ABSTRACT

The *Nuclear Physics and Reactor Theory* Handbook was developed to assist nuclear facility operating contractors in providing operators, maintenance personnel, and the technical staff with the necessary fundamentals training to ensure a basic understanding of nuclear physics and reactor theory. The handbook includes information on atomic and nuclear physics; neutron characteristics; reactor theory and nuclear parameters; and the theory of reactor operation. This information will provide personnel with a foundation for understanding the scientific principles that are associated with various DOE nuclear facility operations and maintenance.

Key Words: Training Material, Atomic Physics, The Chart of the Nuclides, Radioactivity, Radioactive Decay, Neutron Interaction, Fission, Reactor Theory, Neutron Characteristics, Neutron Life Cycle, Reactor Kinetics

OVERVIEW

The *Department of Energy Fundamentals Handbook* entitled *Nuclear Physics and Reactor Theory* was prepared as an information resource for personnel who are responsible for the operation of the Department's nuclear facilities. Almost all processes that take place in a nuclear facility involves the transfer of some type of energy. A basic understanding of nuclear physics and reactor theory is necessary for DOE nuclear facility operators, maintenance personnel, and the technical staff to safely operate and maintain the facility and facility support systems. The information in this handbook is presented to provide a foundation for applying engineering concepts to the job. This knowledge will help personnel understand the impact that their actions may have on the safe and reliable operation of facility components and systems.

The *Nuclear Physics and Reactor Theory* handbook consists of four modules that are contained in two volumes. The following is a brief description of the information presented in each module of the handbook.

Volume 1 of 2

Module 1 - Atomic and Nuclear Physics

Introduces concepts of atomic physics including the atomic nature of matter, the chart of the nuclides, radioactivity and radioactive decay, neutron interactions and fission, and the interaction of radiation with matter.

Module 2 - Reactor Theory (Nuclear Parameters)

Provides information on reactor theory and neutron characteristics. Includes topics such as neutron sources, neutron flux, neutron cross sections, reaction rates, neutron moderation, and prompt and delayed neutrons.

OVERVIEW (Cont.)

Volume 2 of 2

Module 3 - Reactor Theory (Nuclear Parameters)

Explains the nuclear parameters associated with reactor theory. Topics include the neutron life cycle, reactivity and reactivity coefficients, neutron poisons, and control rods.

Module 4 - Reactor Theory (Reactor Operations)

Introduces the reactor operations aspect of reactor theory. Topics include subcritical multiplication, reactor kinetics, and reactor operation.

The information contained in this handbook is not all-encompassing. An attempt to present the entire subject of nuclear physics and reactor theory would be impractical. However, the *Nuclear Physics and Reactor Theory* handbook presents enough information to provide the reader with the fundamental knowledge necessary to understand the advanced theoretical concepts presented in other subject areas, and to understand basic system and equipment operation.

Department of Energy
Fundamentals Handbook

NUCLEAR PHYSICS AND REACTOR THEORY
Module 1
Atomic and Nuclear Physics

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont.)

TABLE OF CONTENTS (Cont.)

LIST OF FIGURES

LIST OF TABLES

REFERENCES

- Foster, Arthur R. and Wright, Robert L. Jr., Basic Nuclear Engineering, 3rd Edition, Allyn and Bacon, Inc., 1977.
- Jacobs, A.M., Kline, D.E., and Remick, F.J., Basic Principles of Nuclear Science and Reactors, Van Nostrand Company, Inc., 1960.
- Kaplan, Irving, Nuclear Physics, 2nd Edition, Addison-Wesley Company, 1962.
- Knief, Ronald Allen, Nuclear Energy Technology: Theory and Practice of Commercial Nuclear Power, McGraw-Hill, 1981.
- Lamarsh, John R., Introduction to Nuclear Engineering, Addison-Wesley Company, 1977.
- Lamarsh, John R., Introduction to Nuclear Reactor Theory, Addison-Wesley Company, 1972.
- General Electric Company, Nuclides and Isotopes: Chart of the Nuclides, 14th Edition, General Electric Company, 1989.
- Academic Program for Nuclear Power Plant Personnel, Volume III, Columbia, MD, General Physics Corporation, Library of Congress Card #A 326517, 1982.
- Glasstone, Samuel, Sourcebook on Atomic Energy, Robert F. Krieger Publishing Company, Inc., 1979.
- Glasstone, Samuel and Sesonske, Alexander, Nuclear Reactor Engineering, 3rd Edition, Van Nostrand Reinhold Company, 1981.

TERMINAL OBJECTIVE

1.0 Given sufficient information, **DESCRIBE** atoms, including components, structure, and nomenclature.

ENABLING OBJECTIVES

1.1 **STATE** the characteristics of the following atomic particles, including mass, charge, and location within the atom:

a. Proton
b. Neutron
c. Electron

1.2 **DESCRIBE** the Bohr model of an atom.

