Magnetism Lecture
1st Stage

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Electromagnetism

Textbooks

- “Introduction to electrodynamics”, David J. Griffiths (1999, 3rd edition)
- “Classical electromagnetism” R.H. Good (1999)
- “Electromagnetic fields and waves”, P. Lorrain, D.R. Corson, F. Lorrain (1987, 3rd edition)

History of Electromagnetism

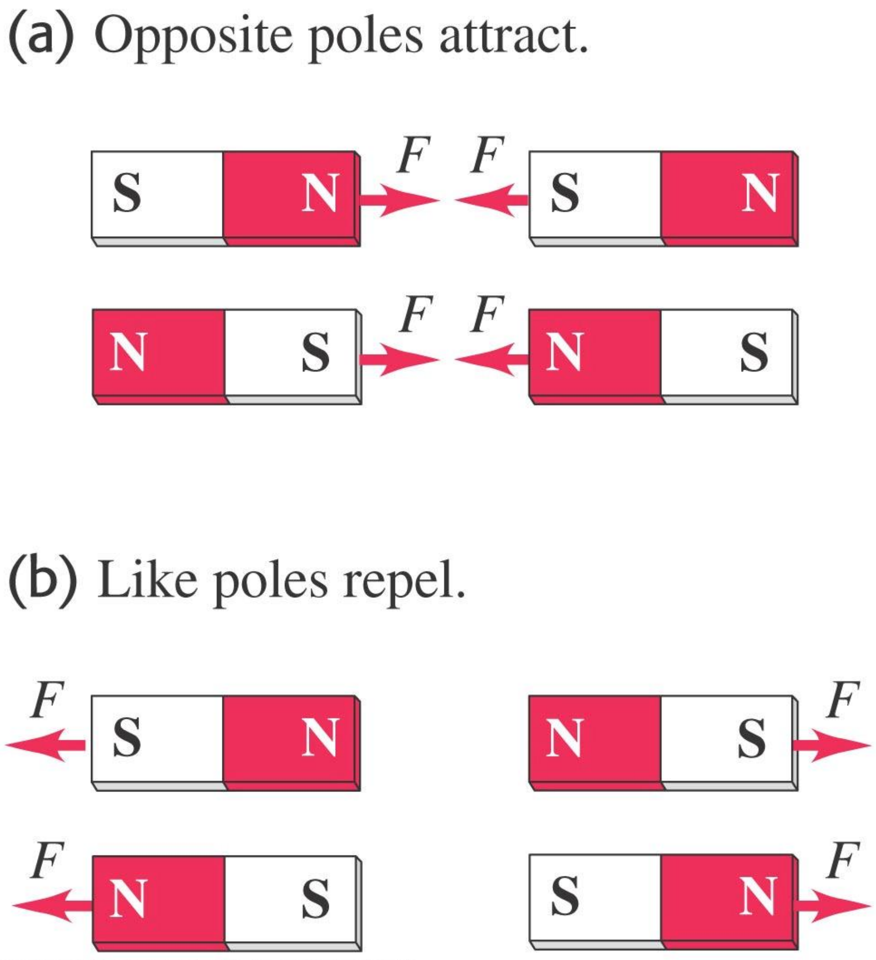
- Static electricity
  - du Fay(France, 1698–1739), Franklin (United States, 1706-1790), Priestly, Cavendish, Coulomb (France, 1736-1806), Maxwell (Scotland, 1831-1879)

    Independently, Franklin (1747) and Watson (1746) stated the principle of conservation of the quantity of electrical charge.
- Static magnetism
  - Gilbert (England, 1540-1603), Descartes, Coulomb, Poisson (France, 1781-1840), Green, Gauss (Germany, 1777-1855)
- Electromagnetism
  - Oersted (Denmark, 1777-1851), Arago, Ampere (France, 1775-1836), Davy, Fresnel, Faraday (England, 1791-1867), Maxwell, Tesla
- Classical field theory
  - Non-relativistic and relativistic
- Quantum mechanics
  - Microscopic understanding

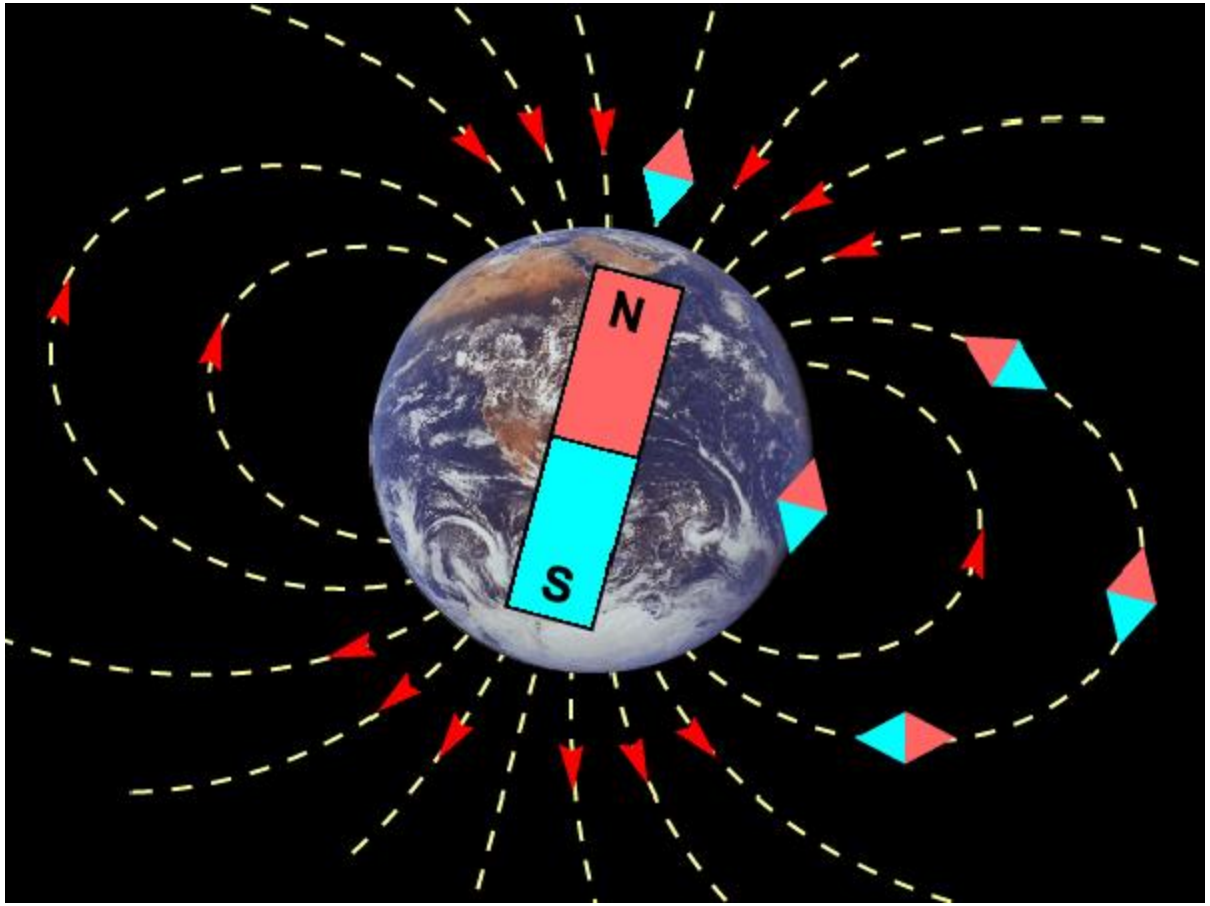
Magnetism

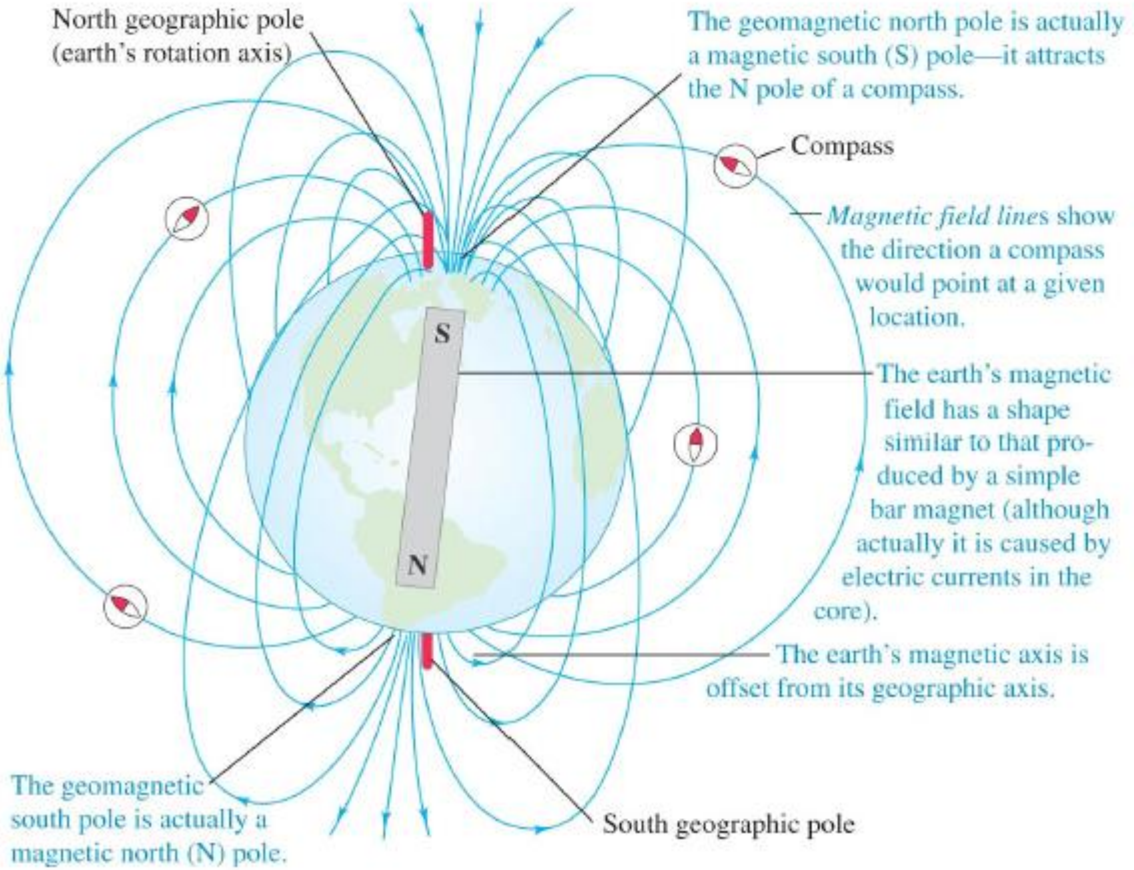
Magnets exert forces on each other just like charges. You can draw magnetic field lines just like you drew electric field lines.

Magnetic north and south pole's behaviour is not unlike electric charges. For magnets, like poles repel and opposite poles attract.



Which one is correct?





Magnetic Field

Electric field:

1) A distribution of electric charge at rest creates an electric field E in the surrounding space.

2) The electric field exerts a force $\vec{F}_E = q\,\vec{E}$ on any other charges in presence of that field.

Magnetic field:

1) A moving charge or current creates a magnetic field in the surrounding space (in addition to $\vec{E}$).

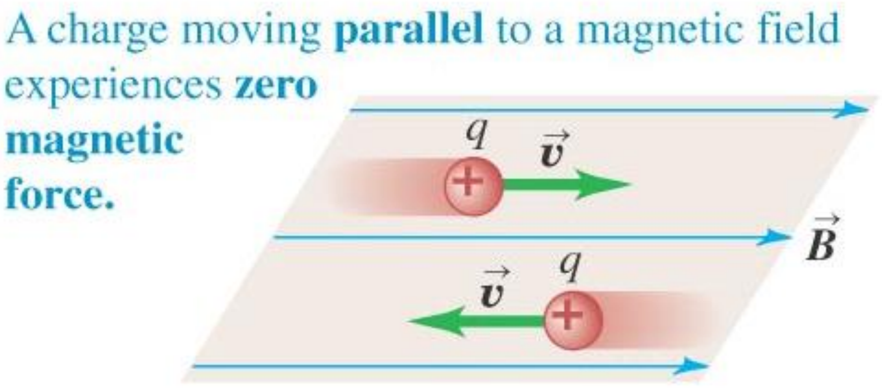
2) The magnetic field exerts a force $\vec{F}_m$ on any other moving charge or current present in that field.

- The magnetic field is a vector field →vector quantity associated with each point in space.

