

## Chapter 3. Modelling the climate system

### 3.1 Introduction

#### 3.1.1 What is a climate model ?

In general terms, a climate model could be defined as a mathematical representation of the climate system based on physical, biological and chemical principles (Fig. 3.1). The equations derived from these laws are so complex that they must be solved numerically. As a consequence, climate models provide a solution which is discrete in space and time, meaning that the results obtained represent averages over regions, whose size depends on model **resolution**, and for specific times. For instance, some models provide only globally or **zonally** averaged values while others have a **numerical grid** whose spatial **resolution** could be less than 100 km. The time step could be between minutes and several years, depending on the process studied.

Even for models with the highest resolution, the **numerical grid** is still much too coarse to represent small scale processes such as turbulence in the **atmospheric and oceanic boundary layers**, the interactions of the circulation with small scale topography features, thunderstorms, cloud micro-physics processes, etc. Furthermore, many processes are still not sufficiently well-known to include their detailed behaviour in models. As a consequence, **parameterisations** have to be designed, based on empirical evidence and/or on theoretical arguments, to account for the large-scale influence of these processes not included explicitly. Because these **parameterisations** reproduce only the first order effects and are usually not valid for all possible conditions, they are often a large source of considerable uncertainty in models.

In addition to the physical, biological and chemical knowledge included in the model equations, climate models require some input from observations or other model studies. For a climate model describing nearly all the components of the system, only a relatively small amount of data is required: the solar irradiance, the Earth's radius and period of rotation, the land topography and bathymetry of the ocean, some properties of rocks and soils, etc. On the other hand, for a model that only represents explicitly the physics of the atmosphere, the ocean and the sea ice, information in the form of boundary conditions should be provided for all sub-systems of the climate system not explicitly included in the model: the distribution of vegetation, the topography of the ice sheets, etc.

Those model inputs are often separated into boundary conditions (which are generally fixed during the course of the simulation) and external forcings (such as the changes in solar irradiance) which drives the changes in climate. However, those definitions can sometimes be misleading. The forcing of one model could be a key state variable of another. For instance, changes in  $CO_2$  concentration could be prescribed in some models, but it is directly computed in models including a representation of the carbon cycle. Furthermore, a fixed boundary in some models, such as the topography of the ice sheet, can evolve interactively in a model designed to study climate variations on a longer time scale.

In this framework, some data are required as input during the simulation. However, the importance of data is probably even greater during the development phase of the model, as they provide essential information on the properties of the system that is being modelled (see Fig. 3.1). In addition, large numbers of observations are needed to test the validity of the models in order to gain confidence in the conclusions derived from their results (see section 3.5.2).

Many climate models have been developed to perform climate projections, i.e. to simulate and understand climate changes in response to the emission of greenhouse gases and aerosols. In addition, models can be formidable tools to improve our knowledge of the most important characteristics of the climate system and of the causes of climate variations. Obviously, climatologists cannot perform experiments on the real climate system to identify the role of a particular process clearly or to test a hypothesis. However, this can be done in the virtual world of climate models. For highly non-linear systems, the design of such tests, often called sensitivity experiments, has to be very carefully planned. However, in simple experiments, neglecting a process or an element of the modelled system (for instance the influence of the increase in  $CO_2$  concentration on the radiative properties of the atmosphere) can often provide a first estimate of the role of this process or this element in the system.

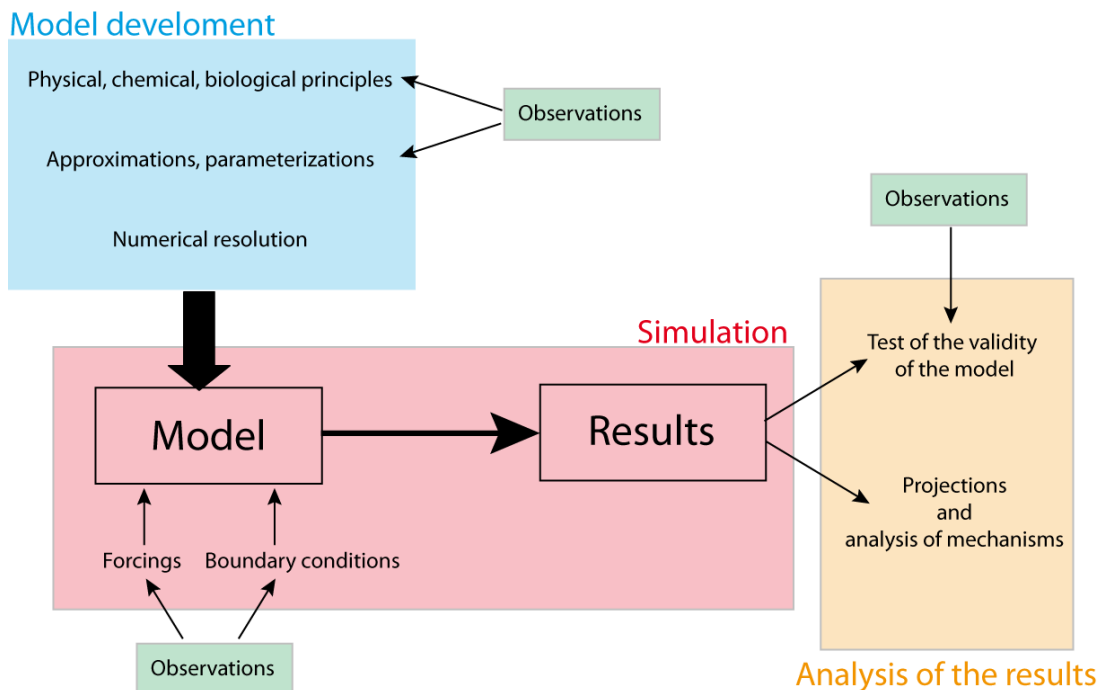


Figure 3.1: Schematic representation of the development and use of a climate model.

### 3.1.2 Types of models

Simplifications are unavoidable when designing a climate model as the processes that should be taken into account range from the scale of centimetres (for instance for atmospheric turbulence) to that of the Earth itself. The involved time scales also vary widely from the order of seconds for some waves, to billions of years when analysing the evolution of the climate since the formation of Earth. It is thus an important skill for a modeller to be able to select the processes that must be explicitly included compared to those that can be neglected or represented in a simplified way. This choice is of course based on the scientific goal of the study. However, it also depends on technical issues since the most sophisticated models require a lot of computational power: even on the largest computer presently available, the models cannot be routinely used for periods longer than a few centuries to millennia. On longer time scales, or when quite a large number of experiments are needed, it is thus necessary to use simpler and faster models. Furthermore, it is often very illuminating to deliberately design a model that includes

### 3. Modelling of the climate system

only the most important properties, so as to understand in depth the nature of a feedback or the complex interaction between the various components of the system. This is also the reason why simple models are often used to analyse the results of more complex models in which the fundamental characteristics of the system could be hidden by the number of processes represented and the details provided.

Modellers have first to decide the variables or processes to be taken into account and those that will be taken as constants. This provides a method of classifying the models as a function of the components that are represented interactively. In the majority of climate studies, at least the physical behaviour of the atmosphere, ocean and sea ice must be represented. In addition, the terrestrial and marine carbon cycles, the dynamic vegetation and the ice sheet components are more and more regularly included, leading to what are called Earth-system models.

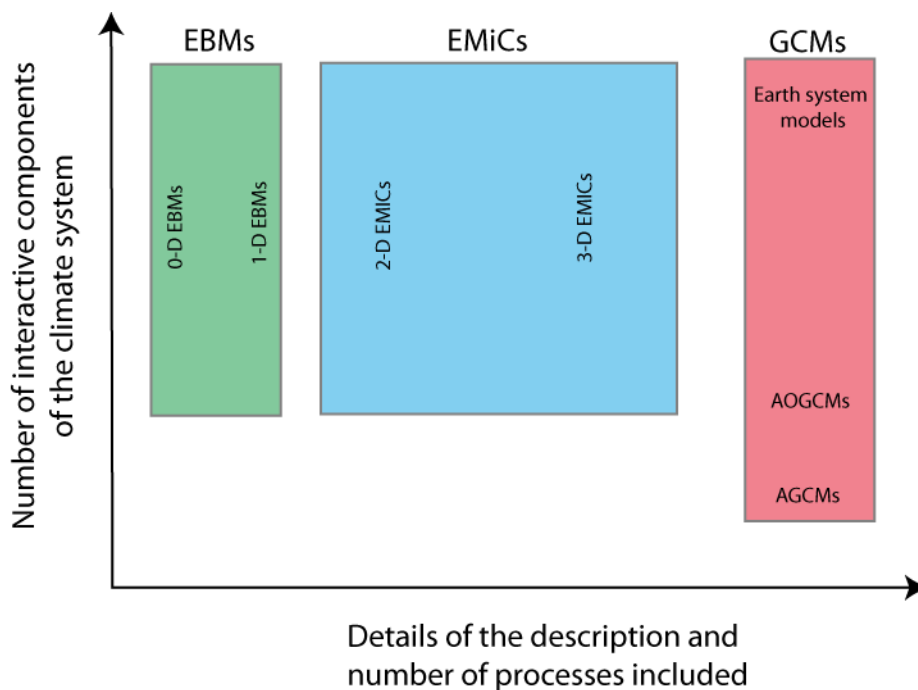


Figure 3.2: Types of climate model.

A second way of differentiating between models is related to the complexity of the processes that are included (Fig. 3.2). At one end of the spectrum, General Circulation Models (GCMs) try to account for all the important properties of the system at the highest affordable resolution. The term GCM was introduced because one of the first goals of these models is to simulate the three dimensional structure of winds and currents realistically. They have classically been divided into Atmospheric General Circulation Models (AGCMs) and Ocean General Circulation Models (OGCMs). For climate studies using interactive atmospheric and oceanic components, the acronyms AOGCM (Atmosphere Ocean General Circulation Model) and the broader CGCM (Coupled General Circulation Model) are generally used.

At the other end of the spectrum, simple climate models (such as the Energy Balance Models, or EBMs, see section 3.2.1) propose a highly simplified version of the dynamic of the climate system. The variables are averaged over large regions, sometimes over the whole Earth, and many processes are not represented or accounted for by the **parameterisations**. EBMs thus include a relatively small number of degree of freedom.

EMICs (Earth Models of Intermediate Complexity) are located between those two extremes. They are based on a more complex representation of the system than EBMs but include simplifications and parameterisations for some processes that are explicitly accounted for in GCMs. The EMICs form the broadest category of models. Some of them are relatively close to simple models, while others are slightly degraded GCMs.

When employed correctly, all the model types can produce useful information on the behaviour of the climate system. There is no perfect model, suitable for all purposes. This is why a wide range of climate models exists, forming what is called the spectrum or the hierarchy of models that will be described in section 3.2. Depending on the objective or the question, one type of models could be selected. The best type of model to use depends on the objective or the question. On the other hand, combining the results from various types of models is often the best way to gain a deep understanding of the dominant processes in action.