1.3 **DEFINE** the following terms:

a. Nuclide
b. Isotope
c. Atomic number
d. Mass number

1.4 Given the standard $^{A}_{Z}X$ notation for a particular nuclide, **DETERMINE** the following:

a. Number of protons
b. Number of neutrons
c. Number of electrons

1.5 **DESCRIBE** the three forces that act on particles within the nucleus and affect the stability of the nucleus.

1.6 **DEFINE** the following terms:

a. Enriched uranium
b. Depleted uranium

1.7 **DEFINE** the following terms:

a. Mass defect
b. Binding energy

1.8 Given the atomic mass for a nuclide and the atomic masses of a neutron, proton, and electron, **CALCULATE** the mass defect and binding energy of the nuclide.

TERMINAL OBJECTIVE

2.0 Given necessary references, **DESCRIBE** the various modes of radioactive decay.

ENABLING OBJECTIVES

2.1 **DESCRIBE** the following processes:

a. Alpha decay
b. Beta-minus decay
c. Beta-plus decay
d. Electron capture
e. Internal conversions
f. Isomeric transitions

2.2 Given a Chart of the Nuclides, **WRITE** the radioactive decay chain for a nuclide.

2.3 **EXPLAIN** why one or more gamma rays typically accompany particle emission.

2.4 Given the stability curve on the Chart of the Nuclides, **DETERMINE** the type of radioactive decay that the nuclides in each region of the chart will typically undergo.

2.5 **DEFINE** the following terms:

a. Radioactivity
b. Curie
c. Becquerel
d. Radioactive decay constant
e. Radioactive half-life

2.6 Given the number of atoms and either the half-life or decay constant of a nuclide, **CALCULATE** the activity.

2.7 Given the initial activity and the decay constant of a nuclide, **CALCULATE** the activity at any later time.

2.8 **CONVERT** between the half-life and decay constant for a nuclide.

2.9 Given the Chart of the Nuclides and the original activity, **PLOT** the radioactive decay curve for a nuclide on either linear or semi-log coordinates.

2.10 **DEFINE** the following terms:

a. Radioactive equilibrium
b. Transient radioactive equilibrium

TERMINAL OBJECTIVE

3.0 Without references, **DESCRIBE** the different nuclear interactions initiated by neutrons.

ENABLING OBJECTIVES

3.1 **DESCRIBE** the following scattering interactions between a neutron and a nucleus:

a. Elastic scattering
b. Inelastic scattering

3.2 **STATE** the conservation laws that apply to an elastic collision between a neutron and a nucleus.

3.3 **DESCRIBE** the following reactions where a neutron is absorbed in a nucleus:

a. Radiative capture
b. Particle ejection

TERMINAL OBJECTIVE

4.0 Without references, **DESCRIBE** the fission process.

ENABLING OBJECTIVES

4.1 **EXPLAIN** the fission process using the liquid drop model of a nucleus.

4.2 **DEFINE** the following terms:

a. Excitation energy
b. Critical energy

4.3 **DEFINE** the following terms:

a. Fissile material
b. Fissionable material
c. Fertile material

4.4 **DESCRIBE** the processes of transmutation, conversion, and breeding.

4.5 **DESCRIBE** the curve of Binding Energy per Nucleon versus mass number and give a qualitative description of the reasons for its shape.

4.6 **EXPLAIN** why only the heaviest nuclei are easily fissioned.

4.7 **EXPLAIN** why uranium-235 fissions with thermal neutrons and uranium-238 fissions only with fast neutrons.

4.8 **CHARACTERIZE** the fission products in terms of mass groupings and radioactivity.

4.9 Given the nuclides involved and their masses, **CALCULATE** the energy released from fission.

4.10 Given the curve of Binding Energy per Nucleon versus mass number, **CALCULATE** the energy released from fission.

TERMINAL OBJECTIVE

5.0 Without references, **DESCRIBE** how the various types of radiation interact with matter.

ENABLING OBJECTIVES

5.1 **DESCRIBE** interactions of the following with matter:

a. Alpha particle
b. Beta particle
c. Positron
d. Neutron

5.2 **DESCRIBE** the following ways that gamma radiation interacts with matter:

a. Compton scattering
b. Photoelectric effect
c. Pair production

Intentionally Left Blank

ATOMIC NATURE OF MATTER

All matter is composed of atoms. The atom is the smallest amount of matter that retains the properties of an element. Atoms themselves are composed of smaller particles, but these smaller particles no longer have the same properties as the overall element.

