

$$(4) \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\int \frac{-\sin x}{\cos x} \, dx = -\ln|\cos x| + C$$

$$(5) \int \frac{dx}{x \ln x} = \int \frac{\frac{dx}{x}}{\ln x} = \ln|\ln x| + C \quad (6) \int \frac{e^x + e^{-x}}{e^x - e^{-x}} \, dx = \ln|e^x - e^{-x}| + C$$

$$(7) \int \frac{\sin 3x}{1 + \cos 3x} \, dx = \frac{-1}{3} \int \frac{-3 \sin 3x}{1 + \cos 3x} \, dx = \frac{-1}{3} \ln|1 + \cos 3x| + C$$

$$(8) \int \frac{dx}{1 + e^x} = \int \frac{e^{-x}}{(1 + e^x)e^{-x}} \, dx = \int \frac{e^{-x}}{e^{-x} + 1} \, dx = -\int \frac{-e^{-x}}{e^{-x} + 1} \, dx = -\ln|e^{-x} + 1| + C$$

$$(9) \int \frac{\sec^2 x}{3 \tan(x) + 5} \, dx = \frac{1}{3} \int \frac{3 \sec^2 x}{3 \tan(x) + 5} \, dx = \frac{1}{3} \ln|3 \tan(x) + 5| + C$$

$$(10) \int (3x+1) \cot(3x^2+2x) \, dx = \int (3x+1) \frac{\cos(3x^2+2x)}{\sin(3x^2+2x)} \, dx$$

$$= \frac{1}{2} \int (6x+2) \frac{\cos(3x^2+2x)}{\sin(3x^2+2x)} \, dx = \frac{1}{2} \ln|\sin(3x^2+2x)| + C$$

### Exercise

Find the integrals

$$(1) \int \frac{(1+e^x)}{e^x} \, dx \quad (2) \int 4x \tan(x^2) \, dx \quad (3) \int \frac{\ln x}{x} \, dx \quad (4) \int \frac{\sec(5x) \tan(5x)}{2 \sec(5x) - 1} \, dx$$

$$(5) \int \frac{2e^{2x} + 1}{e^{2x} + x} \, dx \quad (6) \int \cot x \, dx \quad (7) \int \frac{\sec^2 x}{\tan x} \, dx \quad (8) \int \frac{\sinh 2x}{3 + 5 \cosh 2x} \, dx \quad (9) \int \tanh x \, dx$$

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$$(1) \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C \quad \text{and} \quad \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \left( \frac{x}{a} \right) + C \quad (a > 0)$$

$$(2) \int \frac{dx}{1+x^2} = \tan^{-1} x + C \quad \text{and} \quad \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C \quad (a \neq 0)$$

$$(3) \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C \quad \text{and} \quad \int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1} \left( \frac{x}{a} \right) + C \quad (a > 0)$$