## 3.2 A hierarchy of models

### 3.2.1 Energy balance models

As indicated by their name, energy balance models estimate the changes in the climate system from an analysis of the energy budget of the Earth. In their simplest form, they do not include any explicit spatial dimension, providing only globally averaged values for the computed variables. They are thus referred to as zero-dimensional EBMs. The basis for these EBMs was introduced by both Budyko (1969) and Sellers in (1969). Their fundamental equation is very similar to those analysed in sections 2.1.1 and 2.1.5:

Changes in heat storage = absorbed solar radiation - emitted terrestrial radiation

$$C_E \frac{\partial T_s}{\partial t} = \left( (1 - \alpha_p) \frac{S_0}{4} - A \uparrow \right) \quad (3.1)$$

where, as in Chapter 2,  $C_E$  is the effective heat capacity of the media (measured in  $\text{J m}^{-2} \text{K}^{-1}$ ),  $T_s$  the surface temperature,  $t$  the time,  $\alpha_p$  the planetary albedo,  $S_0$  the **Total Solar Irradiance** (TSI) and  $A \uparrow$  the total amount of energy that is emitted by a  $1 \text{ m}^2$  surface of the Earth.  $A \uparrow$  could be represented on the basis of the **Stefan-Boltzmann law**, using a factor  $\tau_a$  to represent the infrared transmissivity of the atmosphere (including the greenhouse gas effect), as.

$$A \uparrow = \varepsilon \sigma T_s^4 \tau_a \quad (3.2)$$

where  $\varepsilon$  is the emissivity of the surface. Using an albedo of 0.3, an emissivity of 0.97, and a value of  $\tau_a$  of 0.64 leads to an equilibrium temperature  $T_s=287\text{K}$ , which is close to the observed one. In some EBMs, Eq. 3.2 is linearised to give an even simpler formulation of the model. On the other hand,  $\tau_a$  and  $\alpha_p$  are often parameterised as a function of the temperature, in particular to take into account the fact that cooling increases the surface area covered by ice and snow, and thus increases the planetary albedo.

In order to take the geographical distribution of temperature at the Earth's surface into account, zero-dimensional EBMs can be extended to include one (generally the latitude) or two horizontal dimensions (Fig. 3.3). An additional term  $\Delta transp$  is then included in Eq. 3.1 representing the net effect of heat input and output associated with horizontal transport:

### 3. Modelling of the climate system

$$C_E \frac{\partial T_{s,i}}{\partial t} = \left( (1 - \alpha_p) \frac{S_0}{4} - A \uparrow \right) + \Delta transp \quad (3.3)$$

An index  $i$  has been added to the surface temperature to indicate that the variable corresponds to the region  $i$ . The simplest form for the transport is to treat it as a linear function of temperature, but more sophisticated parameterisations are also used, including, for instance, a diffusion term.

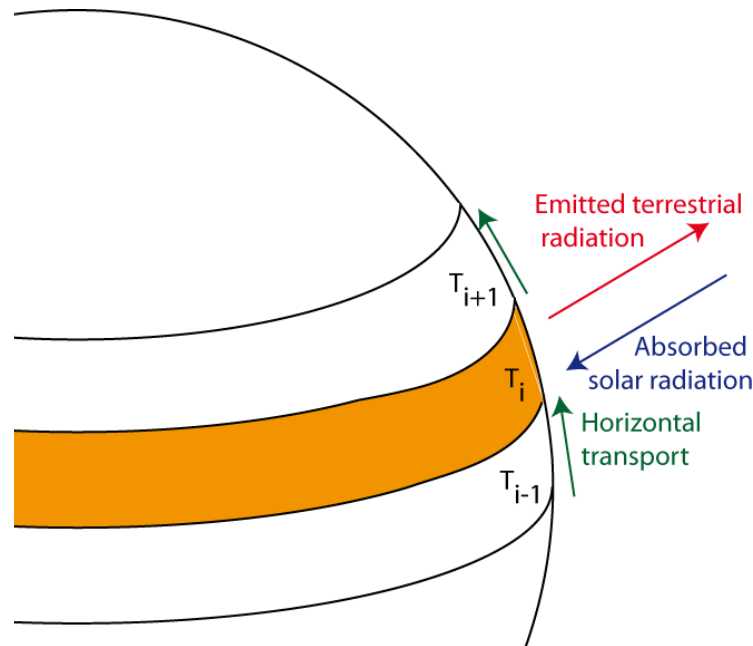


Figure 3.3: Representation of a one-dimensional EBM for which the temperature  $T_i$  is averaged over a band of longitude.

Box models have clear similarities to EBMs as they represent large areas or an entire component of the system by an average which describes the mean over one “box”. The exchanges between the compartments are then **parameterised** as a function of the characteristics of the different boxes. The exact definition of the boxes depends on the purpose of the model. For instance, some box models have a compartment for the atmosphere, the land surface, the ocean surface layers and the deep ocean, possibly making a distinction between the two hemispheres. Others include additional components allowing a description of the carbon cycle and thus have boxes corresponding to the various reservoirs described in section 2.3.

#### 3.2.2 Intermediate complexity models

Like EBMs, EMICs involve some simplifications, but they always include a representation of the Earth’s geography, i.e. they provide more than averages over the whole Earth or large boxes. Secondly, they include many more degrees of freedom than EBMs. As a consequence, the parameters of EMICs cannot easily be adjusted to reproduce the observed characteristics of the climate system, as can be done with some simpler models.

The level of approximation involved in the development of the model varies widely between different EMICs. Some models use a very simple representation of the

geography, with a **zonally** averaged representation of the atmosphere and ocean. A distinction is always made between the Atlantic, Pacific and Indian basins (Fig. 3.4) because of the strong differences between them in the circulation (see section 1.3.2). As the atmospheric and oceanic circulations are fundamentally three-dimensional, some **parameterisations** of the **meridional** transport are required. Those developed for EMICs are generally more complex and physically based than the ones employed in one-dimensional EBMs.

On the other hand, some EMICs include components that are very similar to those developed for GCMs, although a coarser numerical grid is used so that the computations proceed fast enough to allow a large number of relatively long simulations to be run. Some other components are simplified, usually including the atmosphere because this is the component that is most depending on computer time in coupled climate models.

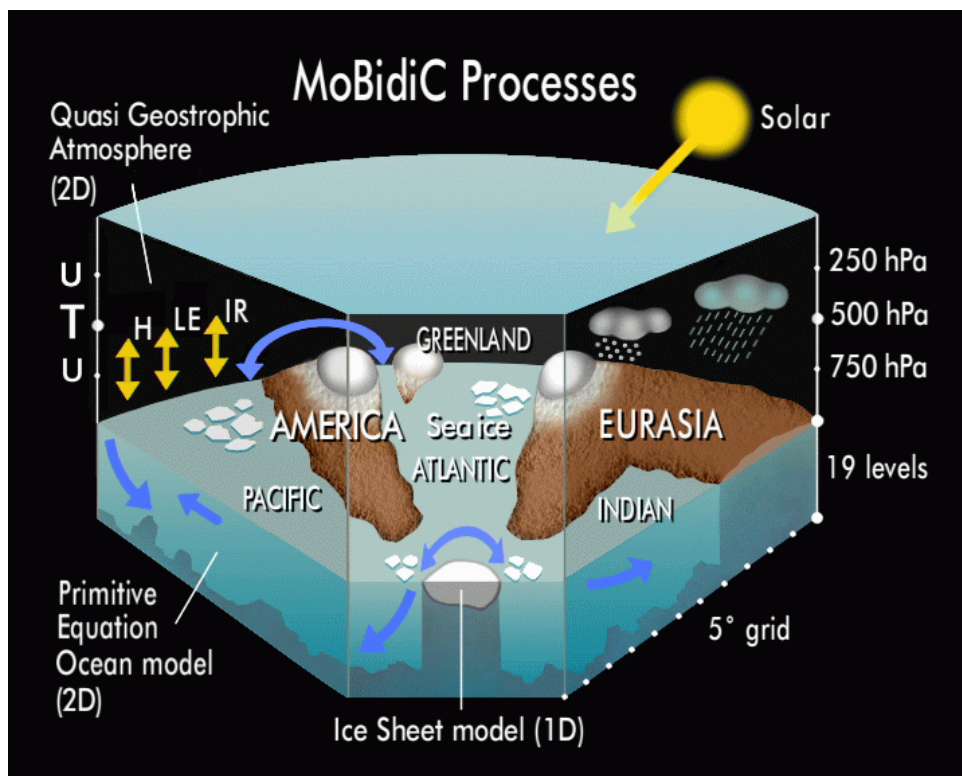


Figure 3.4: Schematic illustration of the structure of the climate model of intermediate complexity MOBIDIC that includes a **zonally** averaged atmosphere, a 3-basin **zonal** oceanic model (corresponding to the Atlantic, the Pacific and the Indian Oceans) and simplified ice sheets. More details about this model are available at <http://www.astr.ucl.ac.be/index.php?page=MoBidiC%40Description>.

### 3.2.3 General circulation models

General circulation models provide the most precise and complex description of the climate system. Currently, their **grid** resolution is typically of the order of 100 to 200 km. As a consequence, compared to EMICs (which have a grid resolution between 300 km and thousands of kilometres), they provide much more detailed information on a regional scale. A few years ago, GCMs only included a representation of the atmosphere, the land surface, sometimes the ocean circulation, and a very simplified version of the sea ice. Nowadays, GCMs take more and more components into account, and many new models

### 3. Modelling of the climate system

now also include sophisticated models of the sea ice, the carbon cycle, ice sheet dynamics and even atmospheric chemistry (Fig. 3.5).

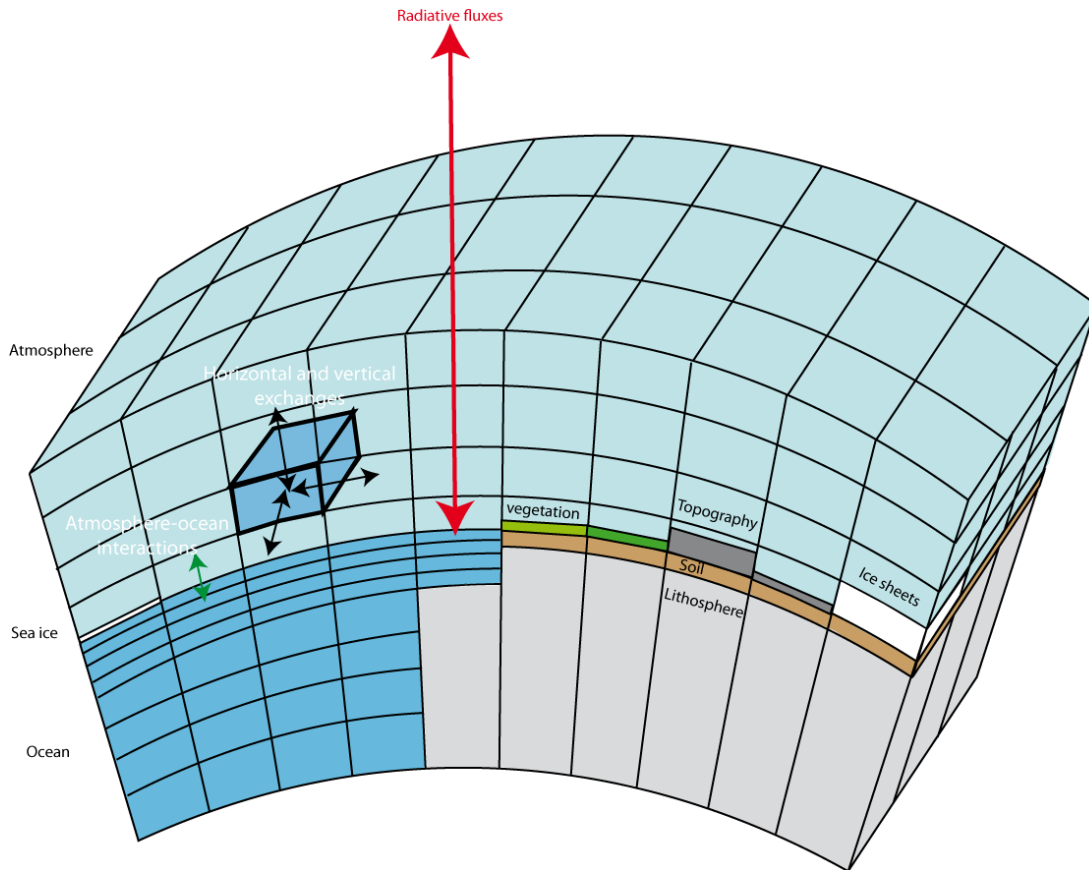


Figure 3.5: A simplified representation of part of the domain of a general circulation model, illustrating some important components and processes. For clarity, the curvature of the Earth has been amplified, the horizontal and vertical coordinates are not to scale and the number of grid points has been reduced compared to state-of-the-art models.