EO 1.1 STATE the characteristics of the following atomic particles, including mass, charge, and location within the atom:

a. Proton
b. Neutron
c. Electron

EO 1.2 DESCRIBE the Bohr model of an atom.

EO 1.3 DEFINE the following terms:

a. Nuclide
b. Isotope
c. Atomic number
d. Mass number

EO 1.4 Given the standard $^{A}_{Z}X$ notation for a particular nuclide, DETERMINE the following:

a. Number of protons
b. Number of neutrons
c. Number of electrons

EO 1.5 DESCRIBE the three forces that act on particles within the nucleus and affect the stability of the nucleus.

Structure of Matter

Early Greek philosophers speculated that the earth was made up of different combinations of basic substances, or elements. They considered these basic elements to be earth, air, water, and fire. Modern science shows that the early Greeks held the correct concept that matter consists of a combination of basic elements, but they incorrectly identified the elements.

In 1661 the English chemist Robert Boyle published the modern criterion for an element. He defined an element to be a basic substance that cannot be broken down into any simpler substance after it is isolated from a compound, but can be combined with other elements to form compounds. To date, 105 different elements have been confirmed to exist, and researchers claim to have discovered three additional elements. Of the 105 confirmed elements, 90 exist in nature and 15 are man-made.

Another basic concept of matter that the Greeks debated was whether matter was continuous or discrete. That is, whether matter could be continuously divided and subdivided into ever smaller particles or whether eventually an indivisible particle would be encountered. Democritus in about 450 B.C. argued that substances were ultimately composed of small, indivisible particles that he labeled atoms. He further suggested that different substances were composed of different atoms or combinations of atoms, and that one substance could be converted into another by rearranging the atoms. It was impossible to conclusively prove or disprove this proposal for more than 2000 years.

The modern proof for the atomic nature of matter was first proposed by the English chemist John Dalton in 1803. Dalton stated that each chemical element possesses a particular kind of atom, and any quantity of the element is made up of identical atoms of this kind. What distinguishes one element from another element is the kind of atom of which it consists, and the basic physical difference between kinds of atoms is their weight.

Subatomic Particles

For almost 100 years after Dalton established the atomic nature of atoms, it was considered impossible to divide the atom into even smaller parts. All of the results of chemical experiments during this time indicated that the atom was indivisible. Eventually, experimentation into electricity and radioactivity indicated that particles of matter smaller than the atom did indeed exist. In 1906, J. J. Thompson won the Nobel Prize in physics for establishing the existence of electrons. *Electrons* are negatively-charged particles that have 1/1835 the mass of the hydrogen atom. Soon after the discovery of electrons, protons were discovered. *Protons* are relatively large particles that have almost the same mass as a hydrogen atom and a positive charge equal in magnitude (but opposite in sign) to that of the electron. The third subatomic particle to be discovered, the neutron, was not found until 1932. The *neutron* has almost the same mass as the proton, but it is electrically neutral.

Bohr Model of the Atom

The British physicist Ernest Rutherford postulated that the positive charge in an atom is concentrated in a small region called a nucleus at the center of the atom with electrons existing in orbits around it. Niels Bohr, coupling Rutherford's postulation with the quantum theory introduced by Max Planck, proposed that the atom consists of a dense nucleus of protons surrounded by electrons traveling in discrete orbits at fixed distances from the nucleus. An electron in one of these orbits or shells has a specific or discrete quantity of energy (quantum). When an electron moves from one allowed orbit to another allowed orbit, the energy difference between the two states is emitted or absorbed in the form of a single quantum of radiant energy called a photon. Figure 1 is Bohr's model of the hydrogen atom showing an electron as having just dropped from the third shell to the first shell with the emission of a photon that has an energy = hν. (h = Planck's constant = 6.63 x 10^{-34} J-s and ν = frequency of the photon.) Bohr's theory was the first to successfully account for the discrete energy levels of this radiation as measured in the laboratory. Although Bohr's atomic model is designed specifically to explain the hydrogen atom, his theories apply generally to the structure of all atoms. Additional information on electron shell theory can be found in the Chemistry Fundamentals Handbook.