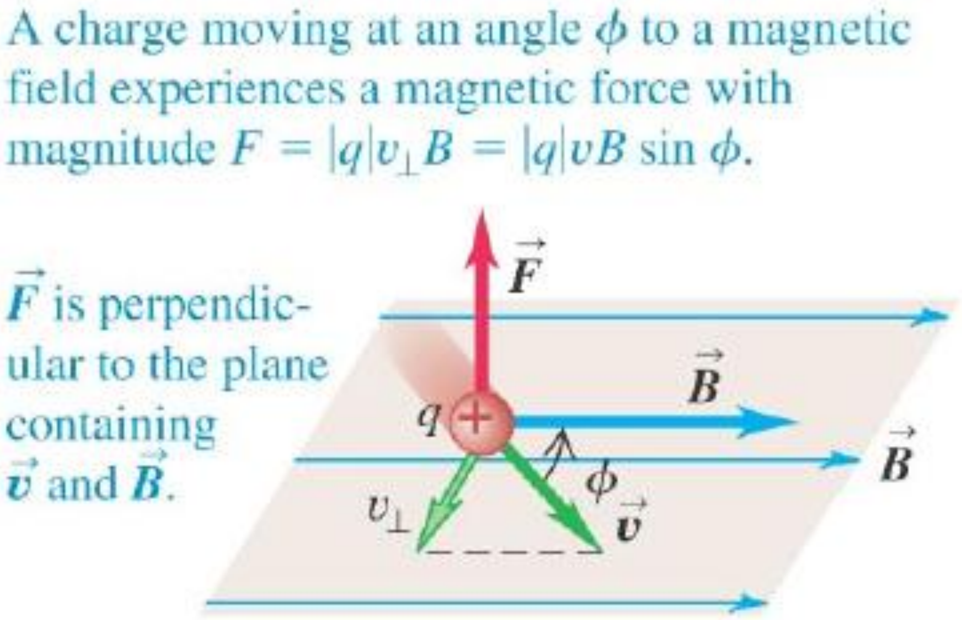
- $\vec{F}_m$ is always perpendicular to $\vec{B}$ and $\vec{v}$.

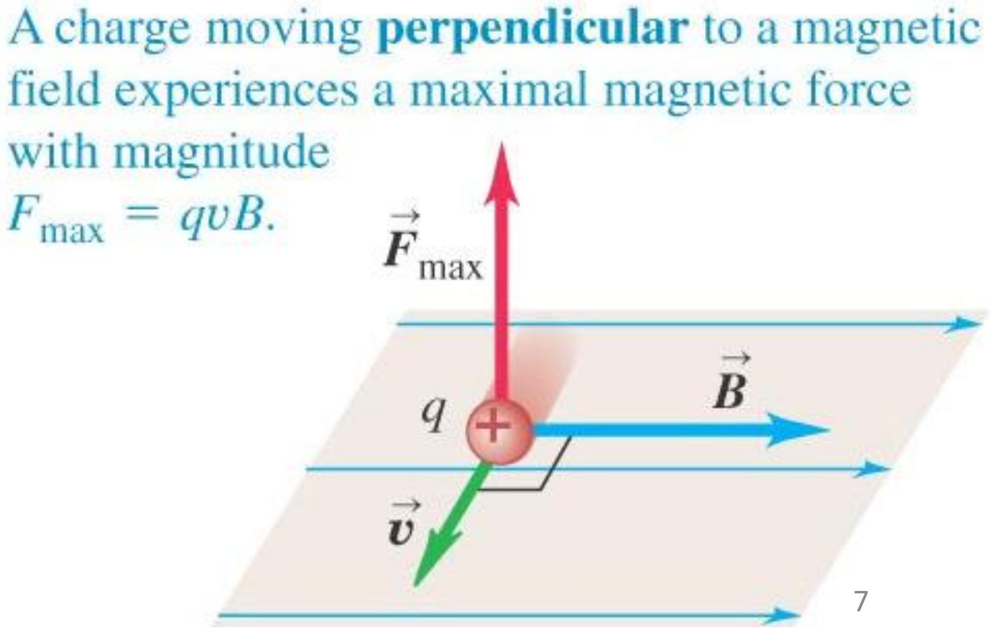
Magnetic Field

- The moving charge interacts with the fixed magnet. The force between them is at a maximum when the velocity of the charge is perpendicular to the magnetic field.



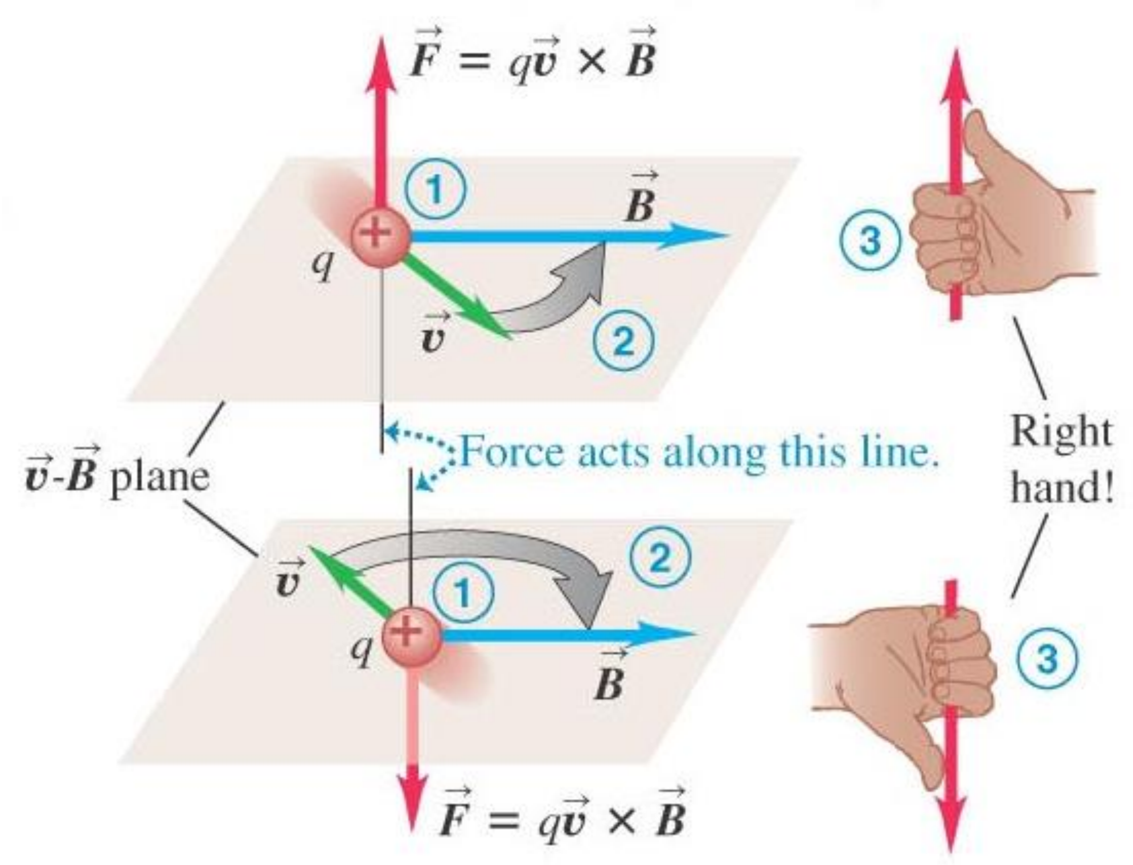
Interaction of magnetic force and charge

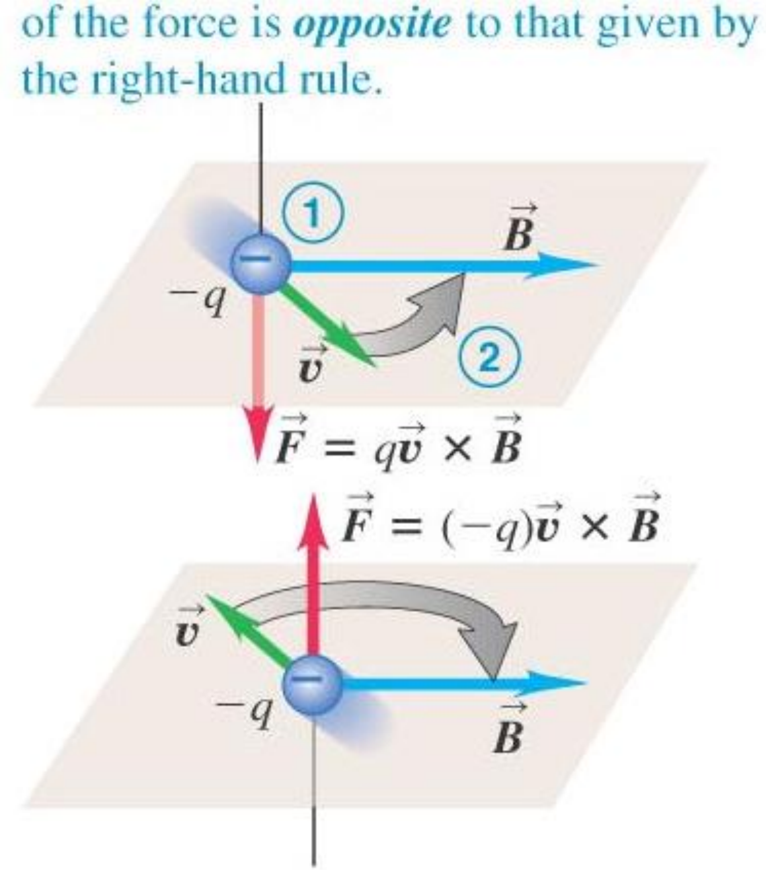




Positive charge moving in magnetic field → direction of force follows right hand rule

Negative charge →F direction contrary to right hand rule.