Because of the large number of processes included and their relatively high resolution, GCM simulations require a large amount of computer time. For instance, an experiment covering one century typically takes several weeks to run on the fastest computers. As computing power increases, longer simulations with a higher resolution become affordable, providing more regional details than the previous generation of models.

## 3.3 Components of a climate model

### 3.3.1 Atmosphere

The basic equations that govern the atmosphere can be formulated as a set of seven equations with seven unknowns: the three components of the velocity  $\vec{v}$  (components  $u$ ,  $v$ ,  $w$ ), the pressure  $p$ , the temperature  $T$ , the specific humidity  $q$  and the density. The seven equations, written for the atmosphere, are:

(1-3) Newton's second law (momentum balance, i.e.  $\vec{F} = m\vec{a}$ , force equals mass times acceleration),

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\vec{\nabla}\rho - \vec{g} + \vec{F}_{fric} - 2\vec{\Omega} \times \vec{v} \quad (3.4)$$

In this equation,  $d/dt$  is the total derivative, including a transport term,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \quad (3.5)$$

$\vec{g}$  is the apparent gravity vector (i.e. taking the centrifugal force into account),  $\vec{F}_{fric}$  is the force due to friction, and  $\vec{\Omega}$  is the angular velocity vector of the Earth (the last term is the **Coriolis force**)

(4) The continuity equation or the conservation of mass

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{v}) \quad (3.6)$$

(5) The conservation of the mass of water vapour

$$\frac{\partial \rho q}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{v} q) + \rho(E - C) \quad (3.7)$$

where  $E$  and  $C$  are evaporation and condensation respectively

(6) The first law of thermodynamics (the conservation of energy)

$$Q = C_p \frac{dT}{dt} - \frac{1}{\rho} \frac{dp}{dt} \quad (3.8)$$

where  $Q$  is heating rate per unit mass and  $C_p$  the specific heat

(7) The equation of state

$$\rho = \rho R_g T \quad (3.9)$$

Before these equations are used in models some standard approximations have to be performed. For instance, assuming **hydrostatic equilibrium**, which is a good approximation at the scale of GCMs, provides a considerable simplification of the equation of motion along the vertical. Secondly, the quasi-Boussinesq approximation states that the time variation of the density could be neglected compared to the other terms of the continuity equation, filtering the sound waves. However, supplementary equations for the liquid water content of atmospheric parcels or other variables related to clouds are often added to this set of equations.

Unfortunately, these seven equations do not form a closed system. First, the frictional force and the heating rate must be specified. Computing the heating rate, in particular, requires a detailed analysis of the radiative transfer in the atmosphere,



### 3. Modelling of the climate system

accounting for both the **longwave** and the **shortwave** radiation in the atmospheric columns (see Fig. 3.5), as well as of the heat transfers associated with evaporation, condensation and sublimation. The influence of clouds in these processes is usually a source of considerable uncertainty. This part of the model is commonly referred to as the model “physics” while the calculation of the transport is called the “dynamics”. Secondly, as discussed in section 3.1.1, the models can only adequately resolve some of the processes that are included in the equations. The important processes that occur at scales that could not be resolved by the model grid must thus be **parameterised**, introducing new terms into the equations 3.5, 3.7 and 3.8. The boundary conditions of the equations describing the interactions between the atmosphere and the other components of the climate system also need to be specified (see section 3.3.7).

#### 3.3.2 Ocean

The major equations that govern the ocean dynamics are based on the same principles as the equations for the atmosphere. The only significant difference is that the equation for the specific humidity is not required for the ocean, while a new equation for the salinity needs to be introduced. The equation of state is also fundamentally different. Unlike the atmosphere, there is no simple law for the ocean and the standard equation of state is expressed as a function of the pressure, the temperature and the salinity as a long polynomial series.

It is much easier to compute the heating rate in the ocean than in the atmosphere. In addition to the heat exchanges at the surface, the only significant heat source in the ocean is the absorption of solar radiation. This is taken into account in the model through an exponential decay of the solar irradiance. The situation for salinity is even more straightforward, as there is no source or sink of salinity inside the ocean. The equations governing these two variables are thus relatively simple:

$$\frac{dT}{dt} = F_{sol} + F_{diff} \quad (3.10)$$

$$\frac{dS}{dt} = F_{diff} \quad (3.11)$$

where  $F_{sol}$  is the absorption of solar radiation in the ocean. Eq. 3.10 does not apply to the *in situ* temperature, but the **potential temperature** in order to account for the effect of the compressibility of seawater. The difference between those two temperatures is relatively low in the upper ocean, but it can reach several tenths of a degree in the deeper layers, an important difference in areas where the gradients are relatively small (see section 1.3.3.2).

In Eqs. 3.10 and 3.11, in contrast to section 3.3.1, we have explicitly added a term to the right hand side representing the influence of the processes at scales that cannot be included in the model. As small-scale processes tend to mix properties, they are generally modelled as a diffusion term  $F_{diff}$ . In its simplest form, a **Laplacian** formulation is retained. This is also often the formulation selected for the friction term (in Eq. 3.4). Because of the very different scales of ocean model grids on the vertical (a few hundred meters) and on the horizontal (tens to hundreds of kilometres), the small-scale processes in these two directions have different properties. As a consequence, the coefficients associated with the **Laplacian** (diffusion coefficient, and viscosity for tracer and momentum equations, respectively) differ by several orders of magnitude in the vertical and the horizontal. Actually, it appears that, rather than separating horizontal and vertical

directions, it is better to use a referential that is aligned with the density surfaces. To this end, **isopycnal** (along surfaces of equal density) and **diapycnal** (normal to surfaces of constant density) diffusion coefficients are calculated. These coefficients can be simply chosen, or they can be computed using sophisticated modules (including turbulence models) that take into account the stirring of the winds, the influence of density gradients, the breaking of surface and internal waves, etc.

However, all small-scale processes cannot be represented by a diffusion term. For instance, dense water formed at high latitudes can flow down the slope in narrow boundary currents called overflows. They have a strong influence on the water mass properties but could not be represented on the model grid scale. In this case, a parameterization of their effects as a transport process rather than a diffusion term appears more appropriate.

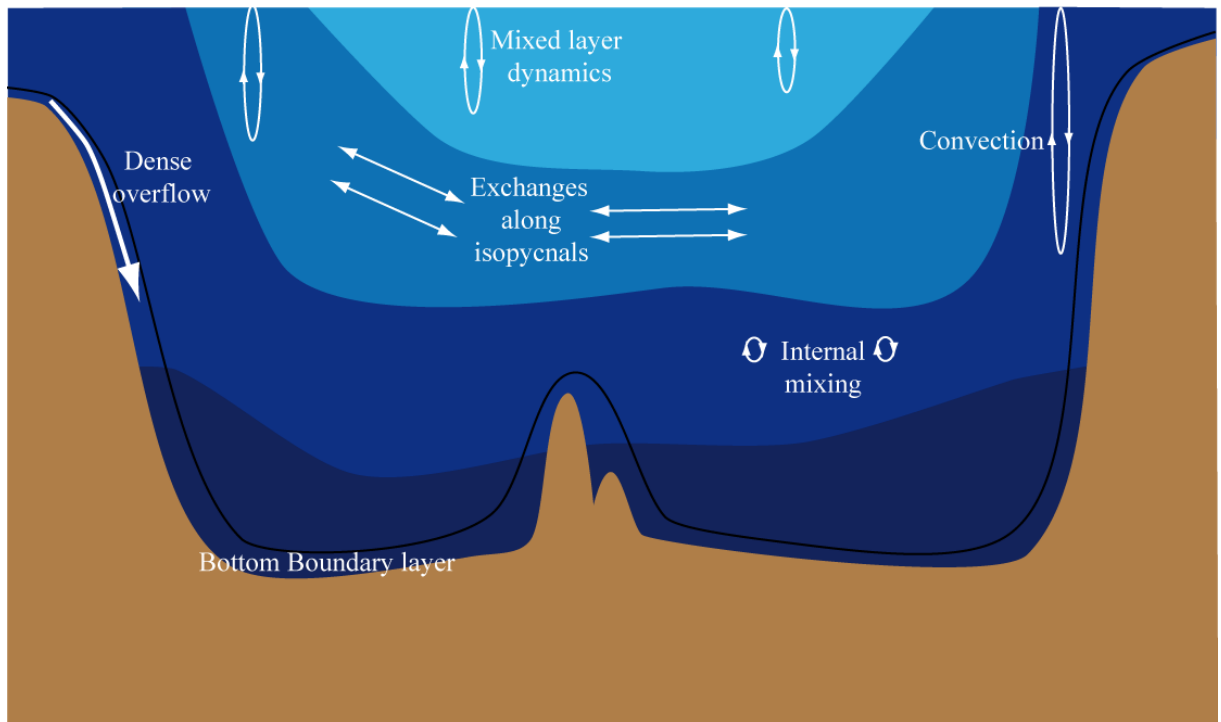


Figure 3.6: Schematic representation of some small-scale processes that have to be parameterized in ocean models. Modified from <http://www.gfdl.noaa.gov/ocean-models-at-gfdl>.

### 3.3.3 Sea ice

The physical processes governing the development of sea ice can be conceptually divided into two parts (Fig. 3.7). The first one covers the thermodynamic growth or decay of the ice, which depend on the exchanges with the atmosphere and the ocean. For those processes, the horizontal heat conduction through ice, can be safely neglected because the horizontal scale is much larger than the vertical one. The thermodynamic code of a sea-ice model is thus basically one-dimensional over the vertical, and the conduction equation can be written:

$$\rho_c c_{pc} \frac{\partial T_c}{\partial t} = k_c \frac{\partial^2 T_c}{\partial z^2} \quad (3.12)$$

### 3. Modelling of the climate system

where  $\rho_c$ ,  $c_{pc}$ , and  $k_c$  are the density, specific heat and thermal conductivity, and  $T_c$  is the temperature. The subscript  $c$  stands for either ice ( $i$ ) or snow ( $s$ ).

The heat balance at the surface (which can be modelled similar to equation 2.36) allows the computation of the surface temperature and of the snow or ice melting. At the bottom of the sea ice, the heat balance provides an estimate of ice melting or formation, the temperature there being considered as equal to the freezing point temperature. Furthermore, the heat budget of the **leads** is used to determine whether new ice will form in the open ocean areas and if lateral melting will occur. Finally, as the ice growth and decay are a function of the ice thickness (the smaller the ice thickness, the faster the growth and decay of the ice), it is necessary to take the distribution of the ice thickness into account in sea-ice models.