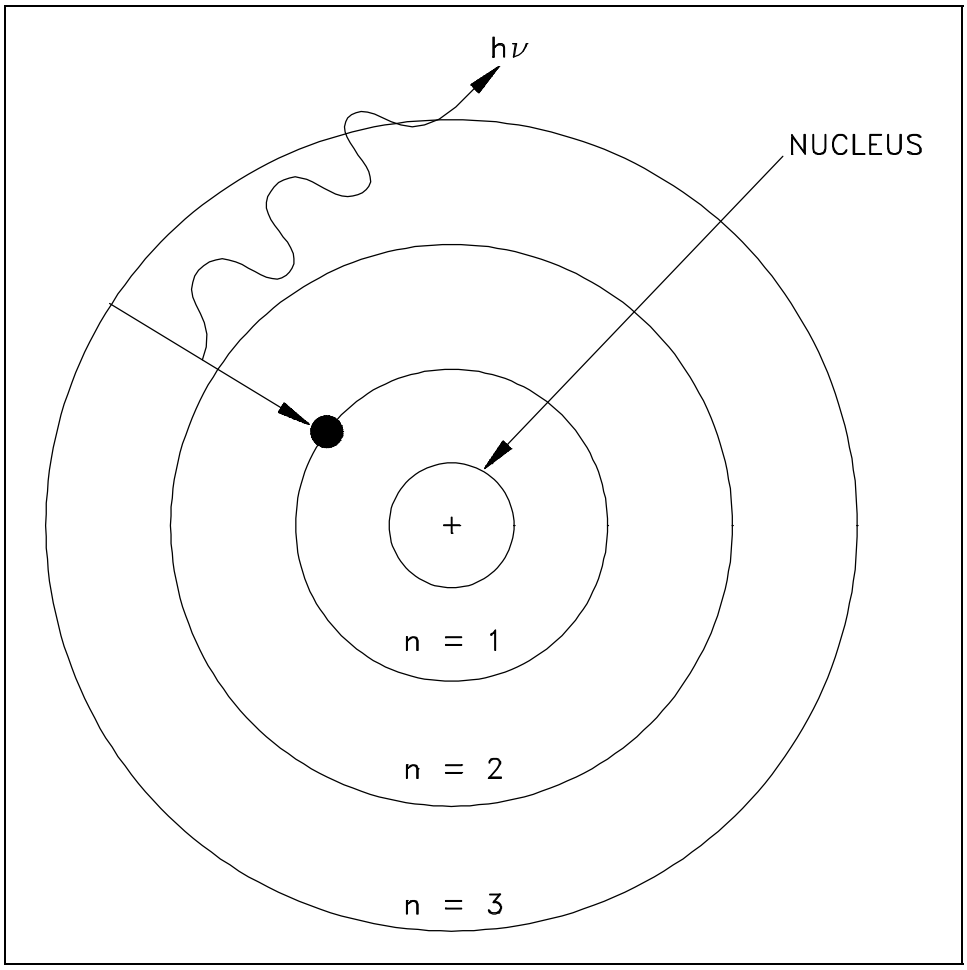


Figure 1 Bohr's Model of the Hydrogen Atom

Properties of the three subatomic particles are listed in Table 1.

TABLE 1
Properties of Subatomic Particles

Particle	Location	Charge	Mass
Neutron	Nucleus	none	1.008665 amu
Proton	Nucleus	+1	1.007277 amu
Electron	Shells around nucleus	-1	0.0005486 amu

Measuring Units on the Atomic Scale

The size and mass of atoms are so small that the use of normal measuring units, while possible, is often inconvenient. Units of measure have been defined for mass and energy on the atomic scale to make measurements more convenient to express. The unit of measure for mass is the atomic mass unit (amu). One atomic mass unit is equal to 1.66×10^{-24} grams. The reason for this particular value for the atomic mass unit will be discussed in a later chapter. Note from Table 1 that the mass of a neutron and a proton are both about 1 amu. The unit for energy is the electron volt (eV). The electron volt is the amount of energy acquired by a single electron when it falls through a potential difference of one volt. One electron volt is equivalent to 1.602×10^{-19} joules or 1.18×10^{-19} foot-pounds.

Nuclides

The total number of protons in the nucleus of an atom is called the *atomic number* of the atom and is given the symbol Z. The number of electrons in an electrically-neutral atom is the same as the number of protons in the nucleus. The number of neutrons in a nucleus is known as the neutron number and is given the symbol N. The *mass number* of the nucleus is the total number of nucleons, that is, protons and neutrons in the nucleus. The mass number is given the symbol A and can be found by the equation $Z + N = A$.

Each of the chemical elements has a unique atomic number because the atoms of different elements contain a different number of protons. The atomic number of an atom identifies the particular element.

Each type of atom that contains a unique combination of protons and neutrons is called a *nuclide*. Not all combinations of numbers of protons and neutrons are possible, but about 2500 specific nuclides with unique combinations of neutrons and protons have been identified. Each nuclide is denoted by the chemical symbol of the element with the atomic number written as a subscript and the mass number written as a superscript, as shown in Figure 2. Because each element has a unique name, chemical symbol, and atomic number, only one of the three is necessary to identify the element. For this reason nuclides can also be identified by either the chemical name or the chemical symbol followed by the mass number (for example, U-235 or uranium-235). Another common format is to use the abbreviation of the chemical element with the mass number superscripted (for example, ^{235}U). In this handbook the format used in the text will usually be the element's name followed by the mass number. In equations and tables, the format in Figure 2 will usually be used.