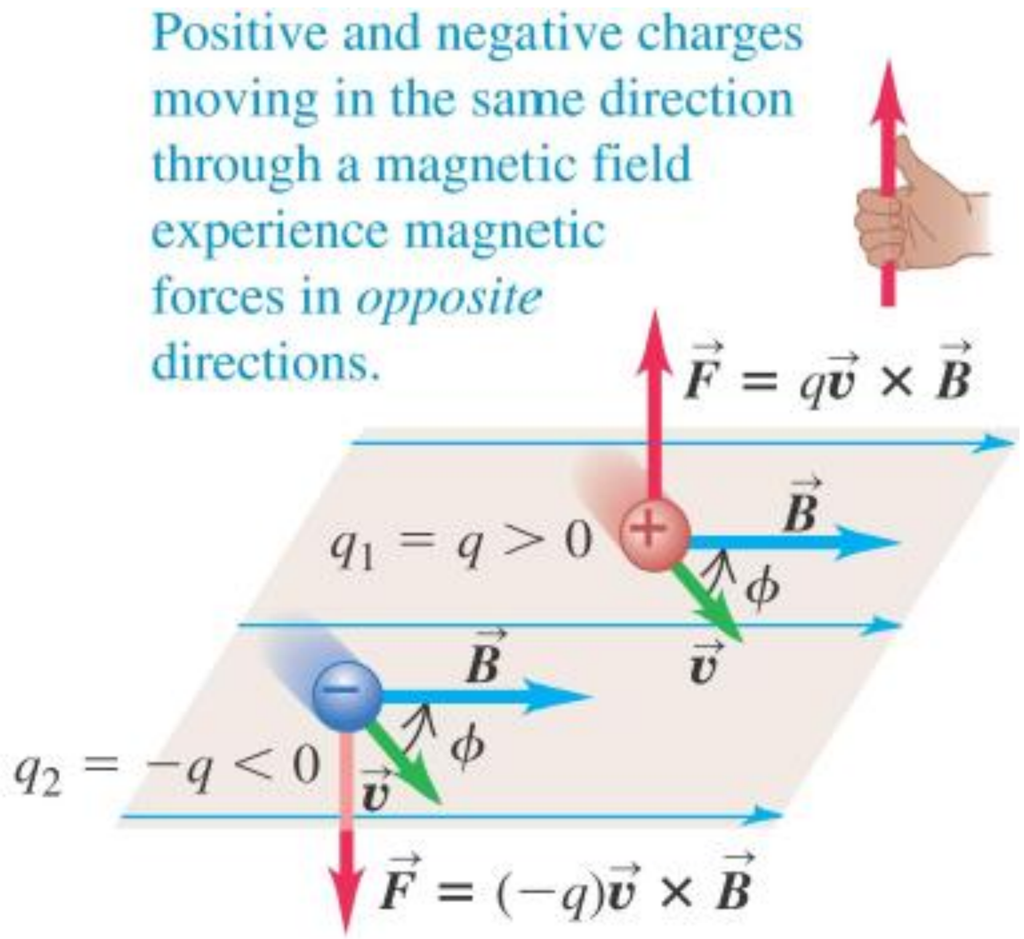




$$F = |q| v B_{\perp}$$

Units: 1 Tesla = 1 N s / C m = 1 N/A m

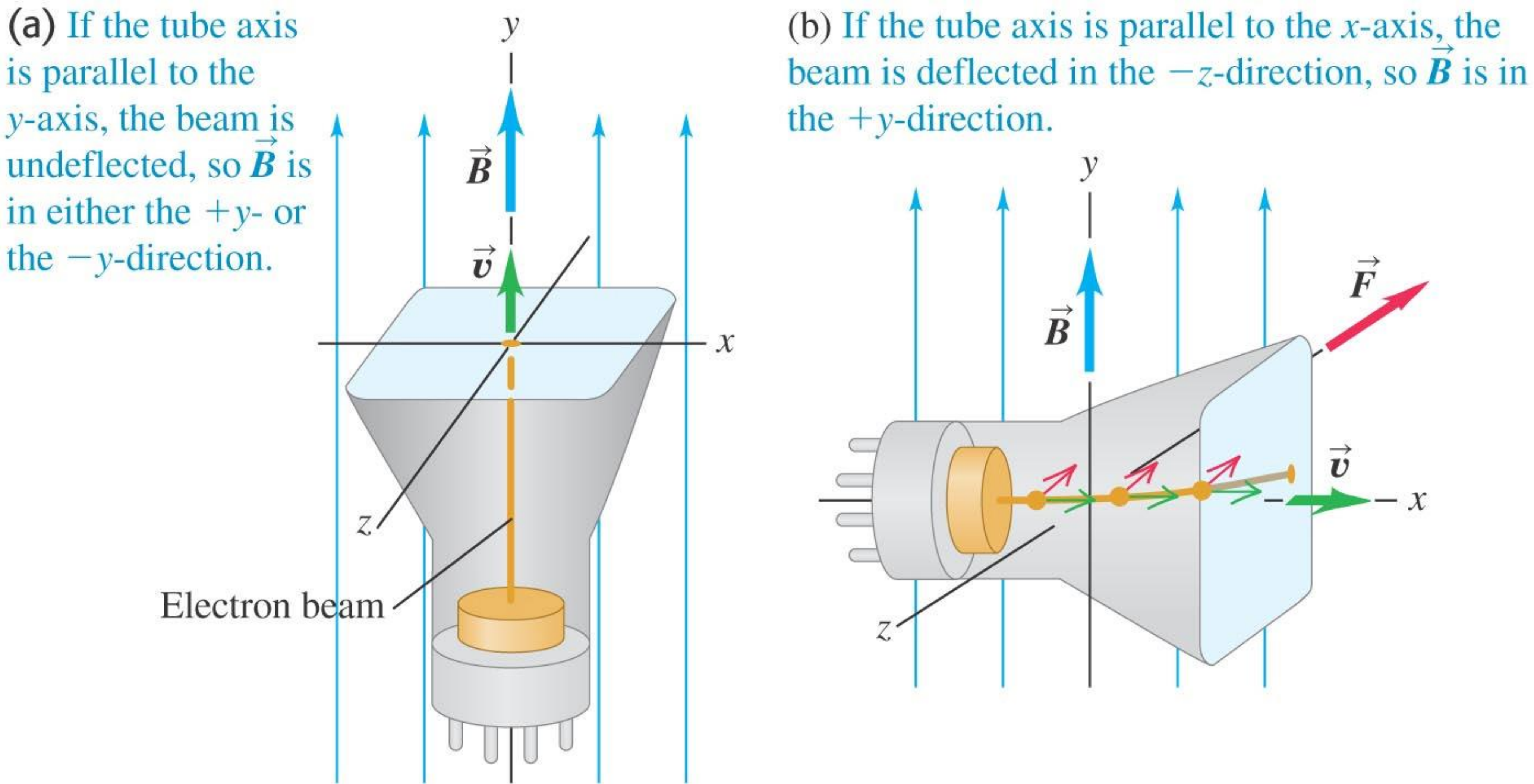
1 Gauss = 10^{-4} T



If charged particle moves in region where both, E and B are present:

Ex: electron beam in a cathode X-ray tube.

- In general, if a magnetic field (B) is present, the electron beam is deflected.
However this is not true if the beam is // to B ($\varphi = 0, \pi \rightarrow F=0 \rightarrow$ no deflection).



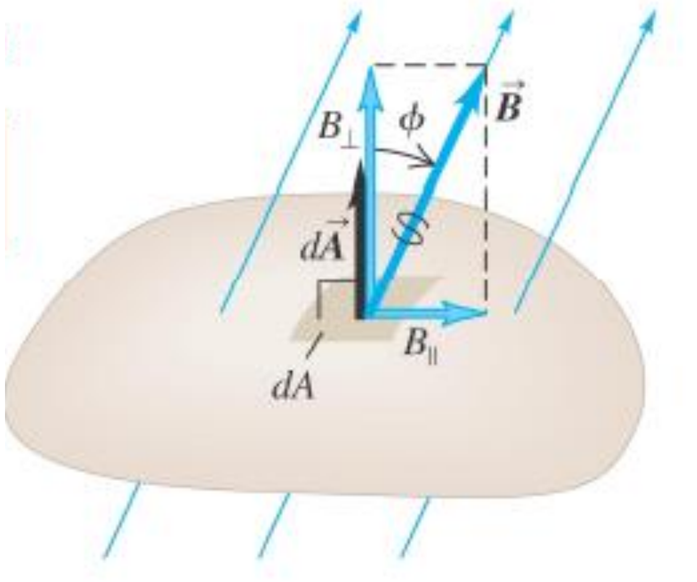
(a) If the tube axis is parallel to the y-axis, the beam is undeflected, so $\vec{B}$ is in either the $+y$- or the $-y$-direction.

(b) If the tube axis is parallel to the x-axis, the beam is deflected in the $-z$-direction, so $\vec{B}$ is in the $+y$-direction.

Magnetic Flux

Consider a uniform magnetic field passing through a surface *S*, as shown in Figure below.

The magnetic flux through the surface is given by



Magnetic flux through a non-planar surface

- Magnetic flux is a scalar quantity.

- If **B** is uniform:

Units: 1 Weber (1 Wb = 1 T m^2 = 1 N m / A)

- Difference with respect to electric flux → the total magnetic flux through a closed surface is always zero. This is because there is no isolated magnetic charge ("monopole") that can be enclosed by the Gaussian surface.

- The magnetic field is equal to the flux per unit area across an area at right angles the magnetic field = magnetic flux density.

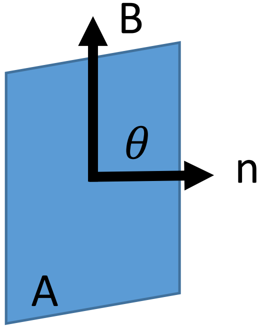
Q/ A uniform magnetic field (B) pointed horizontal. Its magnitude is 1.5 (w/m^2). If a proton with energy 5 (Mev) moves vertically through this field, what is the Force that acts on this proton?

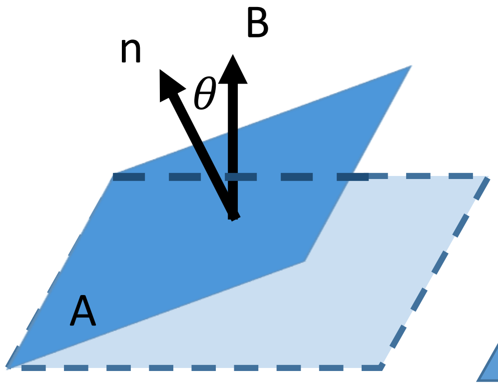
$\varphi = 90$

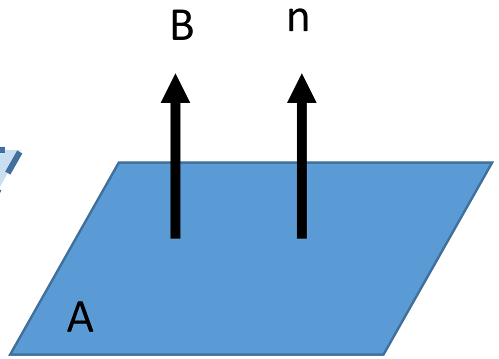
Q/ Prove that $W_b = V.S$?

Q/ A uniform magnetic field (B=1.2T), find the magnetic flux that inter a plane surface of area (2 m^2) if:

1. The surface is perpendicular to the direction of the field.
2. The direction of the surface make an angle 30^o with the field.
3. The surface is parallel to the magnetic field.







1- when the surface is perpendicular to the field

$\theta = 0$

2- when $\theta = 30^\circ$

3- $\theta = 90$

Motion of Charged Particles in a Magnetic Field

- Magnetic force perpendicular to v →it cannot change the magnitude of the velocity, only its direction.

- **F** does not have a component parallel to particle's motion →cannot do work.

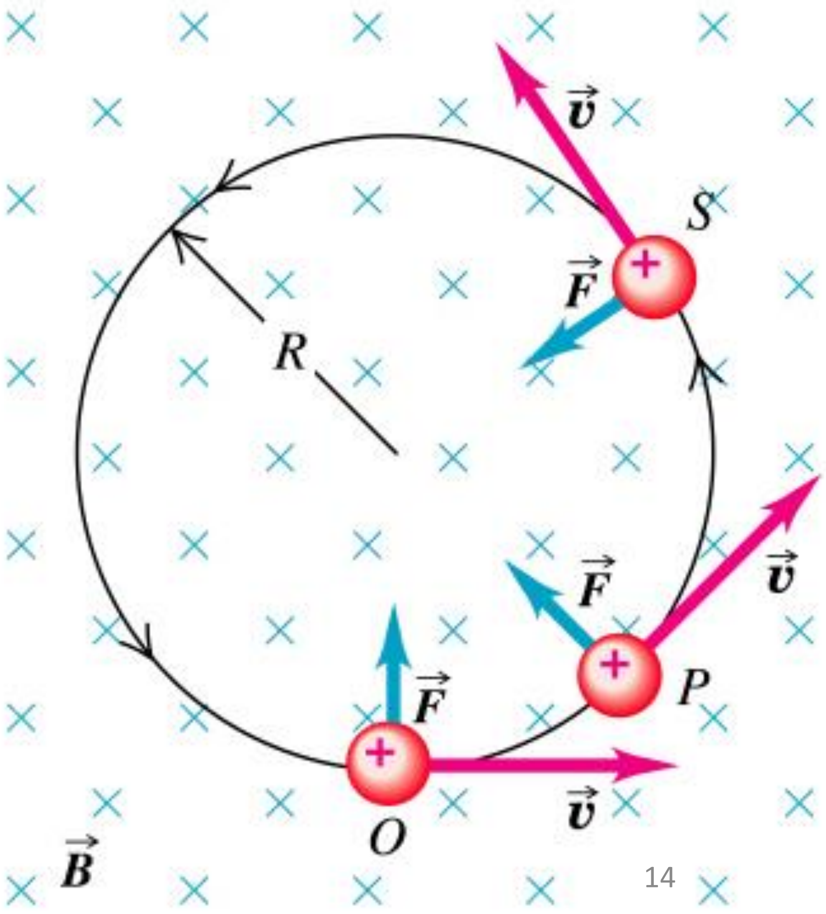
- Motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.

- Magnitudes of F and v are constant (v perp. B) →uniform circular motion.

$$F = |q| \cdot v \cdot B = m\frac{v^2}{R}$$

Radius of circular orbit in magnetic field:

+ particle →counter-clockwise rotation.
- particle →clockwise rotation.

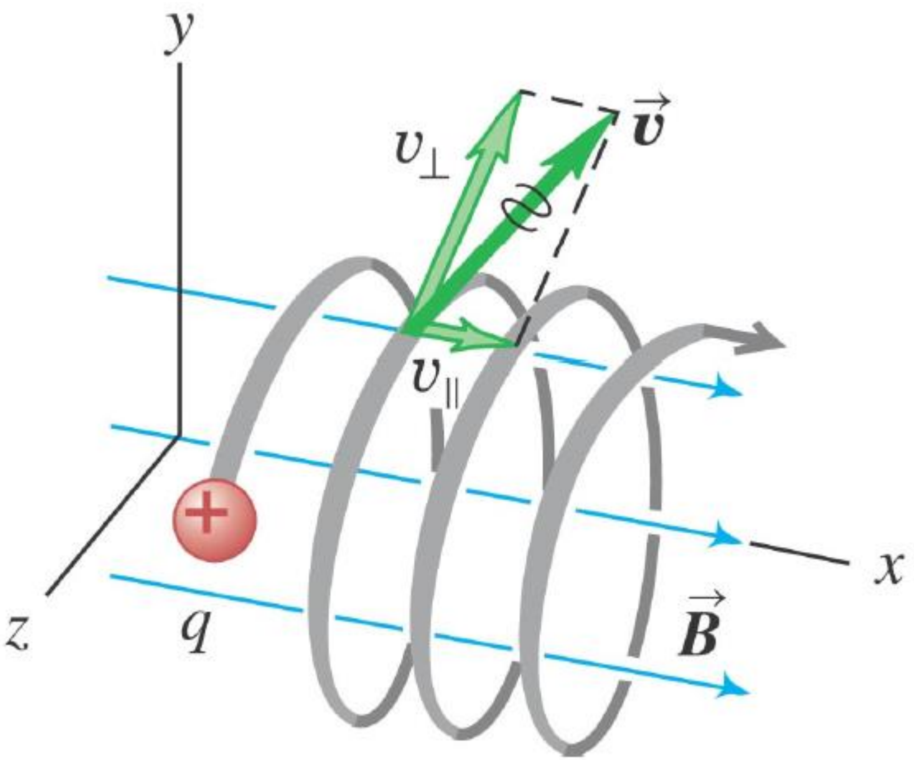


Angular speed: $\omega = v/R$

Cyclotron frequency: $f = \omega/2\pi$

- If v is not perpendicular to B → $v_{//}$ (parallel to B) constant because $F_{//} = 0$ → particle moves in a helix. (R same as before, with $v = v_{\perp}$).

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a helical path.



Since the time of one rotation T=1/f then:

$$T = \frac{2\pi m}{qB}$$

A charged particle will move in a plane perpendicular to the magnetic field.

Q/ An electron with energy 10 ev is circulating in a plane at right angles to a uniform magnetic field of $1x10^{-4}$ Tesla. What is

a) The orbital radius of the electron?

b) The frequency of the electron?

c) Period of rotation?