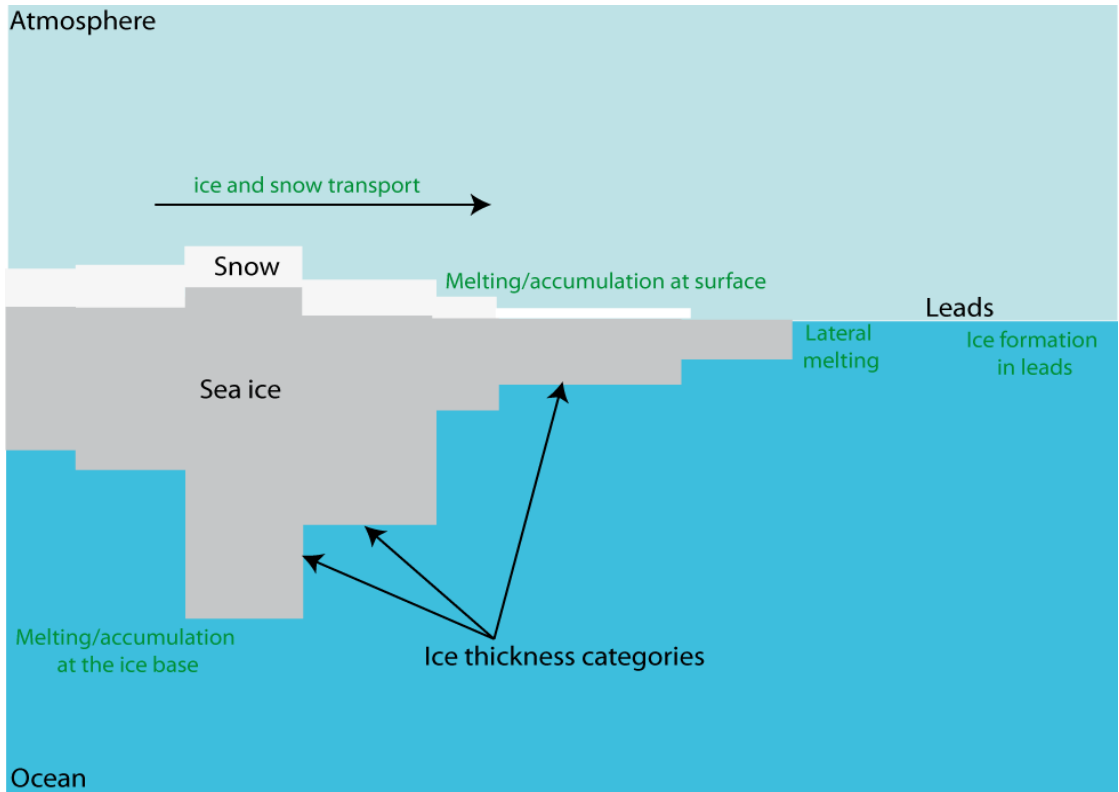


Figure 3.7: The main processes that have to be taken into account in a sea ice model.

When studying the large-scale dynamics of sea ice, the ice is modelled as a two-dimensional continuum. This hypothesis works if, in a model grid box, a large number of ice floes of different sizes and thicknesses are present as well as the **leads**. Newton's second law then gives:

$$m \frac{d\vec{u}_i}{dt} = \vec{\tau}_{ai} + \vec{\tau}_{wi} - m f \vec{e}_z \times \vec{u}_i - mg \vec{\nabla} \eta + \vec{F}_{int} \quad (3.13)$$

where  $m$  is the mass of snow and ice per unit area,  $\vec{u}_i$  is the ice velocity.  $\vec{\tau}_{ai}$  and  $\vec{\tau}_{wi}$  are the forces per unit area from the air and water.  $\vec{F}_{int}$  is the force per unit area due to internal interactions and  $f$ ,  $\vec{e}_z$ ,  $g$ , and  $\eta$  are respectively the Coriolis parameter, a unit vector

pointing upward, the gravitational acceleration, and the sea-surface elevation. The first two terms on the right hand side represent the interactions with the ocean and the atmosphere. The third term is the **Coriolis force** and the fourth term the force due to the oceanic tilt. The internal forces  $\vec{F}_{int}$  are a function of ice thickness and concentration, providing a strong link between dynamics and thermodynamics, while the velocity obtained from equation 3.13 is used in the computation of the transport of the model state variables such as the ice thickness, the concentration of each ice thickness category and the internal sea ice temperature and salinity.

### 3.3.4 Land surface

As with sea ice, horizontal heat conduction and transport in soil can be safely neglected. Therefore, thermodynamic processes are only computed along the vertical (in a similar way to Eq. 3.12). In the first generation of land surface models, only one soil layer was considered. As in Eq. 2.36, soil temperature can then be computed from the energy balance at the surface:

$$\rho c_p h_{su} \frac{\partial T_s}{\partial t} = (1 - \alpha) F_{sol} + F_{IR\downarrow} + F_{IR\uparrow} + F_{SE} + L_f E + F_{cond} \quad (3.14)$$

When a snow layer is present, the computation of the development of the snow depth, density and concentration is part of the surface energy balance. This is very important for the **albedo**, whose **parameterisation** as a function of the soil characteristics (snow depth, vegetation type, etc) is a crucial element of surface models.

The latent heat flux  $F_{LE}$  in Eq. 2.36 has been replaced in Eq. 3.14 by the latent heat of fusion times the evaporation rate ( $L_f E$ ), as is classically done in surface models. This evaporation rate depends on the characteristics of the soil and the vegetation cover as well as on the water availability. It can be expressed with the help of the **moisture availability function**  $\beta$  ( $0 < \beta < 1$ ), defined as the ratio between the evaporation rate of the surface ( $E$ ) and the potential evaporation ( $E_p$ ), i.e. the evaporation that would occur on a homogenous wet surface such as a lake:

$$\beta = E/E_p \quad (3.15)$$

A land surface model also simulates the water content of the soil. In the simple early models, this is represented by a bucket model. The bucket is allowed to fill up to a critical level, corresponding to the equivalent of 15 to 30 cm of water everywhere on a grid cell. If, after taking into account precipitation and surface evaporation, the amount of water in the soil exceeds this threshold, the excess water is transferred to a prescribed oceanic grid point through river runoff.

The easiest way is to simply select the parameters (such as the **albedo**, the surface roughness or the **moisture availability function**) in Eq. 3.14 and 3.15 on the basis of observations of surface characteristics. However, the accurate representation of the energy and water exchanged between the atmosphere and the land surface requires a more sophisticated description of the effects of vegetation and soil. Horizontally, instead of a homogenous description as proposed in Eq. 3.14, the heterogeneous nature of the land surface, covered by different types of vegetation, bare soils, cities, etc, must be explicitly accounted for. Vertically, the interactions between the **canopy**, the soil and the roots also have a clear impact that cannot be adequately computed by an equation as simple as 3.14.

### 3. Modelling of the climate system

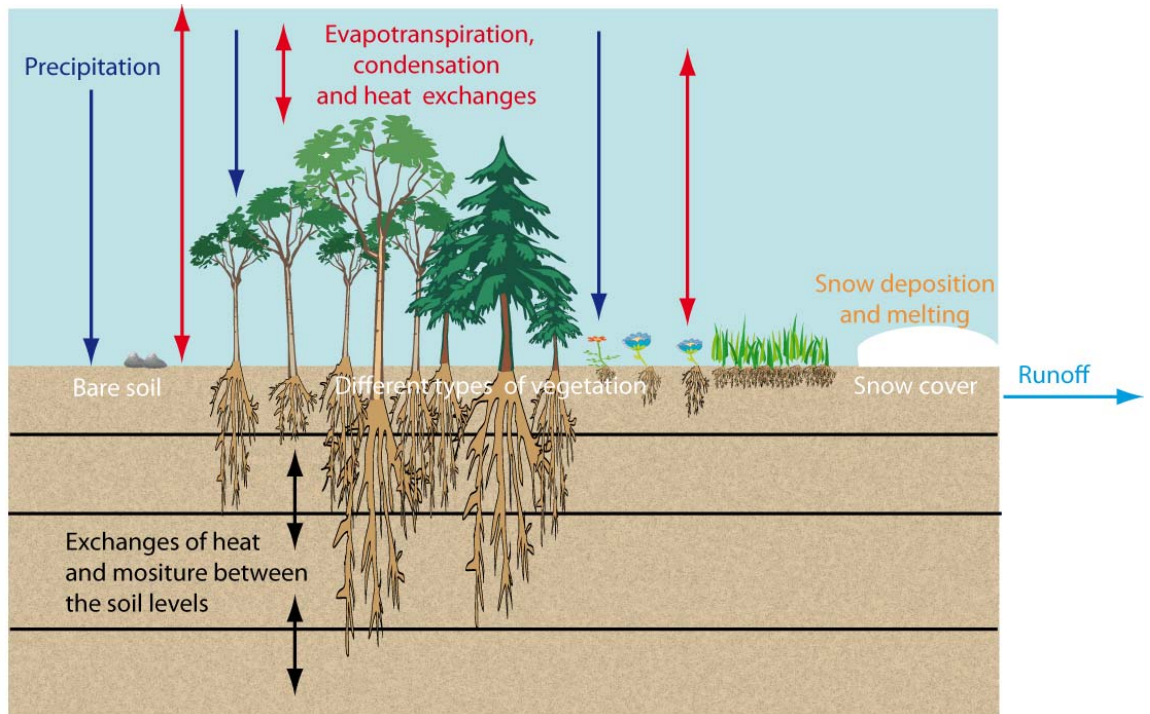


Figure 3.8: The main processes that have to be taken into account in a land surface model. For clarity, the carbon storage in plants and in soils, as well as the exchanges between these reservoirs and with the atmosphere are not shown.

Sophisticated representations of these processes are now included in the state-of-the-art GCMs. In particular, they include a multi-layer soil model, a comprehensive description of the vegetation cover and of the physical and chemical interactions between the plants, the soil and the atmosphere. They also have a sophisticated river-routing scheme which accounts for the duration of the water transport as well as evaporation during the journey to the ocean or the interior sea. These improvements are also essential in an adequate representation of the carbon cycle on land (see section 2.3.3). At present, the majority of climate models do not include a representation of **permafrost**, but this is likely to change because of the large modifications in the extent of permafrost that are expected during the 21<sup>st</sup> century.

Some models take the community composition and vegetation structure as a boundary condition or forcing (if land use changes are specified for instance). They then use this information to determine the physical characteristics of the surface and the soil, as well as the carbon storage over land. However dynamic global vegetation models (DGVMs) explicitly compute the transient dynamics of the vegetation cover in response to climate changes and disturbances such as fires. DGVMs can also provide the distribution of **biomes** that are in equilibrium with climate conditions (Figure 3.9). It is of course impossible to represent the fraction covered by each of the hundreds of thousands of different plant species in DGVMs. The plants are thus grouped into **plant functional types** (PFTs) that share common characteristics. Very simple models only use two PFTs (trees and grass, the area not covered by trees or grass corresponding to the fraction of desert in a **grid** element), while more sophisticated models use more than ten different PFTs.

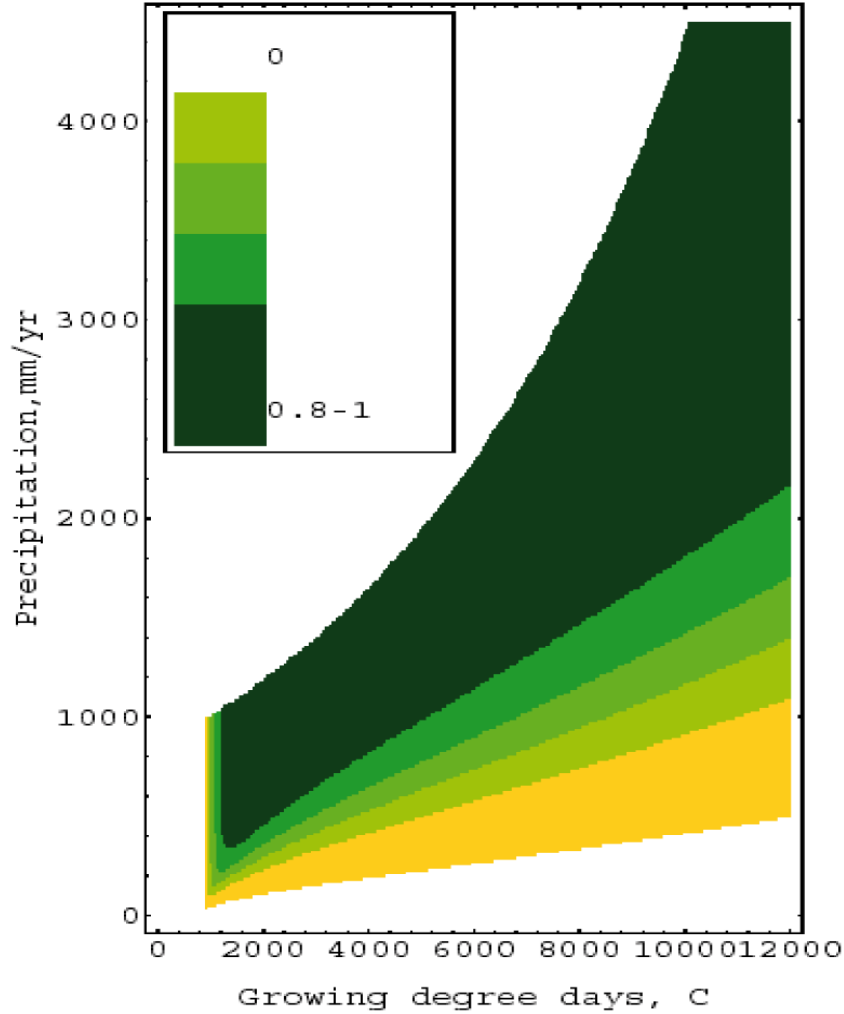


Figure 3.9: The equilibrium fraction of trees in a model that includes two plant functional types and whose community composition is only influenced by precipitation and the growing degree days (GDD) (Brovkin et al. 1997). Growing degree days are defined as the sum of the daily mean near-surface temperatures for the days of a year with a temperature higher than 0°C. Figure Courtesy of V. Brovkin.