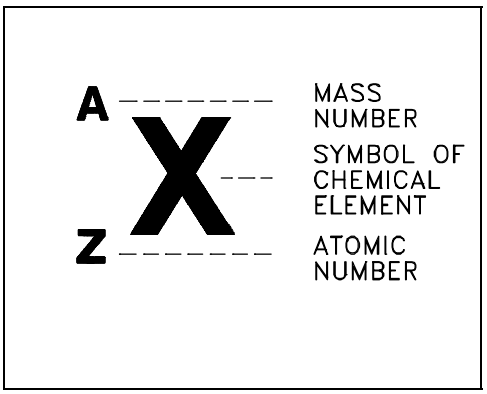


Figure 2 Nomenclature for Identifying Nuclides

Example:

State the name of the element and the number of protons, electrons, and neutrons in the nuclides listed below.

$^{1}_{1}H$

$^{10}_{5}B$

$^{14}_{7}N$

$^{114}_{48}Cd$

$^{239}_{94}Pu$

Solution:

The name of the element can be found from the Periodic Table (refer to Chemistry Fundamentals Handbook) or the Chart of the Nuclides (to be discussed later). The number of protons and electrons are equal to Z. The number of neutrons is equal to Z - A.

Nuclide	Element	Protons	Electrons	Neutrons
$^{1}_{1}H$	hydrogen	1	1	0
$^{10}_{5}B$	boron	5	5	5
$^{14}_{7}N$	nitrogen	7	7	7
$^{114}_{48}Cd$	cadmium	48	48	66
$^{239}_{94}Pu$	plutonium	94	94	145

Isotopes

Isotopes are nuclides that have the same atomic number and are therefore the same element, but differ in the number of neutrons. Most elements have a few stable isotopes and several unstable, radioactive isotopes. For example, oxygen has three stable isotopes that can be found in nature (oxygen-16, oxygen-17, and oxygen-18) and eight radioactive isotopes. Another example is hydrogen, which has two stable isotopes (hydrogen-1 and hydrogen-2) and a single radioactive isotope (hydrogen-3).

The isotopes of hydrogen are unique in that they are each commonly referred to by a unique name instead of the common chemical element name. Hydrogen-1 is almost always referred to as hydrogen, but the term protium is infrequently used also. Hydrogen-2 is commonly called deuterium and symbolized $^{2}_{1}D$. Hydrogen-3 is commonly called tritium and symbolized $^{3}_{1}T$. This text will normally use the symbology $^{2}_{1}H$ and $^{3}_{1}H$ for deuterium and tritium, respectively.

Atomic and Nuclear Radii

The size of an atom is difficult to define exactly due to the fact that the electron cloud, formed by the electrons moving in their various orbitals, does not have a distinct outer edge. A reasonable measure of atomic size is given by the average distance of the outermost electron from the nucleus. Except for a few of the lightest atoms, the average atomic radii are approximately the same for all atoms, about 2×10^{-8} cm.

Like the atom the nucleus does not have a sharp outer boundary. Experiments have shown that the nucleus is shaped like a sphere with a radius that depends on the atomic mass number of the atom. The relationship between the atomic mass number and the radius of the nucleus is shown in the following equation.

$$r = (1.25 \times 10^{-13} \text{ cm})\, A^{1/3}$$

where:

r = radius of the nucleus (cm)
A = atomic mass number (dimensionless)

The values of the nuclear radii for some light, intermediate, and heavy nuclides are shown in Table 2.

TABLE 2
Calculated Values for Nuclear Radii

Nuclide	Radius of Nucleus
$^{1}_{1}H$	1.25×10^{-13} cm
$^{10}_{5}B$	2.69×10^{-13} cm
$^{56}_{26}Fe$	4.78×10^{-13} cm
$^{178}_{72}Hf$	7.01×10^{-13} cm
$^{238}_{92}U$	7.74×10^{-13} cm
$^{252}_{98}Cf$	7.89×10^{-13} cm

From the table, it is clear that the radius of a typical atom (e.g. 2×10^{-8} cm) is more than 25,000 times larger than the radius of the largest nucleus.

Nuclear Forces

In the Bohr model of the atom, the nucleus consists of positively-charged protons and electrically-neutral neutrons. Since both protons and neutrons exist in the nucleus, they are both referred to as nucleons. One problem that the Bohr model of the atom presented was accounting for an attractive force to overcome the repulsive force between protons.

Two forces present in the nucleus are (1) electrostatic forces between charged particles and (2) gravitational forces between any two objects that have mass. It is possible to calculate the magnitude of the gravitational force and electrostatic force based upon principles from classical physics.