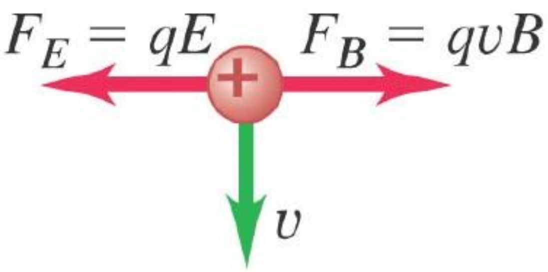
Applications of Motion of Charged Particles

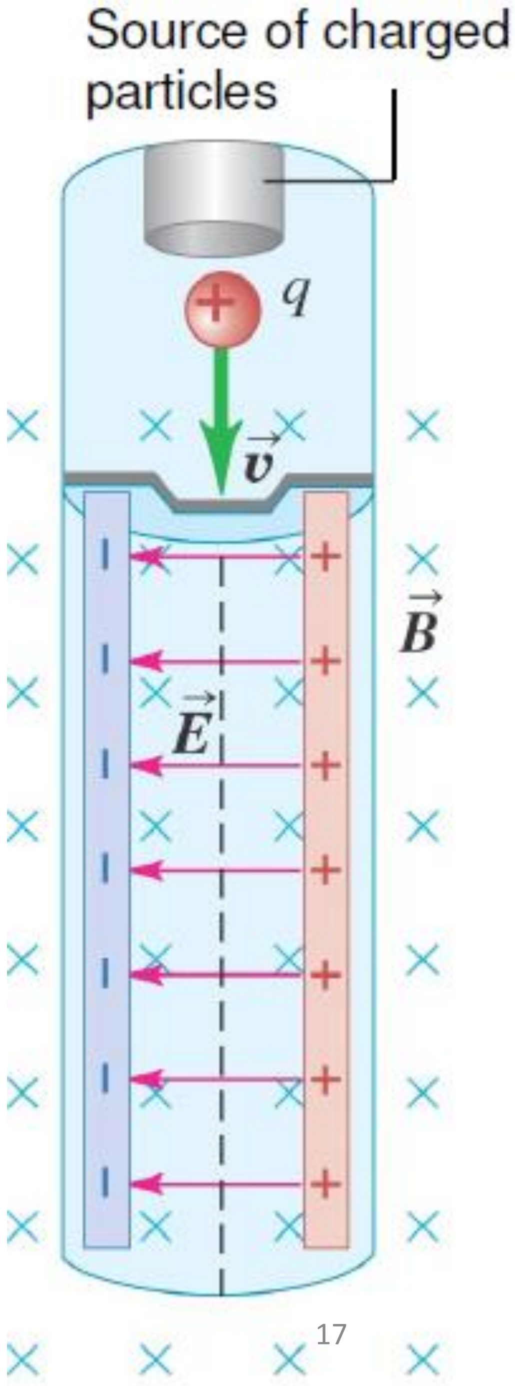
Velocity selector

- Particles of a specific speed can be selected from the beam using an arrangement of E and B fields.

- F_m (magnetic) for + charge towards right (q v B).
- F_E (electric) for + charge to left (q E).
- $F_{net} = 0$ if $F_m = F_E$ → $-qE + q\,v\,B = 0$ →

- Only particles with speed E/B can pass through without being deflected by the fields.





Thomson's *e/m* Experiment

$\Delta E = \Delta K + \Delta U = 0 \rightarrow 0.5\, m\, v^2 = U = e\, V$

$$v = \frac{E}{B} = \sqrt{\frac{2eV}{m}}$$

e/m does not depend on the cathode material or residual gas on tube →particles in the beam (electrons) are a common basic of all matter.

The deflection (d) can be given by:

$$d = \frac{eELD}{mv^2}$$

$$\frac{e}{m} = \frac{Ed}{LDB^2}$$

Where:

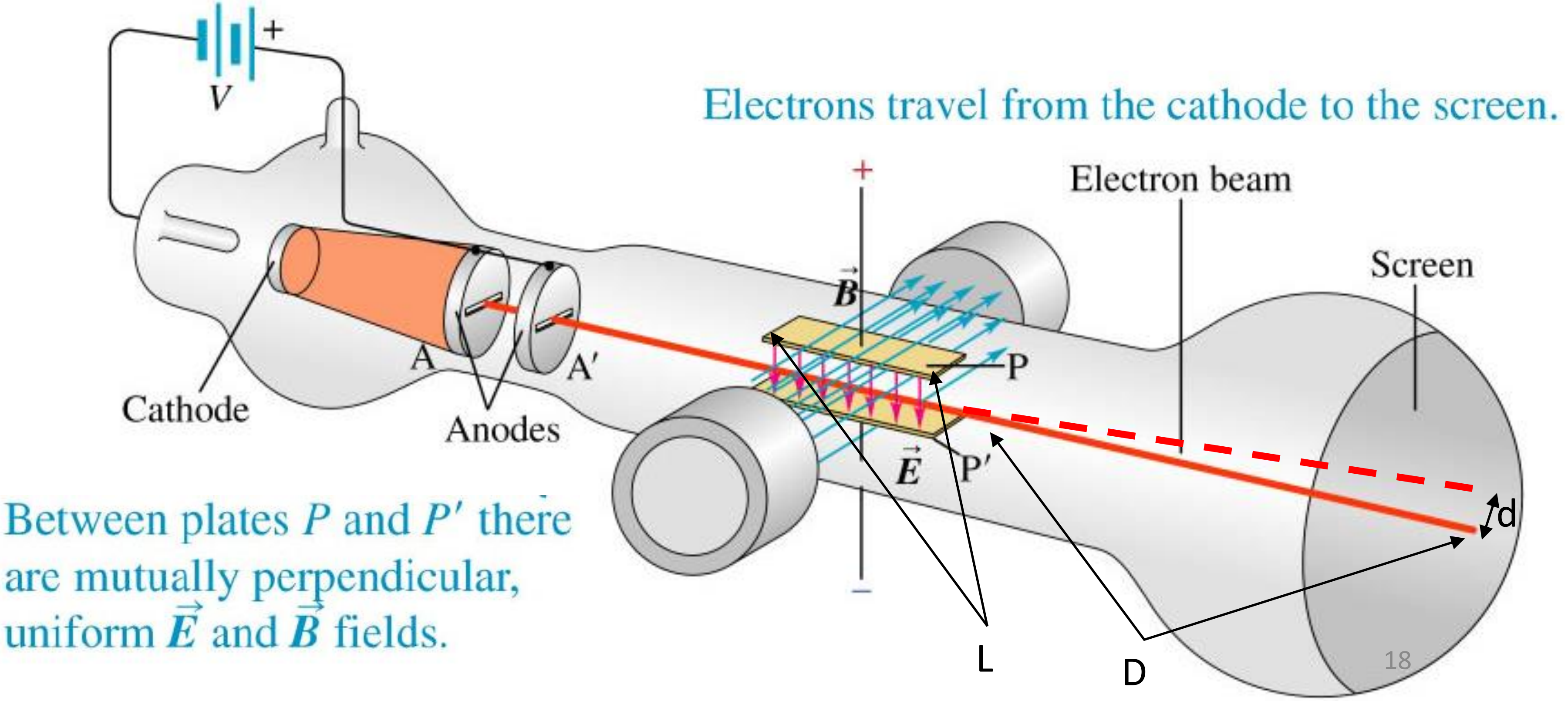
e: charge of electron

E: electric field

L: length of metal

m: mass of electron

V: velocity of electron

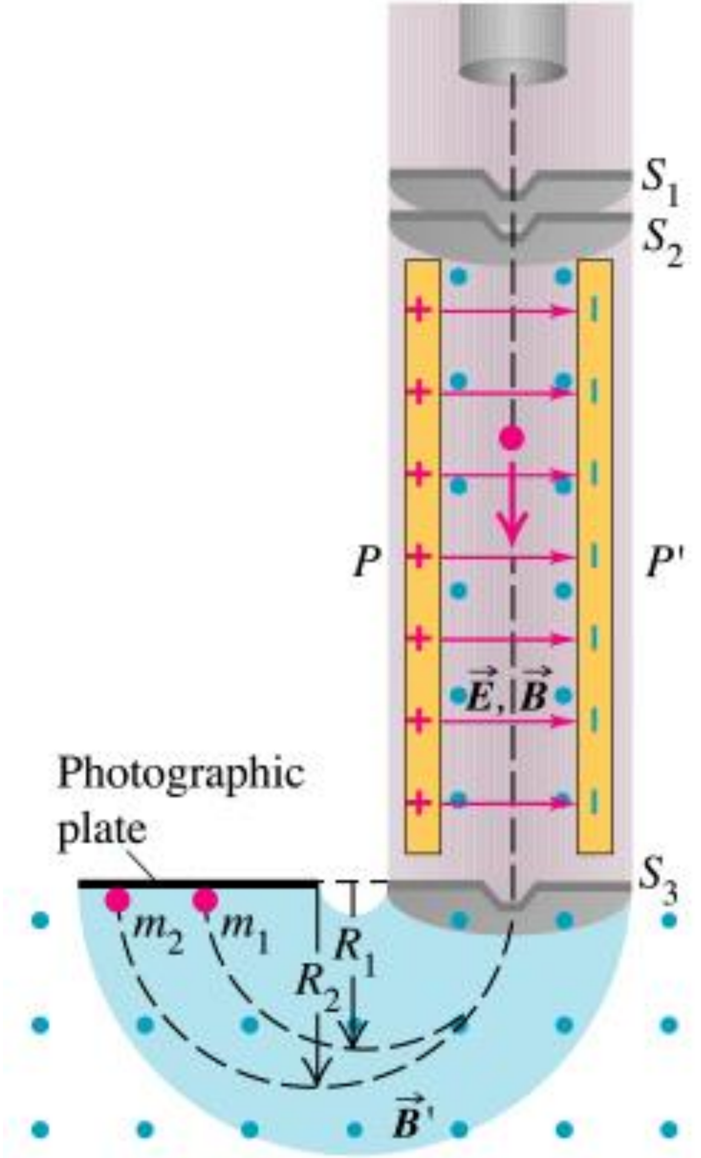


Mass Spectrometer

- Using the same concept as Thompson, Bainbridge was able to construct a device that would only allow one mass in flight to reach the detector.

- Velocity selector filters particles with $v = E/B$.
After this, in the region of B' particles with $m_2 > m_1$ travel with radius ($R_2 > R_1$).

$$R = \frac{mv}{|q|B'}$$



Magnetic Force on a Current-Carrying Conductor

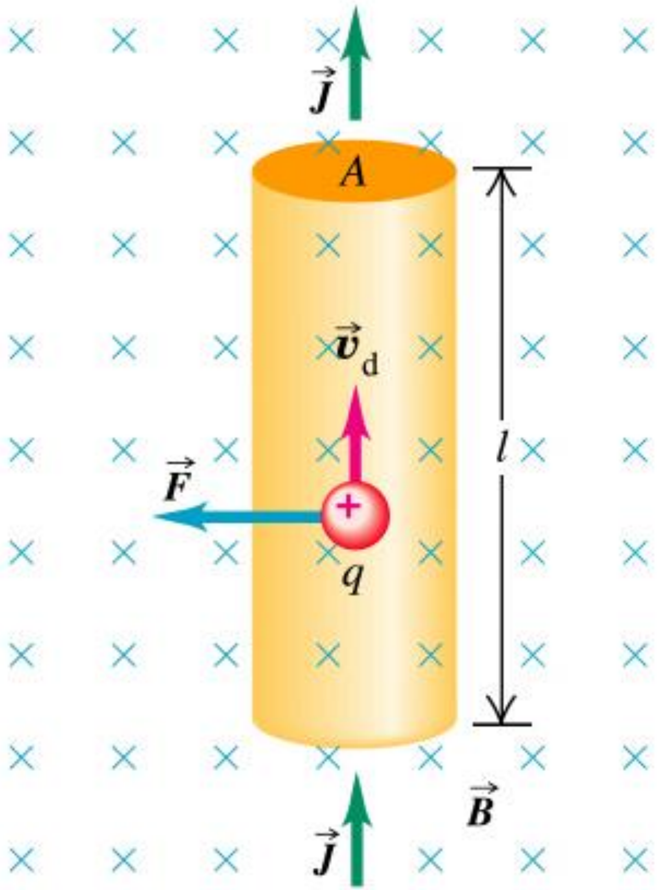
$\vec{F}_m = q\vec{v}_d \times \vec{B}$ $\qquad F_m = qv_dB$ Force on one charge

- Total force: $F_m = (nAl)(qv_dB)$

n = number of charges per unit volume

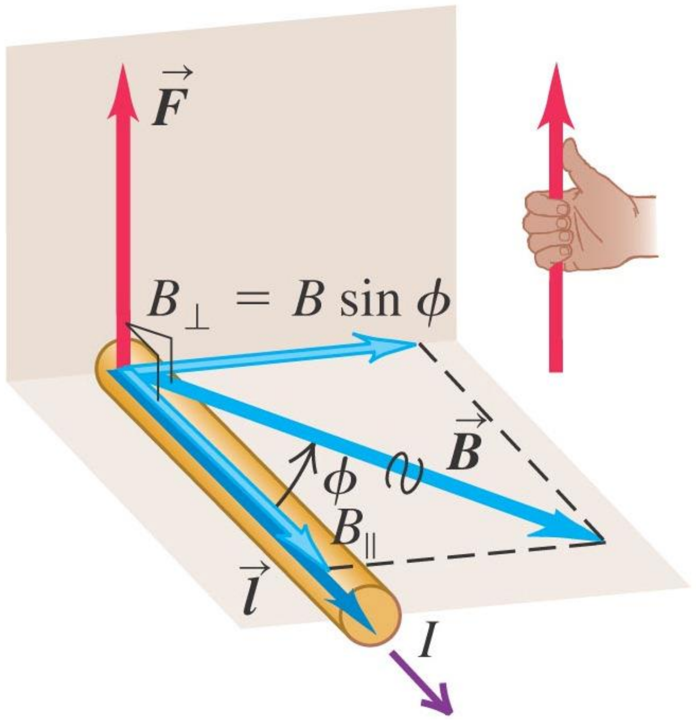
$A\,l$ = volume

$$F_m = (nqv_d)(A)(lB) = (JA)(lB) = IlB \qquad (B \perp \text{wire})$$



In general:

$$F = IlB_\perp = IlB\sin\varphi$$



Magnetic force on a straight wire segment:

Magnetic force on an infinitesimal wire section:

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

$q = nALq$

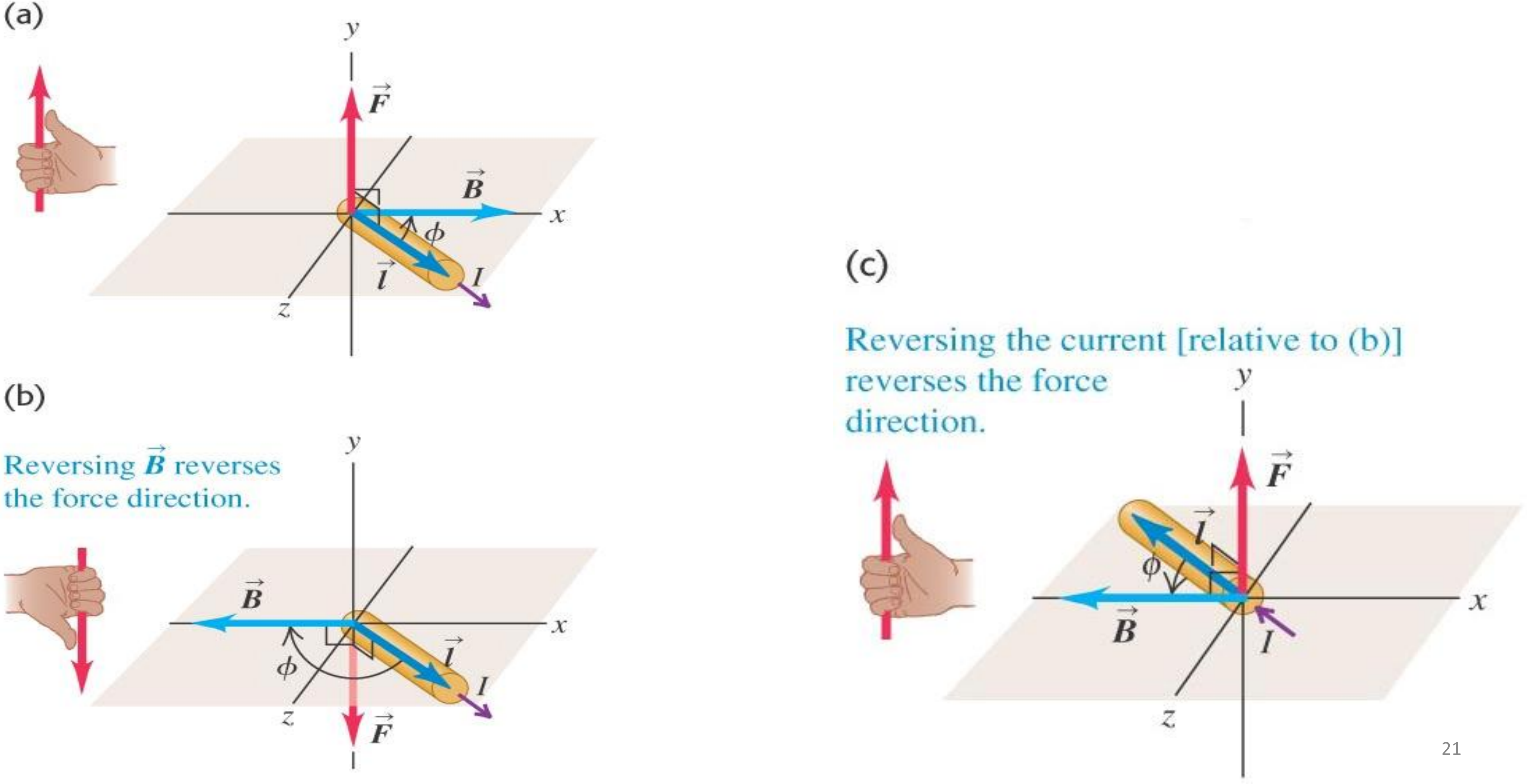
$I = \frac{q}{t} = \frac{nALq}{t}$

$\frac{L}{t} = v_d$

$v_d = \frac{I}{nAq}$

- Current is not a vector. The direction of the current flow is given by $\overrightarrow{dl}$, not I.

$\overrightarrow{dl}$ is tangent to the conductor.



Force and Torque on a Current Loop

- The net force on a current loop in a uniform magnetic field is zero.

Right wire of length "a" $F = I\,a\,B$ ($B \perp I$)
Left wire of length "b" $F' = I\,b\,B \sin(90^{\circ} - \varphi)$ (B forms $90^{\circ}-\varphi$ angle with I)
$F' = I\,b\,B \cos\varphi$

$F_{net} = F - F + F' - F' = 0$

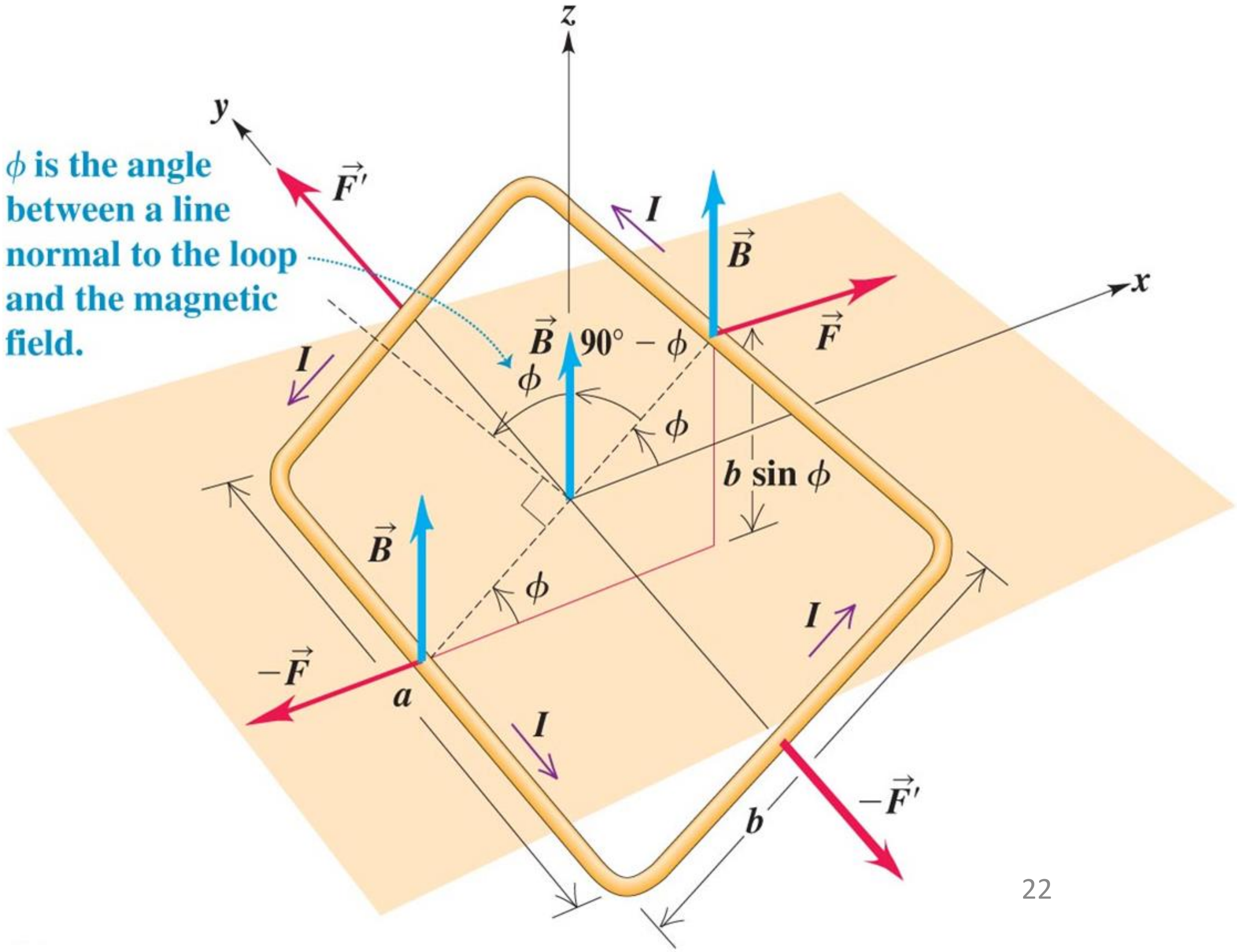
- Net torque ≠ 0 (general).

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = r \cdot F \sin\alpha = r_{\perp} F = r F_{\perp}$$

$$\tau_{F'} = r \cdot F \sin 0^{\circ} = 0$$

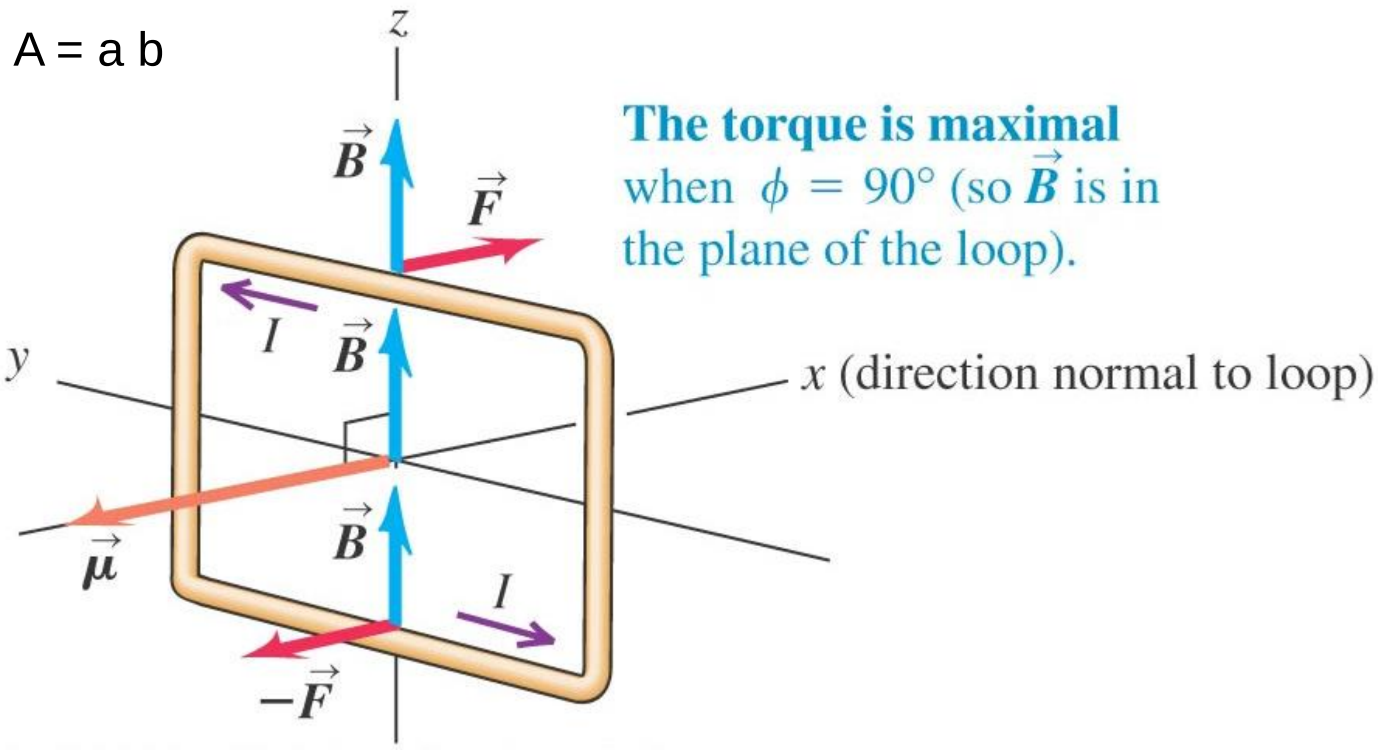
$$\tau_{F} = F(b/2)\sin\varphi$$



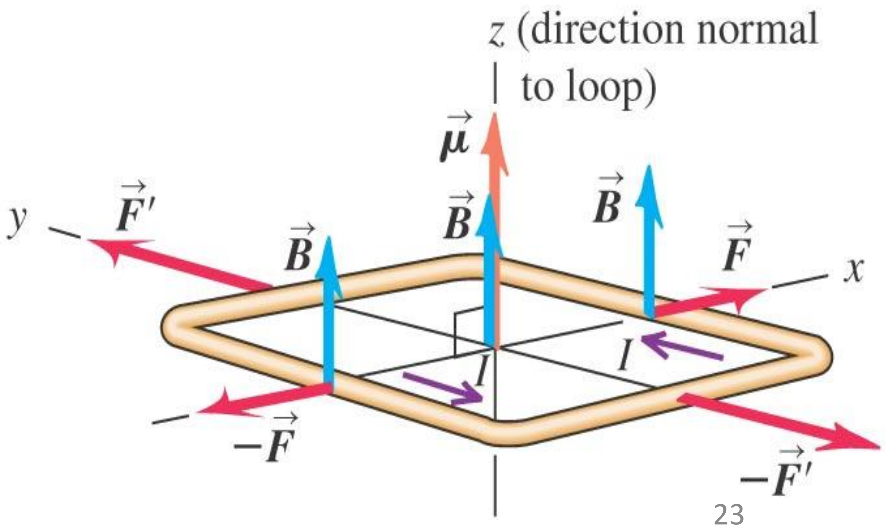
$$\tau_{total} = \tau_{F'} + \tau_{-F'} + \tau_F + \tau_{-F} = 0 + 0 + 2(b/2)F\sin\varphi$$