### 3.3.5 Marine biogeochemistry

Models of biogeochemical cycles in the oceans are based on a set of equations whose formulation is very close to that of equations 3.10 and 3.11 for the ocean temperature and salinity:

$$\frac{dTrac_{bgc}}{dt} = F_{diff} + Sources - Sinks \quad (3.16)$$

where  $Trac_{bgc}$  is a biogeochemical variable. Those variables are often called tracers because they are transported and diffused by the oceanic flow (the left hand side of the equation and the term  $F_{diff}$ ).

### 3. Modelling of the climate system

$Trac_{bgc}$  can represent  $DIC$ ,  $Alk$ , the concentration of various chemical species (including nutrients necessary for phytoplankton growth) or the biomass of different groups of phytoplankton, detritus, zooplankton and (more rarely) higher trophic levels. Simplified carbon cycle models include a few state variables while the most sophisticated biogeochemical models have more than 30 of them. The *Sources* and *Sinks* terms account for the increase or decrease of the tracer concentration in response to biogeochemical processes, including thus a representation of the processes described in section 2.3. For instance, for a particular phytoplankton group, the *Sources* term could be related to the growth in the biomass by photosynthesis, while the *Sinks* are the consumption of phytoplankton by zooplankton as well as the mortality of the cells. In addition to the processes taking place in the water column, some models include a comprehensive ocean sediment component in order to be able to study the long-term changes in the carbon cycle.

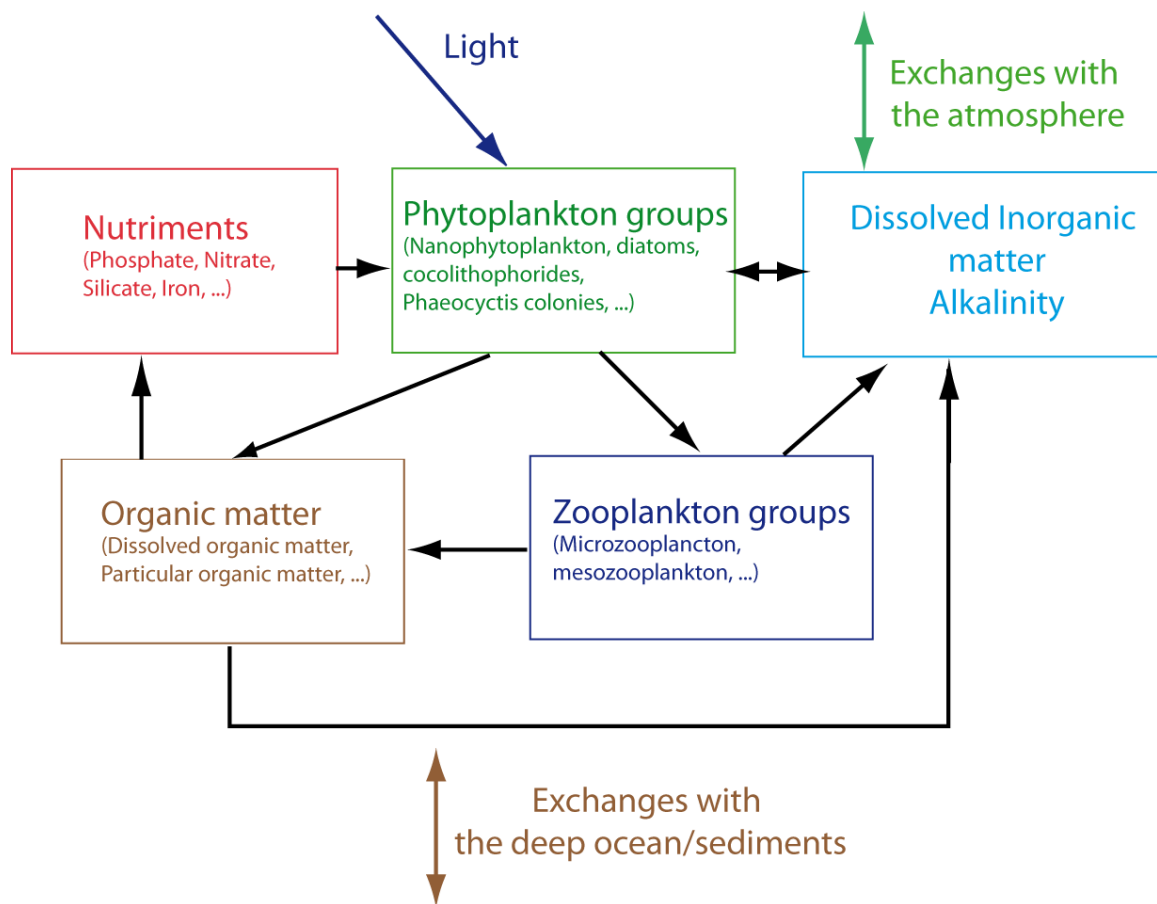


Figure 3.10: A simplified scheme representing some of the variables of a biogeochemical model. The interactions between the groups are complex as the different types of phytoplankton need different nutrients, are grazed by different types of zooplankton etc.

#### 3.3.6 Ice sheets

As already discussed for the atmosphere and the sea ice, ice-sheet models can be decomposed into two major components: a dynamic core that computes the flow of the ice and a thermodynamic part that estimates the changes in ice temperatures, snow accumulation, melting, etc. The ice velocity can be computed using the complete three-

dimensional equation. This is affordable for regional models, focusing on particular regions, but approximations are often necessary for climate models which compute the development of whole ice sheets on long timescales.

The conservation of ice volume can be written as:

$$\frac{\partial H}{\partial t} = -\vec{\nabla} \cdot (\vec{v}_m H) + M_b \quad (3.17)$$

where  $\vec{v}_m$  is the depth-averaged horizontal velocity field and  $M_b$  is the mass balance accounting for snow accumulation as well as basal and surface meltings. Surface melting can be deduced from the energy budget at the surface (similar to Eq. 2.36, see also section 3.3.3). Simpler formulations of surface melting are based on the positive degree-day methods, which relates the melting to the temperature during the days with temperatures above 0°C. An important element in the mass balance at the surface of the ice sheets is the position of the equilibrium line between the regions where, on a yearly average, snow tends to accumulate and the ablation region (where there is net melting of the snow and ice when the surface mass balance is integrated over the whole year). On the Greenland ice sheet, in present-day conditions, ablation occurs in many areas, whereas on the colder Antarctic ice sheet, it is restricted to a few regions only.

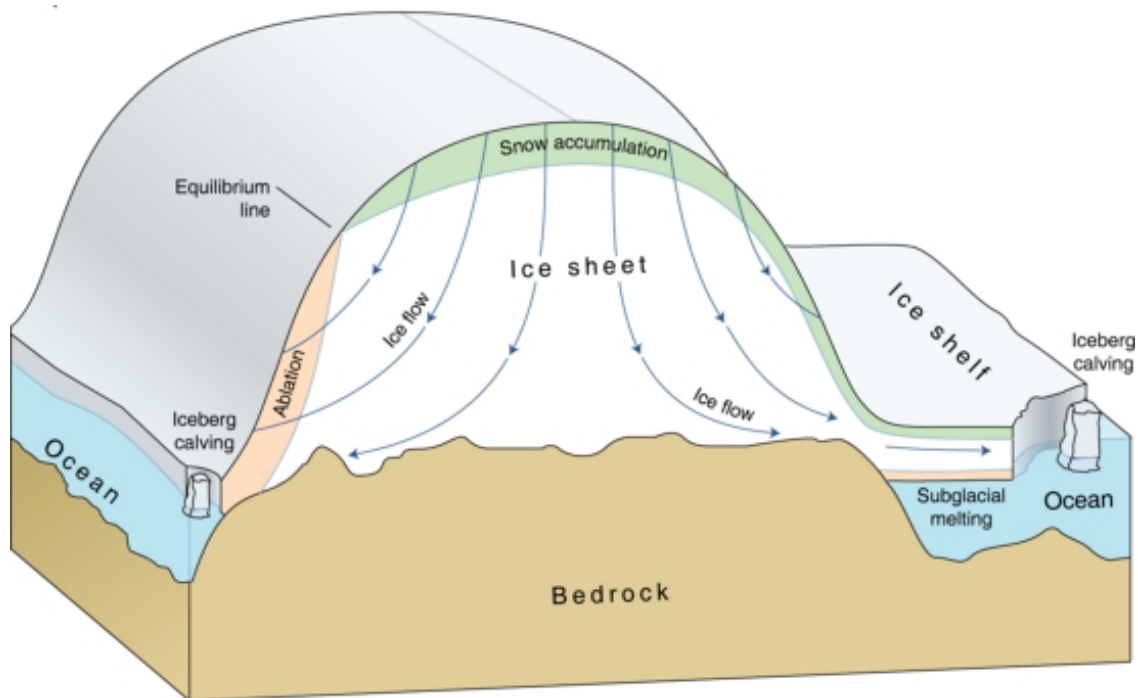


Figure 3.11: The main processes that have to be taken into account in an ice sheet model. Figure reproduced from <http://www.solcomhouse.com/icecap.htm>.

The melting at the ice base is deduced from the balance between the heat conduction in the ice and in the ground, taking into account the geothermal heat flux. Conditions at the ice base, and in particular the presence of water or ice close to the melting point at the corresponding pressure, have a large impact on the ice velocity as they reduce the stresses greatly, compared to the situation where the ice is well below the freezing point.



### 3. Modelling of the climate system

Ice sheets models also need to take into account the interactions between grounded ice and **ice shelves**. Because of local melting and iceberg calving, ice shelves can make a large contribution to the mass balance of the ice sheets, as is currently the case for Antarctica. Furthermore, they generate stresses that tend to slow down the ice flow on land. Indeed, observations have shown that the recent breakdown of some ice shelves has produced, in some regions, an acceleration of the land ice.

An additional element in ice-sheet models is the representation of interactions with the underlying bedrock. In particular, as the load of the **ice sheet** tends to depress the bedrock, a bedrock adjustment model is needed to compute the position of the ground as a function of the ice mass. This then yields the elevation of the ice sheet as a function of the ice thickness.

#### 3.3.7 Coupling between the components - Earth system models

The interactions between the various components of the system play a crucial role in the dynamics of climate. Wind stress, heat and freshwater fluxes at the ocean surface are the main drivers of the ocean circulation (see section 1.3.2). The evaporation at the ocean surface is the largest source of water vapour for the atmosphere, which influences the radiative properties of the air (section 2.1.2) and the atmospheric heat transport (section 2.1.5). Snow falling on ice sheets is an essential element of their mass balance. Many other examples could be cited.