Newton stated that the *gravitational force* between two bodies is directly proportional to the masses of the two bodies and inversely proportional to the square of the distance between the bodies. This relationship is shown in the equation below.

$$F_g = \frac{G\ m_1\ m_2}{r^2}$$

where:

F_g = gravitational force (newtons)
m_1 = mass of first body (kilograms)
m_2 = mass of second body (kilograms)
G = gravitational constant (6.67×10^{-11} N-m^2/kg^2)
r = distance between particles (meters)

The equation illustrates that the larger the masses of the objects or the smaller the distance between the objects, the greater the gravitational force. So even though the masses of nucleons are very small, the fact that the distance between nucleons is extremely short may make the gravitational force significant. It is necessary to calculate the value for the gravitational force and compare it to the value for other forces to determine the significance of the gravitational force in the nucleus. The gravitational force between two protons that are separated by a distance of 10^{-20} meters is about 10^{-24} newtons.

Coulomb's Law can be used to calculate the force between two protons. The *electrostatic force* is directly proportional to the electrical charges of the two particles and inversely proportional to the square of the distance between the particles. Coulomb's Law is stated as the following equation.

$$F_e = \frac{K\ Q_1\ Q_2}{r^2}$$

where:

F_e = electrostatic force (newtons)
K = electrostatic constant (9.0×10^9 N-m^2/C^2)
Q_1 = charge of first particle (coulombs)
Q_2 = charge of second particle (coulombs)
r = distance between particles (meters)

Using this equation, the electrostatic force between two protons that are separated by a distance of 10^{-20} meters is about 10^{12} newtons. Comparing this result with the calculation of the gravitational force (10^{-24} newtons) shows that the gravitational force is so small that it can be neglected.

If only the electrostatic and gravitational forces existed in the nucleus, then it would be impossible to have stable nuclei composed of protons and neutrons. The gravitational forces are much too small to hold the nucleons together compared to the electrostatic forces repelling the protons. Since stable atoms of neutrons and protons do exist, there must be another attractive force acting within the nucleus. This force is called the nuclear force.

The *nuclear force* is a strong attractive force that is independent of charge. It acts equally only between pairs of neutrons, pairs of protons, or a neutron and a proton. The nuclear force has a very short range; it acts only over distances approximately equal to the diameter of the nucleus (10^{-13} cm). The attractive nuclear force between all nucleons drops off with distance much faster than the repulsive electrostatic force between protons.

TABLE 3
Forces Acting in the Nucleus

Force	Interaction	Range
Gravitational	Very weak attractive force between all nucleons	Relatively long
Electrostatic	Strong repulsive force between like charged particles (protons)	Relatively long
Nuclear Force	Strong attractive force between all nucleons	Extremely short

In stable atoms, the attractive and repulsive forces in the nucleus balance. If the forces do not balance, the atom cannot be stable, and the nucleus will emit radiation in an attempt to achieve a more stable configuration.

Summary

The important information in this chapter is summarized on the following page.

Atomic Nature of Matter Summary

- Atoms consist of three basic subatomic particles. These particles are the proton, the neutron, and the electron.
- Protons are particles that have a positive charge, have about the same mass as a hydrogen atom, and exist in the nucleus of an atom.
- Neutrons are particles that have no electrical charge, have about the same mass as a hydrogen atom, and exist in the nucleus of an atom.
- Electrons are particles that have a negative charge, have a mass about eighteen hundred times smaller than the mass of a hydrogen atom, and exist in orbital shells around the nucleus of an atom.
- The Bohr model of the atom consists of a dense nucleus of protons and neutrons (nucleons) surrounded by electrons traveling in discrete orbits at fixed distances from the nucleus.
- Nuclides are atoms that contain a particular number of protons and neutrons.
- Isotopes are nuclides that have the same atomic number and are therefore the same element, but differ in the number of neutrons.
- The atomic number of an atom is the number of protons in the nucleus.
- The mass number of an atom is the total number of nucleons (protons and neutrons) in the nucleus.
- The notation ${}^{A}_{Z}X$ is used to identify a specific nuclide. "Z" represents the atomic number, which is equal to the number of protons. "A" represents the mass number, which is equal to the number of nucleons. "X" represents the chemical symbol of the element.

  Number of protons = Z
  Number of electrons = Z
  Number of neutrons = A - Z

- The stability of a nucleus is determined by the different forces interacting within it. The electrostatic force is a relatively long-range, strong, repulsive force that acts between the positively charged protons. The nuclear force is a relatively short-range attractive force between all nucleons. The gravitational force the long range, relatively weak attraction between masses, is negligible compared to the other forces.

CHART OF THE NUCLIDES

The Chart of the Nuclides, like the Periodic Table, is a convenient format for presenting a large amount of scientific information in an organized manner.