Torque on a current loop

A = a b



Torque is zero, φ = 0º

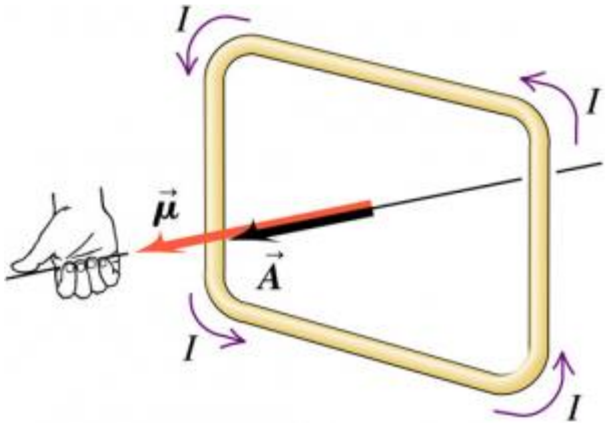


φ is angle between a vector perpendicular to loop and $\vec{B}$

$$\tau_{total} = IBA\sin\varphi$$

Magnetic dipole moment:

Direction: perpendicular to plane of loop (direction of loop's vector area → right hand rule)



$$\tau_{total} = \mu B\sin\varphi$$

Magnetic torque:

Potential Energy for a Magnetic Dipole:

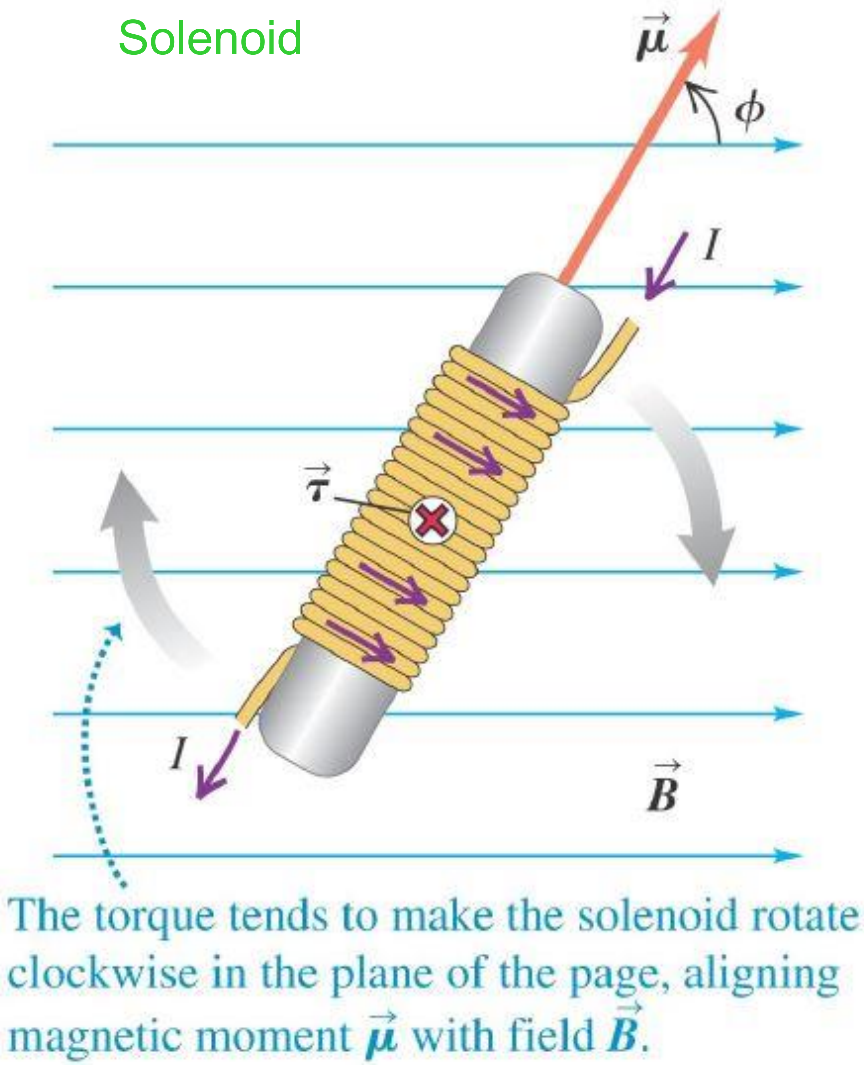
Electric dipole moment:

Electric torque:

Potential Energy for an Electric Dipole:

Magnetic Torque: Loops and Coils

Solenoid



$$\tau = NIBA \sin \varphi$$

N= number of turns

φ is angle between axis of solenoid and B

Max. torque: solenoid axis ⊥ B.

Torque rotates solenoid to position where its axis is parallel to B.

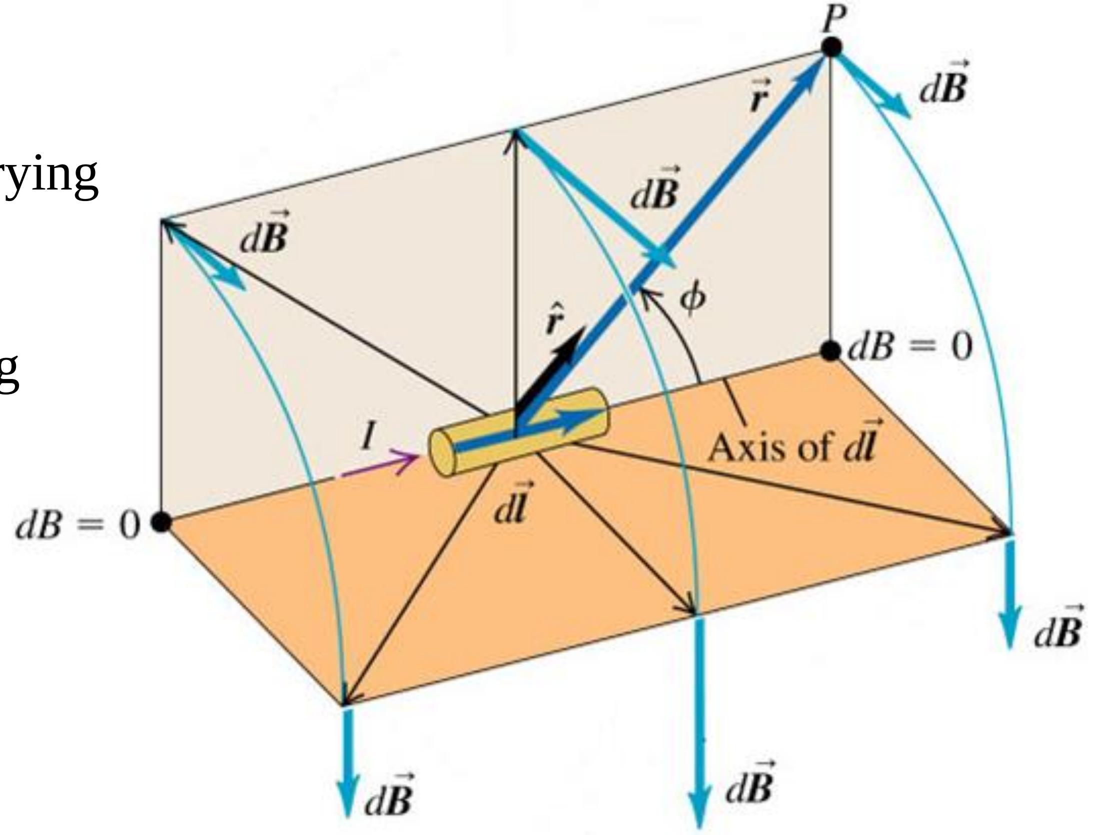
Q/ A wire with length 1m carrying a current 10A placed in a uniform magnetic field of 1.5T. Find the magnitude of the force on the wire if:

(a) wire is placed normally to the field

(b) Wire makes an angle of 30^o with the field

Magnetic Field of a Current and a moving Charge

The magnetic field caused by a short segment $\vec{dl}$ of a current-carrying conductor can be obtained by a short derivation:



We know that the magnetic field of a single point charge q moving with a constant velocity $\vec{v}$ is given by:

Hence

where dQ is the element of charge in the short segment of current-carrying conductor and v_d is the drift velocity of the charge carriers in the conductor.

We need to calculate dQ:

We know that

$$I = n|q|v_d A$$

where n is the number of charge carriers per unit volume

Hence

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\phi}{r^2}$$

or

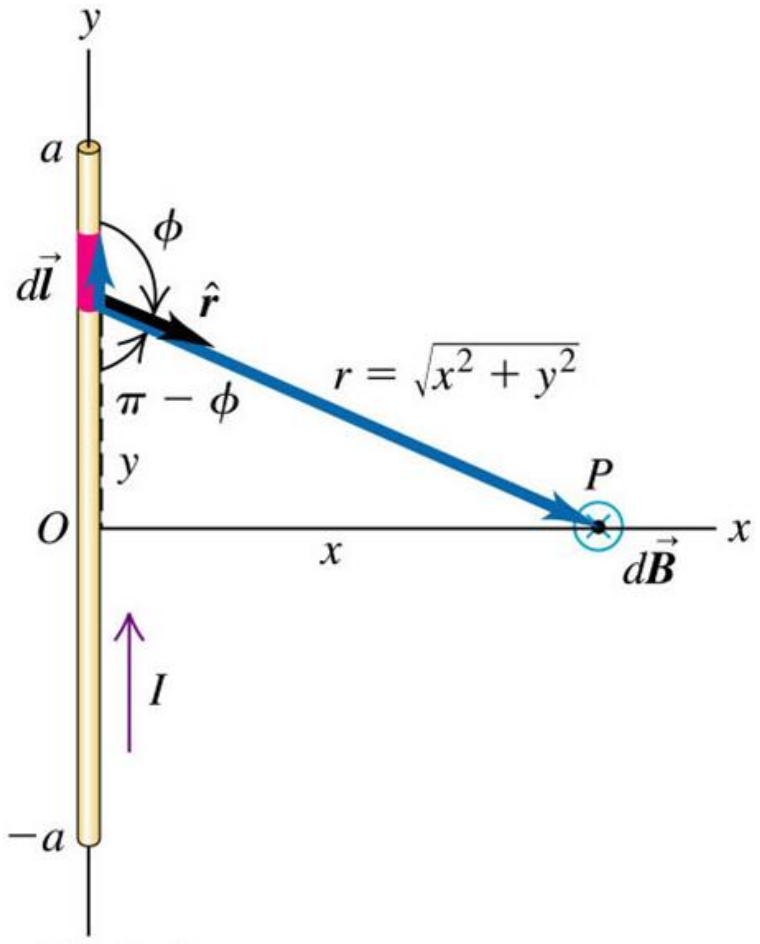
The above equation is known as **Biot-Savart Law**.

The Principle of superposition of magnetic fields states that the total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges.

The total magnetic field at any point in space due to the current in a complete circuit will be:

Magnetic Field of a Long Straight Wire

Consider a straight conductor with length 2a carrying a current I. Find the magnetic field at point P which is located at a distance x from the conductor on its perpendicular bisector.



From Biot-Savart Law, we have:

First, we will need to work out what is $d\vec{l} \times \hat{r}$.

$$d\vec{l} = dl\,\hat{j}$$

$$\begin{aligned}\hat{r} &= \cos(\phi - 90^\circ)\,\hat{i} - \sin(\phi - 90^\circ)\,\hat{j} \\ &= \sin\phi\,\hat{i} + \cos\phi\,\hat{j}\end{aligned}$$

Hence,

$$d\vec{l} \times \hat{r} = -\sin\phi\, dl\,\hat{k}$$

Substituting that into the Biot-Savart Law, we have:

Notice that both r^2 and $\sin\phi$ can be replaced with:

$$r^2 = x^2 + y^2$$

$$\sin\phi = \frac{x}{\sqrt{x^2 + y^2}}$$

We have:

The magnitude of the magnetic field is:

$$B = \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2 + a^2}}$$

What if the current carrying conductor is long (assumed to be infinite in length)?