Some of those interactions are quite straightforward to compute from the models state variables, while more sophisticated **parameterisations** are required for others. For instance, the parameterisation of the wind stress and of the heat flux at the atmospheric base (e.g., Eqs. 2.33 and 2.34) can be derived from theories of the **atmospheric boundary layer**. However, this computation still requires empirical parameters that depend on the characteristics of the surface, introducing some uncertainties into the determination of the flux.

The technical coupling of the various components to obtain a climate- or Earth-system model brings additional difficulties. The numerical codes have generally been developed independently by different groups, using different coding standards, different **numerical grids**, etc. It is thus necessary to design an interface and to use a coupler, i.e. code specially adapted to represent the exchanges between the various components.

The above presentation includes the main elements of the majority of current Earth-system models. However, the description is far from exhaustive. New components (such as models of the methane cycle, of the nitrogen cycle, more sophisticated representation of atmospheric chemistry, etc) are continuously included in order to obtain a more comprehensive representation of the complex interactions in the system.

## 3.4 Numerical resolution of the equations

### 3.4.1 Consistence, convergence and stability

The equations that rule the climate system are **partial differential equations** (PDEs) such as those presented in section 3.3, except when extremely simplified models are used (section 3.2.1). It is first necessary to ensure that those equations are mathematically well-posed, i.e. that the problem has a unique solution that depends on the initial and boundary conditions. This requires that those initial and boundary conditions are properly specified. For instance, to solve the equation for temperature in the ocean knowing the velocity field (Eq. 3.10), we must specify the initial temperature

over the whole domain at a time  $t_0$  as well as one boundary condition over all the points of the spatial boundaries of the domain, which can be the value of the heat flux or of the temperature there. Below, we will consider that all the problems investigated are well-posed.

In order to solve the equations of the mathematical models developed for each component of the climate system (section 3.3), those models have to be transformed into numerical models that can be handled by a computer. The first method, which is probably the easiest to understand, is to approximate the derivatives in the **partial differential equations** by finite differences. This is called the **finite difference method**. The solution is no longer a continuous function (as for the PDEs) but a discrete one, only defined for specific times separated by the time step  $\Delta t$ , and specific locations separated by the spatial step  $\Delta x$  (plus  $\Delta y$  and  $\Delta z$  for a problem with three spatial dimensions).

Imagine for instance, a very simple ordinary differential equation:

$$\frac{du}{dt} = A \cos(t) \quad (3.18)$$

where  $t$  is the time,  $u$  a state variable (for instance the velocity) which depends here only on the time, and  $A$  a constant. The derivative according to time could be approximated by a finite difference, leading to the finite difference equation:

$$\frac{U^{n+1} - U^n}{\Delta t} = A \cos(n\Delta t) \quad (3.19)$$

$U^n$  is the discrete solution of the finite difference equation at time step  $n$ . If  $\Delta t$  is constant,  $t = n \Delta t$ , assuming that the initial time is 0.

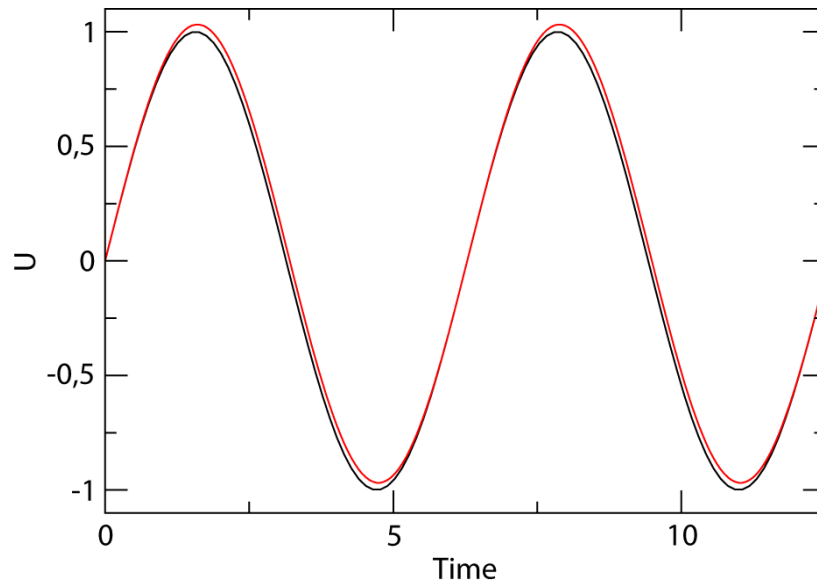


Figure 3.12: The analytical solution of equation 3.18 using  $A=1$  and  $u(t=0)=0$  (black) and the numerical solution using Eq. 3.20 with a time step  $\Delta t = \pi/50$  (red). The discrete solution  $U^n$  at times  $n\Delta t$  have been joined by straight lines. Note that a much more precise solution could be obtained by using  $U^{n+1} = U^n + \Delta t (A \cos((n+1/2)\Delta t))$  instead of Eq.3.20, i.e. by evaluating the cosine at time  $t=(n+1/2)\Delta t$  instead of time  $t=n\Delta t$ .

### 3. Modelling of the climate system

Knowing  $U^n$ , this equation can be easily solved for  $U^{n+1}$  (Fig. 3.12):

$$U^{n+1} = U^n + \Delta t (A \cos(n\Delta t)) \quad (3.20)$$

Such problems are called initial value problems because, when the initial value is specified, values for any time can be obtained by advancing or “marching” in time.

For a numerical method to be adequate, two fundamental properties must hold. First, the finite difference equation must be consistent with the partial differential equation. This means that as  $\Delta t \rightarrow 0, \Delta x \rightarrow 0$ , the finite differential equation coincides with the PDE. This is absolutely essential to ensure that the equation that has been solved numerically is a reasonable approximation to the mathematical model. It can be checked by replacing all the terms by the Taylor series expansion. For the left-hand side of Equation 3.19, this gives:

$$U^{n+1} = U^n + \frac{du}{dt} \Delta t + \frac{1}{2} \frac{d^2u}{dt^2} \Delta t^2 + \text{higher order terms} \quad (3.21)$$

and thus

$$\frac{U^{n+1} - U^n}{\Delta t} = \frac{du}{dt} + \frac{1}{2} \frac{d^2u}{dt^2} \Delta t + \text{higher order terms} \quad (3.22)$$

which effectively tends to  $du/dt$  as  $\Delta t$  tends to 0. This shows that the scheme is consistent.

Secondly, the solution of the finite difference must converge to the solution of the PDE as  $\Delta t \rightarrow 0, \Delta x \rightarrow 0$ . In our example, this means that

$$U(n\Delta t) \rightarrow u(t) \text{ when } \Delta t \rightarrow 0 \quad (3.23)$$

This convergence is related to the computational stability which states that a numerical scheme is computationally stable if the solution of the finite difference equation at a fixed time remains bounded as  $\Delta t \rightarrow 0$ . In more colourful language, it can be said that, in this case, the numerical model does not explode. Indeed, the Lax-Richtmyer theorem, which can be formally demonstrated for a well-posed initial-value problem, states that, for a consistent numerical method, stability and convergence are equivalent.

As a consequence, the practical methods used to test the convergence of a numerical scheme are based on an analysis of the stability of the scheme. In some cases, it is possible to explicitly demonstrate that the solution is bounded, a propriety generally conditioned by a criteria that governs  $\Delta t$  and  $\Delta x$ . A more general criterion to determine the largest time and spatial step allowed is the von Neumann method in which the stability of the finite difference equation is analysed by expressing the solution as an expansion of an appropriate set of basis functions, generally **Fourier series**.

Analyses such as the one performed in Equation 3.22 also allow a **truncation error** (i.e. the difference between the PDE and the finite difference equation) to be defined. This error is characterised by an order, corresponding to the power of the first term of the difference. For the scheme described above, the error is thus of the first order in time. In

addition to uncertainties relating to the physical model itself, and the definition of initial and boundary conditions, the representation of numbers by computers using a finite number of digits is also a source of error in the numerical solution.

### 3.4.2 Time and space discretisations using finite differences

Many options are available for discretising an equation and the choice depends on the properties required in the numerical scheme. In addition to consistency and the stability for reasonably long time steps, the scheme must be precise enough, but not too demanding of computer time. We have presented a first example of time discretisation in section 3.4.1, which is called the upward scheme (or forward Euler method):

$$\frac{U^{n+1} - U^n}{\Delta t} = F(U^n) \quad (3.24)$$

for a right-hand side represented in a general way as a function  $F(U^n)$ .

An alternative scheme is a centred difference (leapfrog scheme):

$$\frac{U^{n+1} - U^{n-1}}{2\Delta t} = F(U^n) \quad (3.25)$$

which has a second order truncation error and is thus in principle more precise than the first order upward scheme. However, this scheme allows the presence and growth of unphysical modes, and is thus generally stabilised by associating it with a time filter.

In implicit schemes, the right-hand side is not only expressed at time step  $n$  but also at time step  $n+1$ .  $F$  then becomes, in general, a function of both  $U^n$  and  $U^{n+1}$ . If  $F$  is only a function of  $U^{n+1}$ , the scheme is called fully implicit or backward. Implicit schemes require an equation or a system of equations to be solved to obtain  $U^{n+1}$ , equations that could be non-linear. Implicit schemes could thus be relatively expensive in computer time. On the other hand, implicit schemes allow longer time steps, which in some circumstances is a clear advantage.

The same variety of numerical schemes is available for space discretisation. Consider the diffusion equation:

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \quad (3.26)$$

where  $k$  is a constant. This can be discretised as:

$$\frac{U_j^{n+1} - U_j^n}{\Delta t} = k \frac{U_{j+1}^n - 2U_j^n + U_{j-1}^n}{\Delta x^2} \quad (3.27)$$

The index  $j$  refers to point number  $j$  of the spatial grid, which is at a distance  $(j-1)\Delta x$  from the first grid point if the grid spacing  $\Delta x$  is constant. It can easily be shown that this scheme is consistent and that the truncation error is first order in time and second order in space. It is stable if:

### 3. Modelling of the climate system

$$k \frac{\Delta t}{\Delta x^2} \leq \frac{1}{2} \quad (3.28)$$

illustrating the link usually present between  $\Delta t$  and  $\Delta x$ . Using this scheme, the solution at point  $j$  is updated at each time step  $n+1$  from the values computed at time step  $n$  for points  $j-1, j$  and  $j+1$  (Fig. 3.13).