EO 1.6 DEFINE the following terms:

a. Enriched uranium
b. Depleted uranium

Chart of the Nuclides

A tabulated chart called the *Chart of the Nuclides* lists the stable and unstable nuclides in addition to pertinent information about each one. Figure 3 shows a small portion of a typical chart. This chart plots a box for each individual nuclide, with the number of protons (Z) on the vertical axis and the number of neutrons (N = A - Z) on the horizontal axis.

The completely gray squares indicate stable isotopes. Those in white squares are *artificially radioactive*, meaning that they are produced by artificial techniques and do not occur naturally. By consulting a complete chart, other types of isotopes can be found, such as naturally occurring radioactive types (but none are found in the region of the chart that is illustrated in Figure 3).

Located in the box on the far left of each horizontal row is general information about the element. The box contains the chemical symbol of the element in addition to the average atomic weight of the naturally occurring substance and the average thermal neutron absorption cross section, which will be discussed in a later module. The known isotopes (elements with the same atomic number Z but different mass number A) of each element are listed to the right.

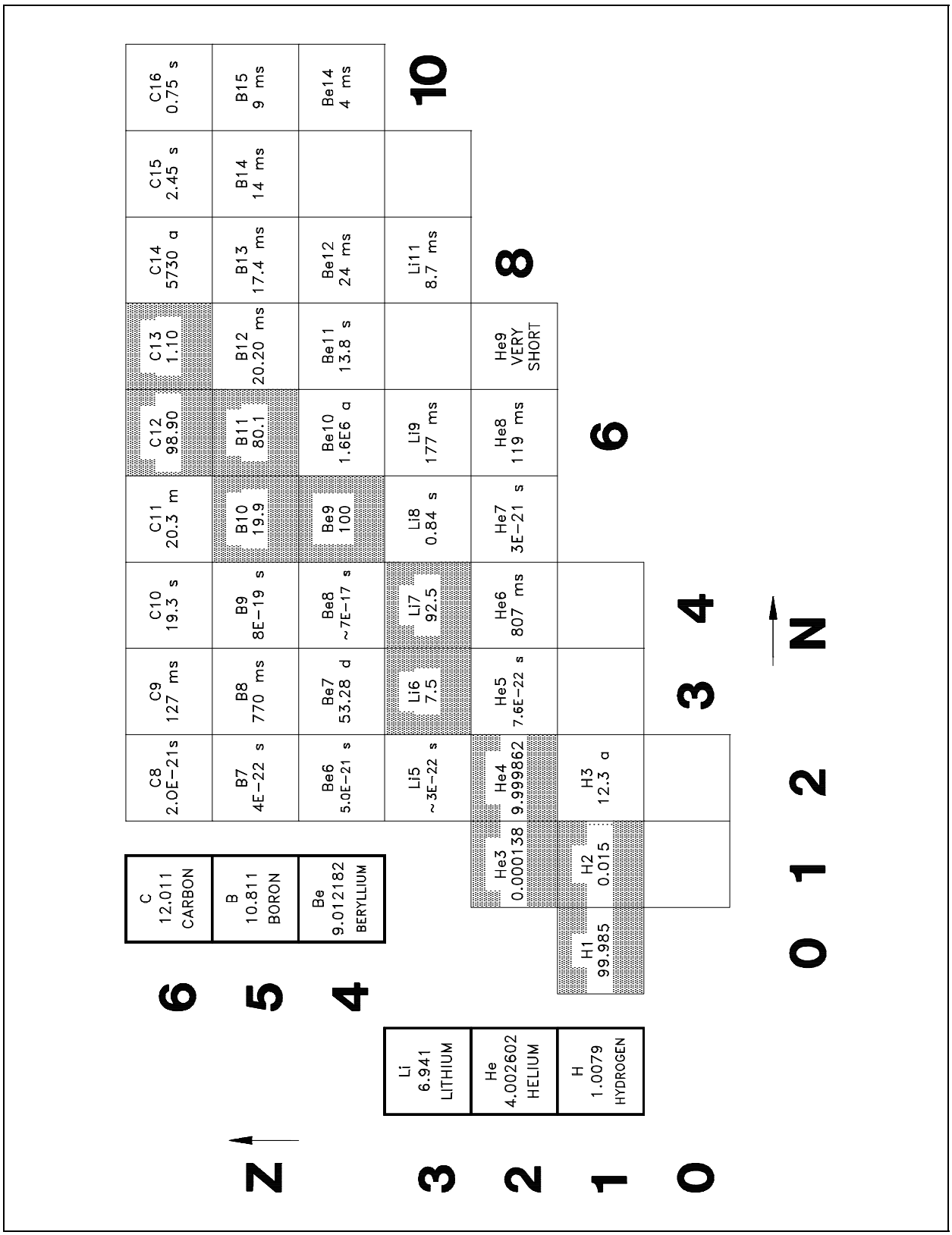


Figure 3 Nuclide Chart for Atomic Numbers 1 to 6

Information for Stable Nuclides

For the stable isotopes, in addition to the symbol and the atomic mass number, the number percentage of each isotope in the naturally occurring element is listed, as well as the thermal neutron activation cross section and the mass in atomic mass units (amu). A typical block for a stable nuclide from the Chart of the Nuclides is shown in Figure 4.