From the above equation, we have:

$$B = \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2 + a^2}}$$
$$= \frac{\mu_0 I}{2\pi} \frac{1}{x\sqrt{1 + \frac{x^2}{a^2}}}$$

For infinite length of wire, $a \to \infty$, $\frac{x^2}{a^2} \to \infty$., We have:

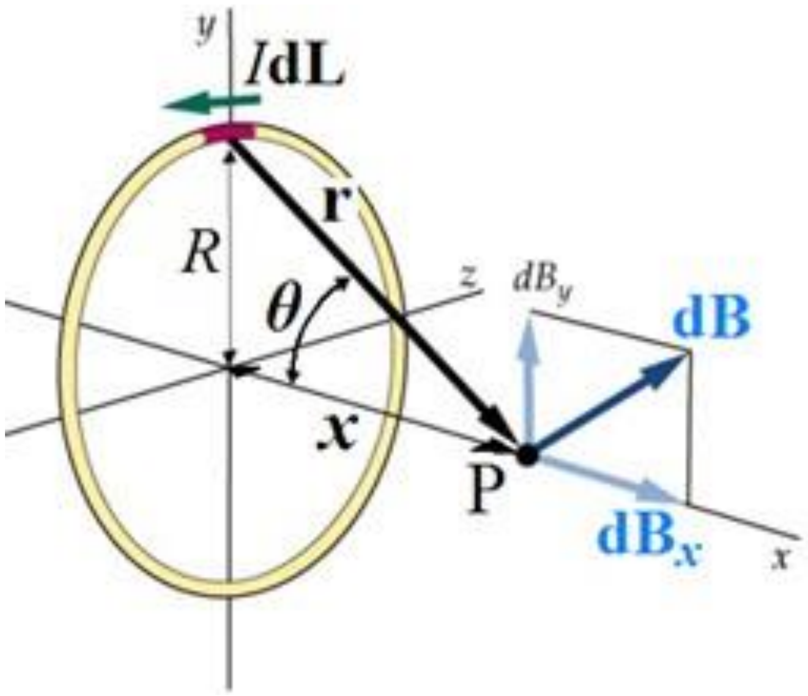
$$B = \frac{\mu_0 I}{2\pi x}$$

x is normally denoted by *r*, which is the distance from the conductor to the point.
Hence,

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic Field for a Circular Wire

The figure shows a circular wire of radius (R) carrying a current (I). In order to find the magnetic field at point (P) located at distance (x) from the centre of the wire, we choose a differential element of the wire (dL). According to Biot_Savart law:



$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

$$dL \times \vec{r} = dL \sin 90 = dL$$

لان r دائما عمودي على dL

The vector ($\vec{r}$) is usually perpendicular to (dL) and the direction of (dB) is perpendicular to (r)

$$dB_x = dB \sin\theta \qquad\qquad dB_y = dB \cos\theta$$

From symmetry dB_y=0, so the magnitude of B will be:

$$B = \int dB_x = \frac{\mu_0 I}{4\pi} \int \frac{dL \sin\theta}{r^2} \qquad\qquad \sin\theta = \frac{R}{r}$$

L=2πR

$$B = \frac{\mu_0 I R^2}{2r^3}$$

$$r = \sqrt{x^2 + R^2} \quad \Rightarrow \quad r^3 = (x^2 + R^2)^{3/2}$$

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

1) If point (P) is too far from the centre x>>R, so,

$$B = \frac{\mu_0 I R^2}{2x^3} = \frac{\mu_0 I \pi R^2}{2\pi x^3}$$

$$\pi R^2 = A$$

$$B = \frac{\mu_0 I A}{2\pi x^3}$$

IA= m (magnetic moment)

$$B = \frac{\mu_0 m}{2\pi x^3}$$

At the centre of the wire (x=0)

$$B = \frac{\mu_0 I}{2R}$$

If the wire is a coil of (N) turns then

$$B = \frac{\mu_0 I N}{2R}$$

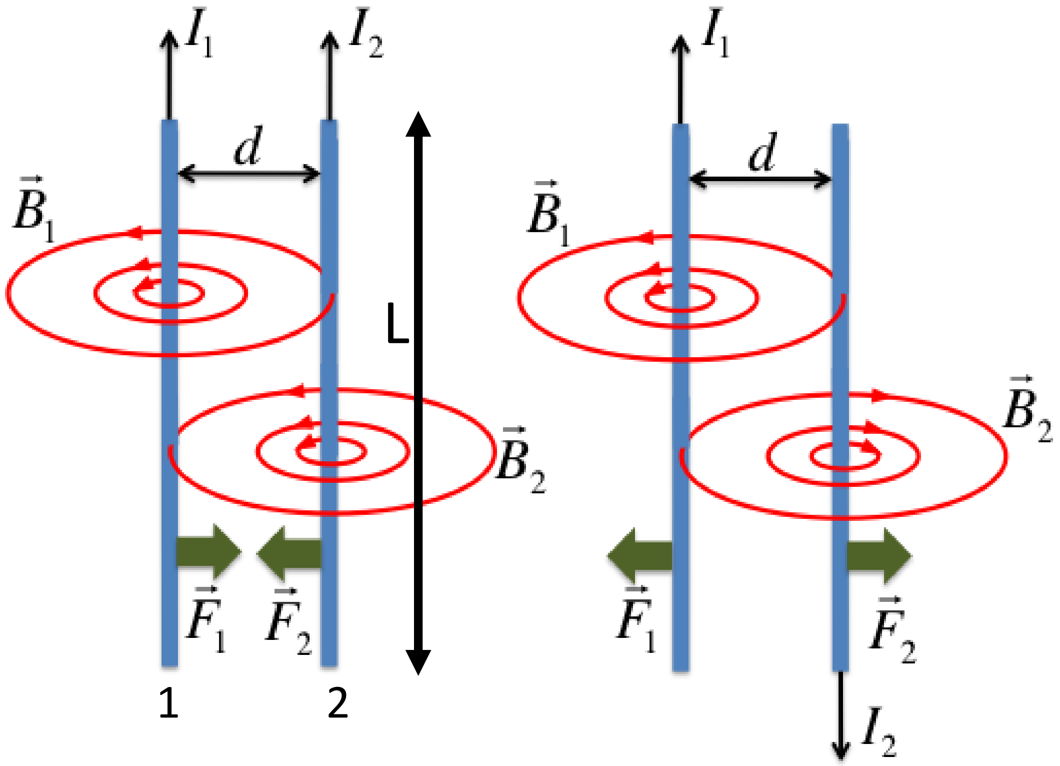
Force between parallel current currying conductor

It is experimentally established fact that two current carrying conductors attract each other when the current is in same direction and repel each other when the current are in opposite direction.

Figure shows two long parallel wires separated by distance d and carrying currents I_1 and I_2

Consider wire 1 will produce a field B_1 at all near by points .The magnitude of B_1 due to current I_1 at a distance d i.e. on wire 2 is

$B_1=\mu_0 I_1/2\pi d$



The direction of B_1 is according to the right hand rule

Consider length L of wire 2 and the force experienced by it will be ($I_2L\mathbf{B}$) whose magnitude is

Direction of F_2 can be determined using vector rule .F_2 Lies in the plane of the wires and points to the left.

Force per unit length of wire B is

The force between current-carrying wires is used as part of the operational definition of the Ampere. For parallel wires placed one meter away from one another, each carrying one ampere, the force per meter is:

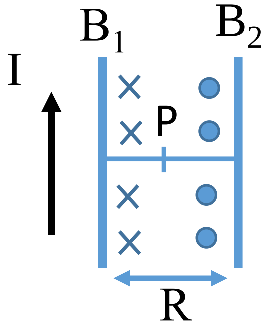
$$\frac{F}{L} = \frac{(4\pi \times 10^{-7} T.m/A)\,(1A)^2}{2 \times 3.14 \times 1m} = 2 \times 10^{-7}\ N/m$$

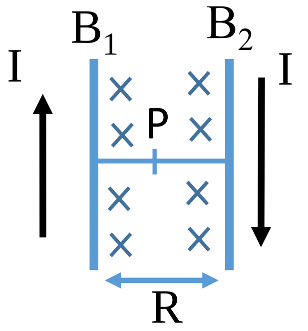
Incidentally, this value is the basis of the operational definition of the ampere. This means that one ampere of current through two infinitely long parallel wires (separated by one meter in empty space and free of any other magnetic fields) causes a force of 2×10^{-7} N/m on each conductor.

Ex. Two horizontal parallel wires are separated by a distance (R). Calculate the magnetic field at midpoint between the two wires:

1- if the wires carrying current (I) in opposite direction.

2- if the wires carrying current (I) in same direction.





Answer:

1- The wires carrying current (I) in opposite direction B_1 and B_2 are in the same direction

$B=B_1+B_2$ ➡

2- The wires carrying current (I) in same direction B_1 and B_2 are in the opposite direction

Classification of Magnetic Materials

Materials can be classified according to their reaction to an external magnetic field. The relationship between the magnetisation of a material and an external applied field can be described by the following equation

where χ is the dimensionless magnetic susceptibility, μ_0 is the permeability of vacuum, **M** is the sample magnetisation and **H** is the magnetic field. Depending upon the magnetic susceptibility materials fall into the following categories.

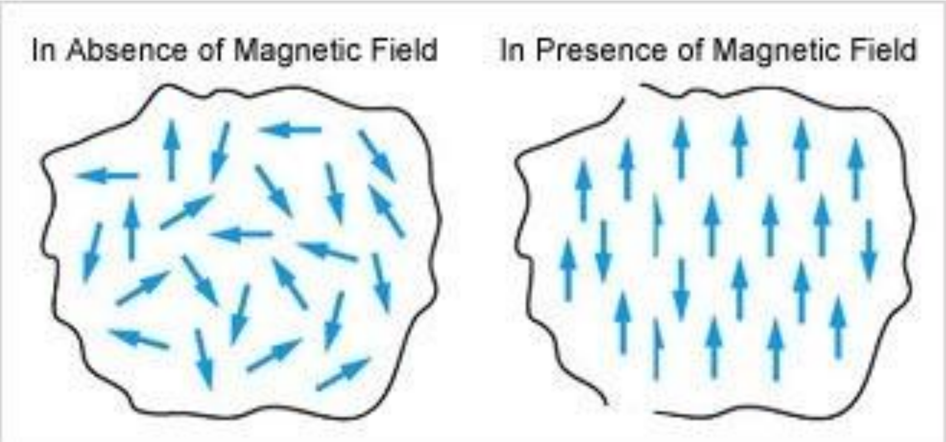
1- Diamagnetic Materials

Diamagnetic materials have negative magnetic susceptibility. Diamagnetic atoms and ions have completely filled electron shells. Application of a magnetic field induces changes in the electrons' orbital motion that leads to a magnetic moment of opposite orientation to that of the magnetic field. Classically this can be thought of as a consequence of Lenz's law. For a diamagnetic material, the susceptibility is independent of temperature. Examples of diamagnetic materials include the monoatomic gases (He, Ne and Ar), molecules such as NaCl, and solids such as C, Si, Ge.

2- Paramagnetic Materials

Paramagnetic materials have unpaired electrons in incomplete sub-shells, which gives a non-zero magnetic moment. The magnetic moment of the paramagnetic material can be aligned with an external magnetic field, while in the absence of an external magnetic field the magnetic moments are randomly oriented, which gives rise to an average magnetisation of zero. Paramagnetic materials have a susceptibility that is positive and small. The relationship between temperature *T* and susceptibility is expressed by Curie's law:

where *C* is Curie's constant. Examples of paramagnetic materials are the rare earth and transition metal ions.



3- Ferromagnetic Materials

A ferromagnetic material has unpaired electron spins aligned parallel with each other, giving long range ordering of the magnetic moments in the absence of an external magnetic field. Ferromagnetic materials have higher values of magnetic susceptibility compared with the paramagnetic and diamagnetic materials and the susceptibility depends upon on temperature. At higher temperature, the susceptibility decreases until a ferromagnetic material undergoes a transition to a paramagnetic state.

Ferromagnetic materials have strong magnetism because of the strong coupling of the spin vectors of adjoining atoms forming regions called domains. In 1906, P. Weiss introduced a first attempt to describe the classical origin of ferromagnetism within these domains, which is called molecular field theory or Weiss theory. His idea based on the existence of an internal molecular field $\mathbf{B}_{in}$, which acts to align the magnetic moments, with an externally applied magnetic field $\mathbf{B}_{ex}$. However, he developed his theory by assuming that the field was proportional to the magnetisation in the ferromagnetic material as:

$$\mathbf{B}_{in} = \lambda \mathbf{M}$$

where λ is the molecular field constant.

