname{sech}^{-1}|x| + c \quad \text{and} \quad \int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a} \operatorname{sech}^{-1}\left|\frac{x}{a}\right| + c$$

$$\textcircled{6} \int \frac{dx}{x\sqrt{1+x^2}} = -\operatorname{csch}^{-1}|x| + c \quad \text{and} \quad \int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a} \operatorname{csch}^{-1}\left|\frac{x}{a}\right| + c$$

Examples: Find the integrals

$$\textcircled{1} \int \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\left(\frac{x}{3}\right) + c$$

$$\textcircled{2} \int \frac{dx}{\sqrt{1-4x^2}} = \int \frac{dx}{\sqrt{1-(2x)^2}} \rightarrow \text{Let } u=2x \rightarrow du=2dx \rightarrow \frac{du}{2}=dx$$

$$\therefore \int \frac{dx}{\sqrt{1-4x^2}} = \int \frac{\frac{du}{2}}{\sqrt{1-u^2}} = \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2} \sin^{-1} u + c = \frac{1}{2} \sin^{-1}(2x) + c$$

$$\textcircled{3} \int \frac{e^x}{\sqrt{1-e^{2x}}} dx = \int \frac{e^x}{\sqrt{1-(e^x)^2}} \rightarrow u=e^x \rightarrow du=e^x dx$$

$$\therefore \int \frac{e^x}{\sqrt{1-e^{2x}}} dx = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + c = \sin^{-1}(e^x) + c$$

$$\textcircled{4} \int \frac{e^x}{\sqrt{4-e^{2x}}} dx = \int \frac{e^x}{\sqrt{4-(e^x)^2}} \rightarrow \text{Let } u=e^x \rightarrow du=e^x dx$$