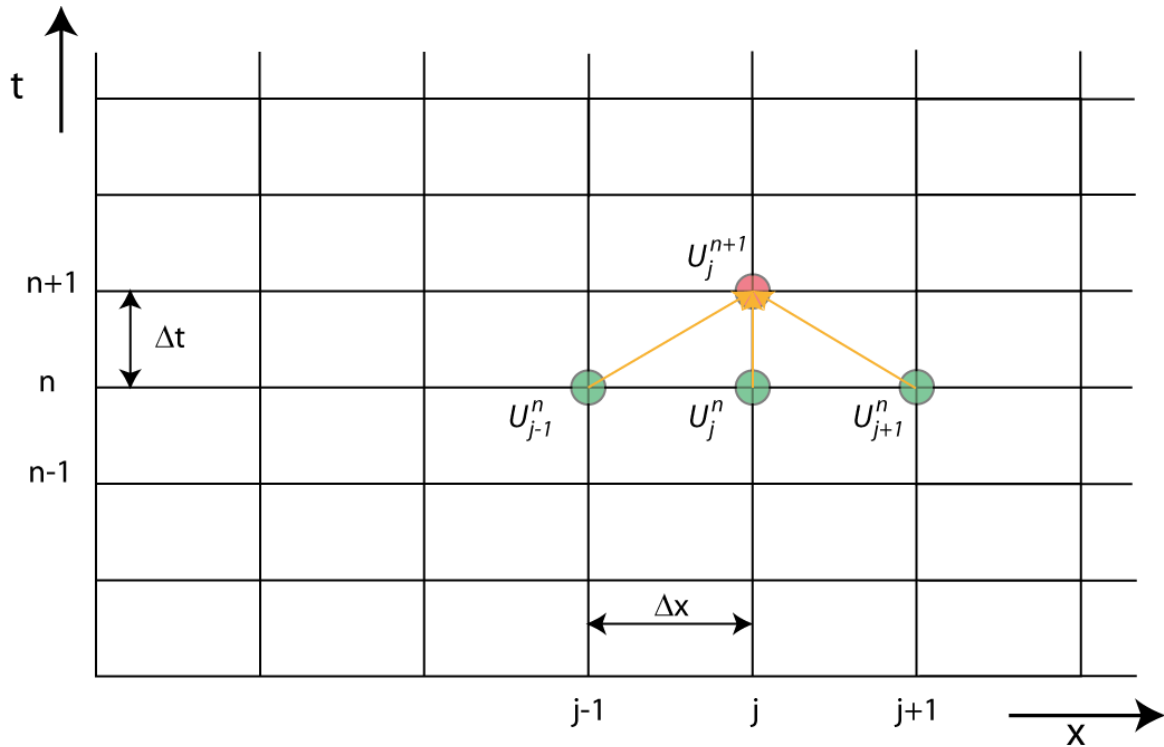


Figure 3.13: Schematic representation of the grid structure in space and time with one spatial dimension for the numerical scheme proposed in Eq. 3.27, showing that  $U_j^{n+1}$  depends on  $U_{j-1}^n, U_j^n, U_{j+1}^n$ .

In the two examples above (Eqs. 3.18 and 3.26), only one equation for one variable was solved. However, when all the components of the climate system are considered, equations for several variables must be solved simultaneously. For numerical reasons, those variables are not necessarily located at the same place on the grid, leading to what are called staggered grids. Arakawa and Lamb (1977) proposed a classification of these grids. Two popular ones are the B and the C grid. If we consider an elementary square of the grid for an ocean model, for instance, for the B grid, the temperature  $T$  (as well as the salinity, the pressure, and the density) are computed at the centres of the grid while the velocity components  $u$  and  $v$  are obtained at the corners of the grid elements (Figure 3.14). For the C grid, the velocities are computed on the side of the elements. Staggered grids are also widely used for vertical elements, with the velocity usually computed at the boundary between the layers, while the temperature is defined at the centre of the layers.

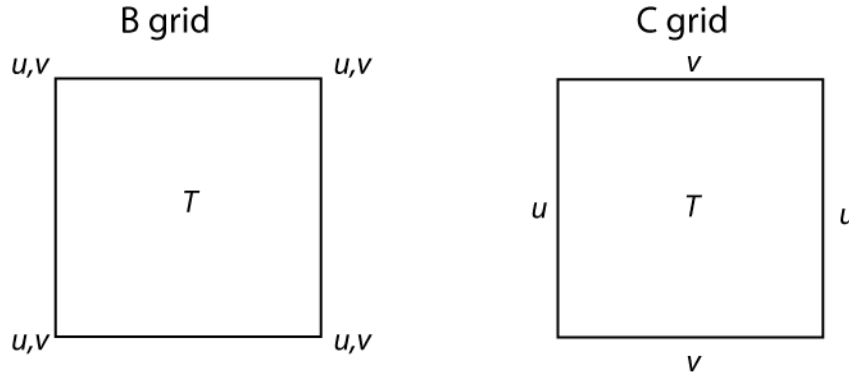


Figure 3.14: The location of some variables on the staggered grids B and C according to Arakawa and Lamb's (1977) classification.

### 3.4.3 Spectral representation and finite element methods

In addition to finite differences, several other methods can be used to discretise equations. One method is to integrate the basic equation of the system over a finite volume before the discretisation. This finite-volume method, has the advantage that it explicitly and easily ensures the conservation of some important properties. If particular hypotheses are made about the changes in the variables inside the volumes, numerical schemes similar to those described using finite-difference methods can be obtained.

In the Galerkin approach, the numerical solution is discretised in space as a sum of  $k$  basis functions  $\varphi_k(x)$  using coefficients  $A_k$  that depend on the time, leading to a spectral representation of the solution:

$$U(x, t) = \sum_{k=1}^K A_k(t) \varphi_k(x) \quad (3.29)$$

The goal of the resolution of the problem is then to determine the coefficients  $A_k$ . The larger the number of basis functions retained (i.e. the larger  $k$  is), the more precise is the spatial representation of the solution. The big advantage of this method is that the space derivation of  $U(x, t)$  can be analytically computed from  $d\varphi_k(x)/dx$  without any additional approximation. The choice of the basis function must be adequate. For a one-dimensional problem, **Fourier series** are a natural choice. For a problem with spherical geometry, in particular for global atmospheric models, spherical harmonics are used. They are the product of **Fourier series** in longitude and associated **Legendre polynomials** in latitude.

The spectral and grid-point (also referred to as physical space, see for instance Fig. 3.13) representations are complementary. The first provides an accurate computation of horizontal derivatives while the second forms a convenient framework to evaluate non-linear terms as well as the physics of the model (for example, the computation of the radiative transfer). When the two approaches are combined, some transfers of the variables from the physical to the spectral space are required in the transform method. There must be an exact correspondence between the number of basis function and the horizontal resolution of the grid to avoid numerical problems during this transformation.

The finite element approach is based on a similar approximation to Eq. 3.29 but instead of functions  $\varphi_k(x)$  that cover the whole domain as in the spectral method, local basis functions are used. For example,  $\varphi_k(x)$  can be a piecewise linear function equal to 1 at a grid point and 0 at all the other points.

## 3.5 Testing the validity of models

### 3.5.1 Verification, validation, testing

Despite very careful design, there is no guarantee that a computer model will be adequate for its intended use: some processes treated as negligible can turn out to be more important than initially thought; a **parameterisation** may not be valid in the particular conditions of interest or may be incompatible with other hypotheses employed; the selection of parameters can be far from optimal; and so on. As a consequence, climate models have to be tested to assess their quality and evaluate their performance. In this framework, it is always necessary to keep in mind the scientific objectives of the study (or studies) that will be conducted using a particular model. Although the principles remain the same, the tests performed with a model developed to analysing the development of the global carbon cycle over the last million years (see section 5.3.2) are clearly different from those for a model providing projections of future climate changes at the highest possible **resolution** (see Chapter 6).

A first step is to ensure that the numerical model solves the equations of the physical model adequately. This procedure, often referred to as **verification** (Fig. 3.15), only deals with the numerical resolution of the equations in the model, not with the agreement between the model and reality. It checks that no coding errors have been introduced into the program. The numerical methods used to solve the model equations must also be sufficiently accurate. Different methods are available to achieve this goal. A standard one is to compare the numerical solution with the analytical one for highly idealised test cases for which an exact solution is available. It is also possible to formally state that some parts of the code are correct, for instance, the one that solves large systems of  $n$  linear algebraic equations with  $n$  unknowns (which are often produced as part of the numerical resolution of the **partial differential equations** on the model **grid**).

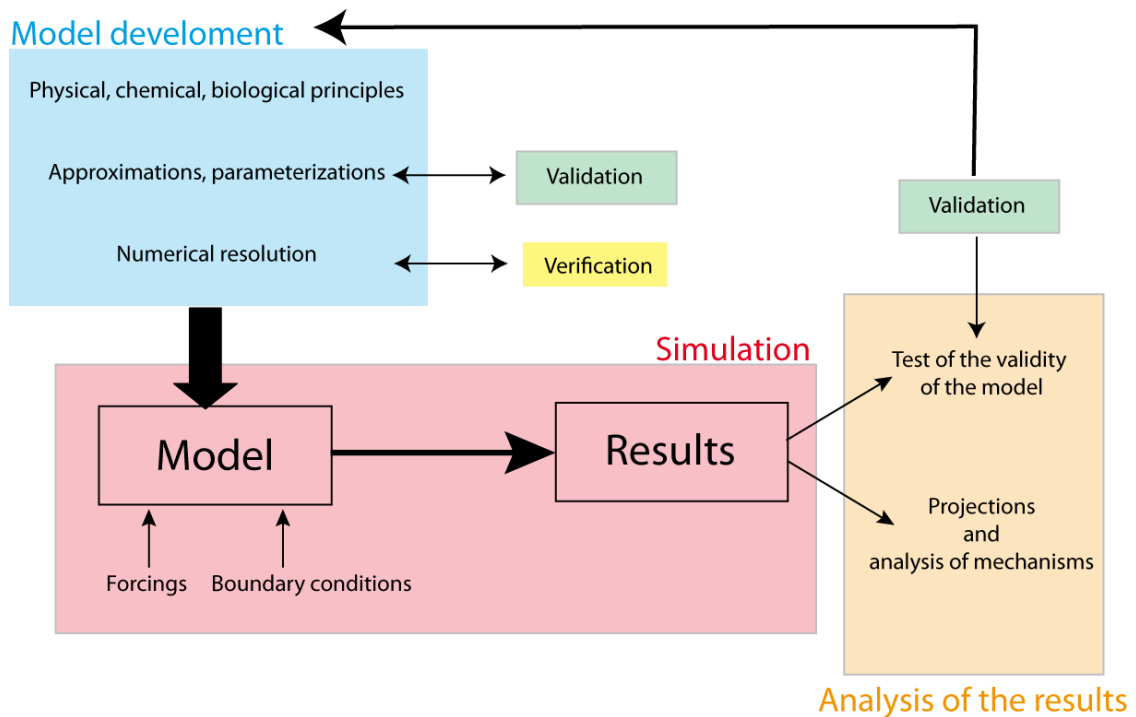


Figure 3.15: A modified version of Fig. 3.1 illustrating the verification and validation processes. An additional arrow from the analysis of the results towards model development has been added to show that **validation** is a continuous process.

The next step is the **validation** process, i.e. determining whether the model accurately represents reality. To do this, the model results have to be compared with observations obtained in the same conditions. In particular, this implies that the boundary conditions and forcings must be correctly specified to represent the observed situation. **Validation** must first be performed on the representation of individual physical processes, such as the formulation of the changes in the snow **albedo** in response to surface melting and temperature change. This is generally achieved for particular locations, during field campaigns specifically designed to study this process. They provide a much larger amount of very specific data than global data bases, allowing a detailed evaluation of the performance of the model on this topic. On a larger scale, the different components of the model (atmosphere, ocean, sea ice, etc, see section 3.3) have to be tested independently, ensuring that the boundary conditions at the interface with the other components are well defined. Finally, the results of the whole coupled model have to be compared with observations. All those steps are necessary because bad surprises are always possible after the different elements are coupled together, due to non-linear interactions between the components. Some problems with the model can also be masked by the formulation of the boundary conditions when components are run individually. However, having a coupled model providing reasonable results is not enough. In order to test whether the results occur for the correct reason, it is necessary to check that all the elements of the model are doing a good job, and that the satisfactory overall behaviour of the model is not due to several errors in its various elements cancelling each other out.

When discussing **verification** and **validation**, we must always recognize that both of them can only be partial for a climate model, except maybe in some trivial cases. The accuracy of the numerical solution can only be estimated for small elements of the code or in very special (simplified) conditions. Indeed, if it were possible to obtain a very accurate solution to compare with the numerical model results in all possible cases, there would be no point in developing a numerical model! The comparison of model results with observations is also limited to some particular conditions and completely validating a climate model in all the potential situations would require an infinite number of tests. *A climate model could thus never be considered as formally verified or validated.* A model is sometimes said to be validated if it has passed a reasonable number of tests. In such a case, the credibility of model projections performed with such a model could be very high. However, there is no way to formally guarantee that the results of the model will be correct even if the conditions are only slightly different from those used in the validation process, in particular for a very complex system like the climate. Furthermore, there is no agreement in climatology as to what a reasonable number of tests is.