Then it is possible to treat a ferromagnet as a paramagnet subject to a field. So, the total field for the ferromagnetic material can be written as:

$$\mathbf{B}_{tot} = \mathbf{B}_{ex} + \mathbf{B}_{in}$$

where $\mathbf{B}_{in}$ is the Weiss field. Hence, the above equation can be rewritten in the new form:

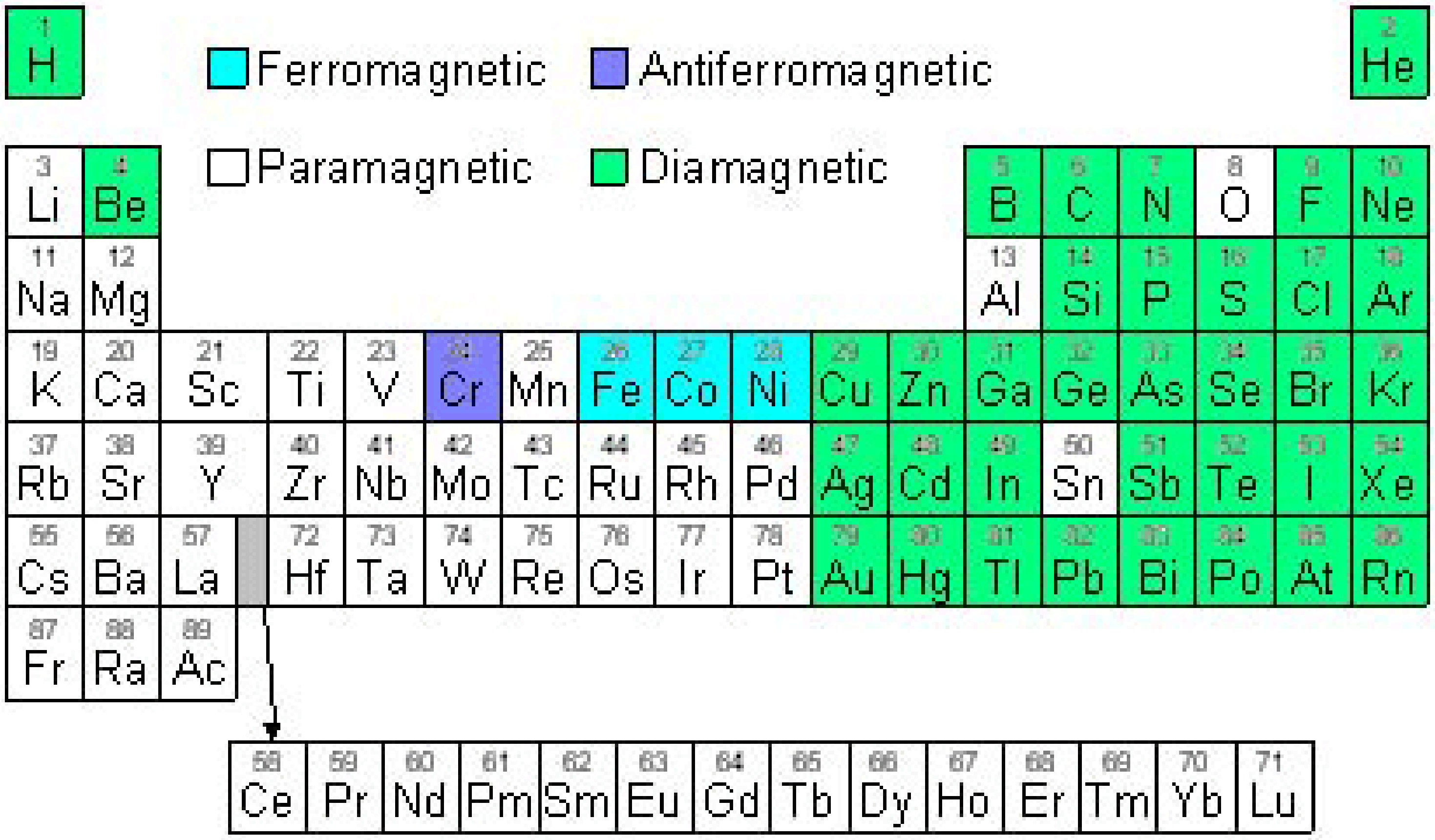
$$\mathbf{B}_{tot} = \mathbf{B}_{ex} + \lambda \, \mathbf{M}$$

Using Curie's law of paramagnetism the magnetic susceptibility can be re-written by combining equations as:

The final form of the magnetic susceptibility of a ferromagnetic material is written in terms of temperature *T*, Curie temperature T_c and the Curie constant ($C = \mu_\circ N m^2/3k_B$), where *N* is the number of magnetic atoms per unit volume, as:

$$\chi = \frac{C}{T - T_c} \; .$$

This equation, known as the Curie-Weiss law, describes the behaviour of ferromagnetic materials above the Curie temperature, where $T_c = C\lambda$. The Curie temperature, for iron is roughly 1000 K, therefore the internal molecular field $B_{in} \approx 10^7$ Oe. The origin of such a strong field could not be explained by Weiss theory.



In 1928, Heisenberg explained the origin of this molecular field using a quantum mechanical description of the exchange interaction in a formulation known as the Heisenberg Hamiltonian. He demonstrated that the electrons of neighbouring atoms would have a lower energy if their spins are aligned parallel. This interaction is the result of a combination of the Pauli exclusion principle and the Coulomb repulsion between charges. The Heitler- London model of the hydrogen molecule is used to describe the concept of direct exchange between two electron spins. This model considers two hydrogen molecules (the proton 'a' has electron '1' and proton 'b' has electron '2'), where two electrons are described by $\psi(\boldsymbol{r}_1, s_1)$ and $\psi(\boldsymbol{r}_2, s_2)$, where $\boldsymbol{r}_{1,2}$ are the spatial coordinates and $s_{1,2}$ are the spin states. There is an attractive and repulsive force between the electrons and protons of the two hydrogen atoms, which is electrostatic in origin and can be explain classically by Coulomb's Law. Electrons are indistinguishable fermions. If the distance between the two hydrogen atoms become closer, the wavefunctions of the electrons overlap so that there is a total wavefunction representing both electrons. The Pauli exclusion principle prevents two electrons from having the same spatial wavefunction and the same spin orientation. The total wavefunction must be antisymmetric with respect to exchange of the electron coordinates and its sign must reverse i.e. $\psi(\boldsymbol{r}_1, \boldsymbol{r}_2) = -\psi(\boldsymbol{r}_2, \boldsymbol{r}_1)$.

The symmetry of the total electron wavefunction to $\psi(r) = \phi(r)\,\chi(r)$ depends upon that of both the spin function $\chi(r)$ and the spatial function $\phi(r)$. This antisymmetry of the function $\psi(r)$ can be achieved either by having a symmetric spatial wave function and anti-symmetric spin wave function, or vice versa. Therefore, there are two possible states for the hydrogen molecule:

$$\psi_S = \frac{1}{\sqrt{2}}\,[\phi_a(\boldsymbol{r}_1)\,\phi_b(\boldsymbol{r}_2) + \phi_a(\boldsymbol{r}_2)\,\phi_b(\boldsymbol{r}_1)]\,\chi_S\,,$$

where the subscripts *S* and *T* refer to the spin singlet (s = 0) and triplet (s =1) respectively. In the singlet state the total spin quantum number s = 0, while in the symmetric triplet state, s = 1. The degeneracy of the singlet and triplet states is equal to 2s + 1. The Hamiltonian used to describe the interaction of the two electron systems:

where r_{ab} is the distance between the nuclei in the first term on the right hand side which represents the interaction of the two nuclei *a* and *b*, r_{12} is the distance between the electrons which represent the interaction of the two electrons 1 and 2, and r_{1b}, r_{2a} are the distances between a given nucleus and the electron on the other atom which represents the interaction between an electron with the nucleus of the other hydrogen atom. The interaction energy can be calculated from the equation as:

$$E = \int \psi^* \mathcal{H}_{12} \psi \, d\tau.$$

Therefore, for the wavefunctions of the singlet and triplet states the energies are:

$$K_{12} = \int \phi_a^* (1) \, \phi_b^*(2) \mathcal{H}_{12} \, \phi_a \, (1) \, \phi_b \, (2) \, d\tau_1 d\tau_2,$$

If J_{12} has positive sign then the ground state exhibits parallel spins, while in the hydrogen molecule the electron spins are antiparallel in the ground state, therefore there is no net spin moment and J_{12} has negative sign.

For transition metals, like Fe, Ni, and Co, the $3d$ and $4s$ bands overlap at the Fermi level. The exchange interaction is due to delocalised electrons. Because of the Coulomb repulsion and kinetic energy of the electrons, the bands with antiparallel spin orientation are exchange split. Therefore, the total magnetic moment is non-zero, which leads to the appearance of ferromagnetism. The Stoner model can be used to describe itinerant ferromagnetism, where the exchange splitting energy can be written as $\Delta E_{ex} = IM$, where I is the Stoner exchange parameter and M is the average magnetisation. According to the Stoner criterion, ferromagnetism occurs when:

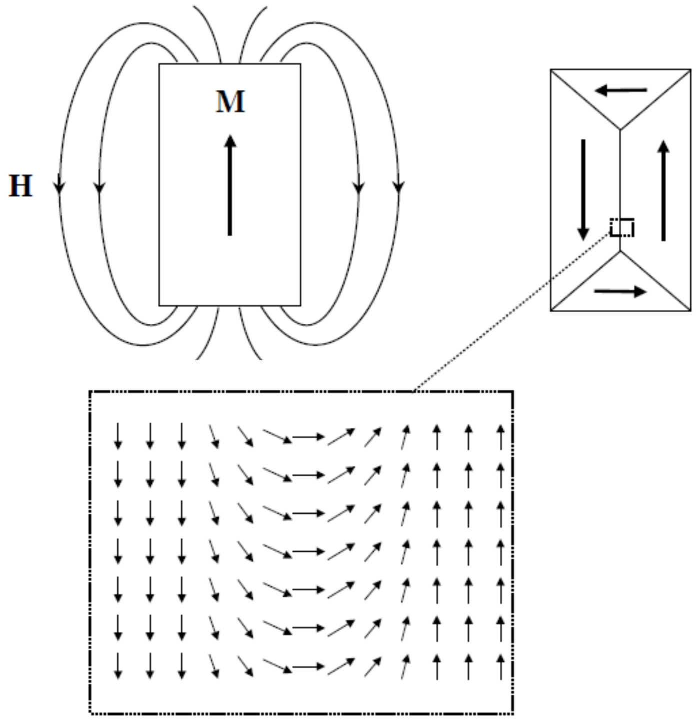
where $N(E_F)$ is the density of states at Fermi level. Therefore, magnetic order is favoured in $3d$ and $4f$ elements that have high $N(E_F)$ and strong exchange splitting, because the change in the exchange energy is bigger than the gain in the kinetic energy when the ferromagnetic state is formed.

Magnetic domains

Everyday experience tells us that there are permanent magnetic materials, and then others that can sometimes be magnetized. How are they different?

A uniformly magnetized material produces a stray magnetic field in which energy is stored - *a high energy state*

This energy may be reduced by the formation of **domains**



But an abrupt reversal of neighbouring magnetic moments would greatly increase the exchange energy, so it is more energetically favorable for the magnetization to rotate gradually within a domain wall In ultrathin films the magnetization tends to rotate within the film forming a Néel wall In thicker films the magnetization rotates out of the plane in a Bloch wall

In permanent magnetic materials domain walls are strongly pinned and so cannot move to form a demagnetized state

Total Magnetic Free Energy

The total magnetic free energy (E_{tot}) of a ferromagnet contains a number of terms. The competition between these energies determines the minimum energy state and can explain many aspects of the material's behaviour. The most significant contributions to the free energy are the exchange interaction, the Zeeman interaction, the magnetostatic energy and the anisotropy energy. The total magnetic energy can be written as:

1. Exchange energy (E_{ex})

The exchange interaction energy was formulated by Heisenberg for two neighbouring spins within a magnetic system. Therefore, this energy is the summation for all pairs of magnetic moments in the magnetic system. However, this interaction is considered to be short-range, therefore with increasing separation between the neighbouring spins, it becomes negligible. This energy accounts for the nearest neighbour spins s_i and s_j in the system as:

The exchange integral (J) in ferromagnetic materials has positive sign, giving parallel alignment of neighbouring spins in the minimum energy configuration.

2. Zeeman Energy (E_z)

The Zeeman energy results from the interaction between the magnetic system (M) and the external applied field (H). This energy is a minimum when all the magnetic moments within a sample are aligned parallel with the applied field and is maximum when the magnetic moments are aligned anti-parallel to the applied field. This energy density is given by:

3. Magnetostatic Energy (E_{ms})

The magnetostatic energy, known as the self-energy, originates from the energy of interaction between magnetic dipoles, which generate an internal field instead of an external applied field. The magnetostatic field direction is opposite to the magnetisation, and is known as the demagnetisation field (H_d). Therefore, while the magnetostatic interaction is long range, it is weaker than the exchange interaction. The energy density resulting from this interaction is:

The factor of half on the RHS appears so that dipoles are not counted twice. The magnetostatic energy is minimised when stray fields from the sample are reduced. The magnitude and distribution of the stray field related to the shape and size of the sample.

4. Anisotropy Energy (E_{ani})

The magnetic anisotropy energy accounts for the fact that a material is more easily magnetised along one axis than another. The origins of some common types of magnetic anisotropy are discussed below.

4.1. Shape Anisotropy

The geometry of the sample plays an important part in increasing or decreasing the anisotropy energy. This is known as shape anisotropy. Shape anisotropy gives rise to free poles at the edge of the sample that generate a stray field. For a sample with non-spherical shape, the magnetisation prefers to align along one axis (easy axis) to reduce the magnetostatic energy of the system, while it costs additional energy for the magnetisation to align along a hard axis or plane. Shape anisotropy can be described using the demagnetisation field (H_d) which is proportional to the sample magnetisation (M) as:

where N is the demagnetisation factor. This factor is estimated from the aspect ratio of the sample, and its value is different depending upon the shape of the sample.

4.2. Magnetocrystalline anisotropy

The origin of the magnetocrystalline anisotropy can be found in the spin-orbit interaction within the sample. This effect is considered to be an intrinsic property of the sample because it is related to the structure of the crystal lattice. The interaction of the spin and orbital angular momenta leads to favourable crystallographic directions in which to magnetise the sample. These favourable and unfavourable directions for the magnetisation are known as the easy and hard axes respectively. The realignment of the magnetisation from an easy to hard axis requires an additional energy known as the magnetocrystalline anisotropy energy.

There are two common types of magnetocrystalline anisotropy known as uniaxial and cubic anisotropy. The energy associated with the uniaxial anisotropy, like hexagonal cobalt, is a function of the angle between the c-axis and the magnetisation, θ, and can be written as:

where $\mathbf{u}=(1/M)\ \mathbf{M}$ is the unit vector parallel to the magnetisation, and K_1 and K_2 are the first and second order anisotropy constants. The constants are sensitive to the temperature.

The expression for the anisotropy energy of cubic crystals such as iron and nickel is:

The values of the anisotropy constants for Ni, Fe and Co may be either negative or positive. For polycrystalline samples, there is no net crystalline anisotropy because of the random orientation of the crystallites.