The term “a validated model” and phrases like “the model has been validated” must therefore be avoided. Rather, the **verification** and **validation** should be considered as processes that never lead to a final, definitive product. The model should be continuously re-tested as new data or experimental results become available. The building of a model could then be viewed in the same way as a scientific theory. Hypotheses are formulated and a first version of the model developed. The results of the model are then compared to observations. If the model results are in good agreement with the data, the model could be said as to be confirmed for those conditions, so increasing its credibility. Nevertheless, this does not mean that the model is validated for all possible cases. If the model results do not compare well with observations, the model should be improved. This could lead to new hypotheses, to additional terms in the governing equations, or to the inclusion of new processes by new equations or new parameterisations.

Alternatively, a disagreement between the model and observations can be related to an inadequate selection of the values of some parameters that are not precisely known (for instance the exchange coefficients in Eqs. 2.33 and 2.34). Adjusting those parameters is part of the **calibration** of the model, also referred to as tuning. Model developers and



### 3. Modelling of the climate system

users also may decide that, if the model cannot reproduce the observations in some special cases, this indicates that it is not valid for such conditions, although it can still be used in other situations where the tests indicate better behaviour. For instance, we can imagine a climate model that cannot simulate the climate of Mars correctly without some modifications; however, this does not invalidate it for modelling conditions on Earth. On the other hand, if it works well for both Mars and Earth, this is a good test of its robustness.

The **calibration** of physical parameters is generally required and is perfectly justified as there is no a priori reason to select one particular value in the observed range of the parameters. It is also valid to calibrate the numerical parameters in order to obtain the most accurate numerical solution of the equations. However, care has to be taken to ensure that the **calibration** is not a way of artificially masking some deficiencies in the model. If this does occur, there is a high probability that the selected parameters will not provide satisfactory results for other conditions (e.g. the climate at the end of the 21<sup>st</sup> century). Performing many tests for widely different situations and for various elements of the model should limit the risk, but the number of observations is often too small to ensure that the problem has been completely avoided. An additional problem with the constant improvement of the model and of its calibration as soon as new data becomes available is the absence of independent data to really test the performance of the model. Ideally, some of the available information should be used for the model development and calibration, and some should be kept to assess its accuracy. Another good model practise is to choose or design models components for which the selection of one particular value of the parameters has only a small impact on model results, so reducing importance of the **calibration**.

In all the tests performed with the model, it is necessary to estimate the agreement between model results and observations. This is a complex and sometimes under-valued task. Indeed, the comparisons between the results of various models have shown that a single model is never the best for all the regions and variables analysed. Introducing a new parameterisation or changing the value of a parameter usually improves the results in some areas and worsens them in others. The agreement should then be related to the intended use of the model. This could be done more or less intuitively by visually comparing maps or plots describing both the model results and the observations. However, a much better solution is to define an appropriate metric. For a single field, such as the annual mean surface temperature  $T_s$ , a simple root mean square (RMS) error may be appropriate:

$$RMS = \sqrt{\frac{1}{n} \sum_{k=1}^n (T_{s,mod}^k - T_{s,obs}^k)^2} \quad (3.30)$$

where  $n$  is the number of grid points for which observations are available,  $T_{s,mod}^k$  is the model surface temperature at point  $k$  and  $T_{s,obs}^k$  is the observed surface temperature at point  $k$ . This estimate could be improved by taking into account the area of each grid point or by giving greater weight to the regions of most interest. If many variables have to be included in the metric, the RMS errors of different variables can be combined in various ways. The model data-comparison should also take into account the errors or uncertainties in both the model results and the observations. Errors in the observations can be directly related to the precision of the instruments or of the indirect method used to retrieve the climate signal (see for instance section 5.3.3). The uncertainties could also be due to the internal variability of the system (see sections 1.1 and 5.2), because

observations and model results covering a relatively short period are not necessarily representative of the mean behaviour of the system.

### 3.5.2 Evaluating model performance

Section 3.5.1 has stressed the absolute necessity of testing the quality of the model results fully. Here we will present some of the standard simulations that could be performed. However, we will not discuss the tests specifically designed to analyse the accuracy of numerical methods or of a particular parameterisation.

The first requirement is that the model is able to simulate reasonably well the climate in recent decades for which we have good estimates (Fig. 3.16). This implies performing simulations including the evolution of both natural and anthropogenic forcings (section 5.5.2) over that period. Numerical experiments with a constant forcing set at the mean for recent decades or for pre-industrial conditions (i.e. before any significant anthropogenic forcing, generally 1750 or 1850) can also be conducted in order to characterise a quasi-equilibrium behaviour of the model. In this case, it is necessary to take into account the difference between the pre-industrial conditions simulated by the model and present-day observations.

In these simulations, the long-term average of various variables, in all the model components, is compared with observations, generally interpolated on a common **grid**. Furthermore, the ability of the model to reproduce the observed climate variability on all time scales must be checked. This ranges from the relatively high frequency variations characteristic of temperature extremes such as heat waves to the most important modes of large-scale variability such as the El Niño-Southern Oscillation and the North Atlantic Oscillation (see section 5.2). Finally when driven by an adequate forcing, the climate models must be able to reproduce the observed warming of the Earth's surface over the last 150 years as well as the other recent climate changes.

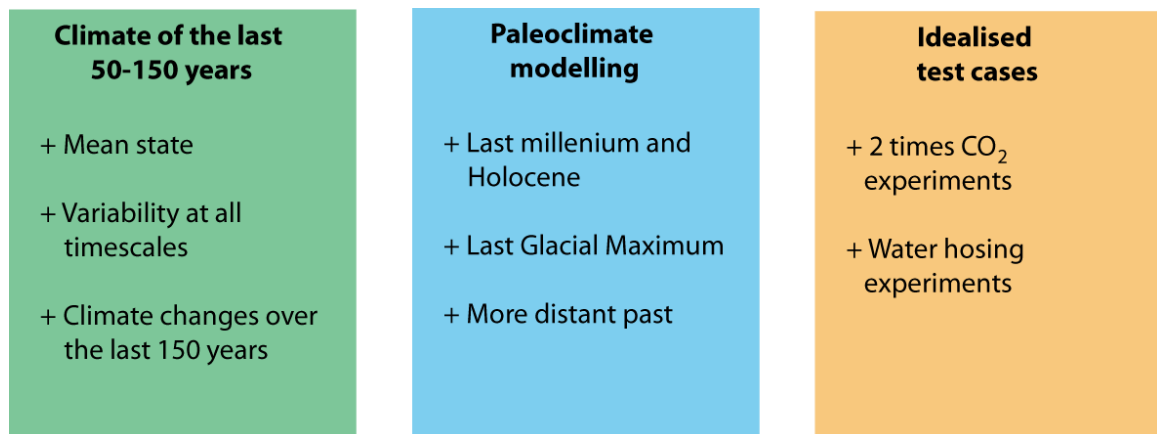


Figure 3.16: Classical tests performed on climate models.

Recent decades only cover a small fraction of the climate variations observed since the Earth's formation (see Chapter 5) and expected in the future (see Chapter 6). To test the ability of models to reproduce different climates, it is thus necessary to try to simulate some past conditions. The quality of the available observational data is (much) lower than that for recent decades and it may sometimes be hard to draw reliable conclusions from model/data comparisons for some past periods. Nevertheless, that is the only sample of possible states of the climate system that is available to us.

### 3. Modelling of the climate system

The second natural test period (Fig. 3.16) is the Holocene and the last millennium, for which we have a reasonably good knowledge of climate variations (see section 5.5). Although significant uncertainties are present, the forcing is much better known than for earlier periods. Furthermore, the boundary conditions (such as the topography or ocean bathymetry, see section 1.5) are similar to the present ones. The last glacial maximum is also a key period because it represents a relatively recent climate clearly different from that of recent decades (see section 5.4.2). In order to perform such simulations, unless the variables are computed interactively, it is necessary to specify variables such as the position and shape of the large ice sheets present on continents, the changes in the land/sea boundaries and ocean depth due to the lower sea levels, the modification in the vegetation cover and in the radiative properties of the atmosphere (in particular due to the higher dust content). All these elements can be sources of uncertainty for the climate simulation. Pre-Quaternary climates (see section 5.3) offer an even wider range of climate variations but the uncertainties on the forcing, boundary conditions and the climate itself are larger. As a consequence, these periods are not currently used as standard tests for climate models, although this will probably change in the near future as new information becomes available.

Finally some idealised experiments are performed with climate models (Fig. 3.16). These could not be directly compared to observations as they do not correspond to any past or present situation. However, they are very useful to document the model response to a simple, well-defined perturbation. Two standard thought-experiments are generally conducted. The first is a doubling of the atmospheric CO<sub>2</sub> concentration in the model, a test required to estimate the climate sensitivity of the model (see section 4.1.3). In the second (water hosing), large amounts of freshwater are poured into the North Atlantic to analyse the climate changes induced by the associated modification of the oceanic circulation (see section 5.5.1). These tests also allow the behaviour of different models to be compared in exactly the same experimental conditions. This leads to model inter-comparison exercises whose goals are a better understanding of the causes of the different responses of the various models. The results of such inter-comparisons are archived in data bases to ensure wide access. The results of other simulations (for example, mid-Holocene or last glacial maximum climates, climate change during recent decades, future climate change) are also stored in public or semi-public databases so that they can be analysed independently by large numbers of scientists.

### Cited references and further reading

- Arakawa A. and V. Lamb (1977). Computational design of the basic dynamical processes in the UCLA general circulation model. In "General circulation models of the atmosphere", methods in computational physics, 17, J. Chang, editor, Academic Press, 174-264.
- Bonan G. (2008). Ecological climatology, concepts and applications. Cambridge University Press, 550pp.
- Brovkin, V., A. Ganopolski, and Y. Svirezhev (1997). A continuous climate-vegetation classification for use in climate-biosphere studies. *Ecological Modelling* 101: 251-261.
- Budyko M.I. (1969). The effect of solar radiation variations on the climate of the Earth. *Tellus* 21: 611-619.
- Claussen M., L.A. Mysak, A. J. Weaver, M. Crucifix, T. Fichefet, M.F. Loutre, S.L. Weber, J. Alcamo, V.A. Alexeev, A. Berger, R. Calov, A. Ganopolski, H. Goosse, G. Lohman, F. Lunkeit, I. I. Mohkov, V. Petoukhov, P. Stone, and Z. Wang (2002). Earth

System Models of Intermediate Complexity : closing the gap in the spectrum of climate system models. *Climate Dynamics* 18: 579-586.

McGuffie K. and A. Henderson-Sellers (2005). *A climate modeling primer* (third edition). John Wiley & Sons, 280pp.

Oberkampf W.L., T.G. Trucano, and C. Hirsch (2002). *Verification, Validation, and Predictive Capability in computational engineering and physics*, Paper presented in the Foundation for verification and validation in the 21st century workshop, October 22-23, 2002, Johns Hopkins University/Applied Physics Laboratory Laurel, Maryland (Available at <http://sokocalo.engr.ucdavis.edu/~jeremic/UsefulReadings/Oberkampf-Trucano-Hirsch.pdf> )

Oreskes N., K. Shrader-Frechette, and K. Belitz (1994). Verification, validation, and confirmation of numerical models in the Earth sciences. *Science* 263: 641-646.

Sellers W.D. (1969). A global climatic model based on the energy balance of the Earth-atmosphere system. *J. Appl. Met.* 8: 392-400.

Trenberth K.E. (Editor), (1992). *Climate System Modeling*. Cambridge University Press. 787pp.

Washington W.M. and C.L. Parkinson (2005). *An introduction to three-dimensional climate modeling* (second edition). University Science Books, 353 pp.

## **Exercises**

Exercises are available on the textbook website (<http://www.climate.be/textbook>) and on iCampus for registered students.