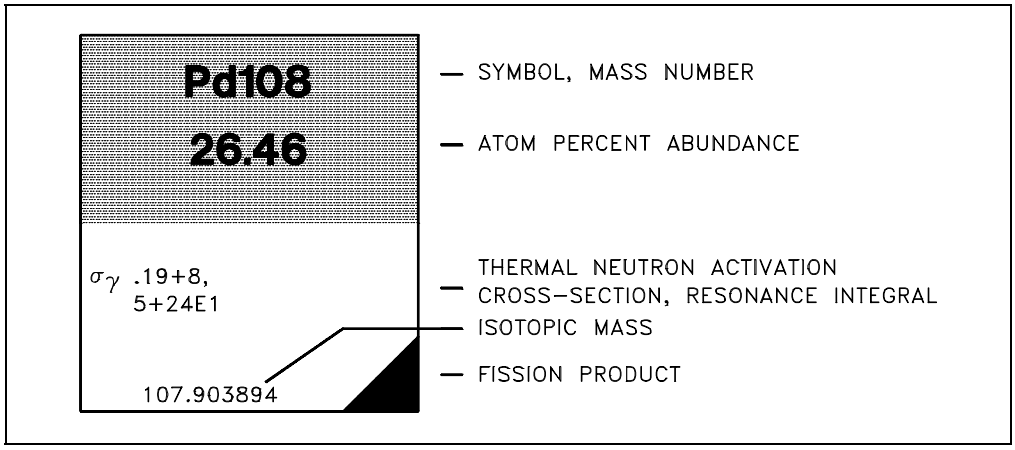


Figure 4 Stable Nuclides

Information for Unstable Nuclides

For unstable isotopes the additional information includes the half life, the mode of decay (for example, β^-, α), the total disintegration energy in MeV (million electron volts), and the mass in amu when available. A typical block for an unstable nuclide from the Chart of the Nuclides is shown in Figure 5.

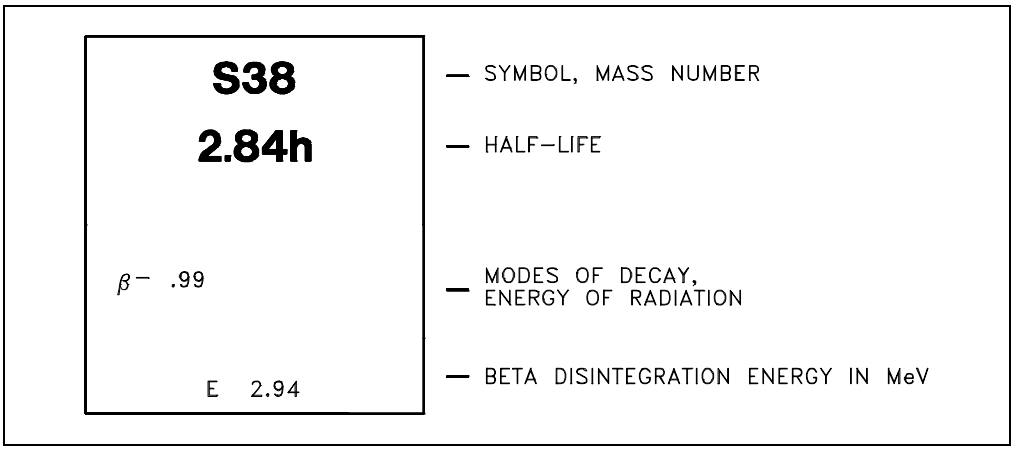


Figure 5 Unstable Nuclides

Neutron - Proton Ratios

Figure 6 shows the distribution of the stable nuclides plotted on the same axes as the Chart of the Nuclides. As the mass numbers become higher, the ratio of neutrons to protons in the nucleus becomes larger. For helium-4 (2 protons and 2 neutrons) and oxygen-16 (8 protons and 8 neutrons) this ratio is unity. For indium-115 (49 protons and 66 neutrons) the ratio of neutrons to protons has increased to 1.35, and for uranium-238 (92 protons and 146 neutrons) the neutron-to-proton ratio is 1.59.

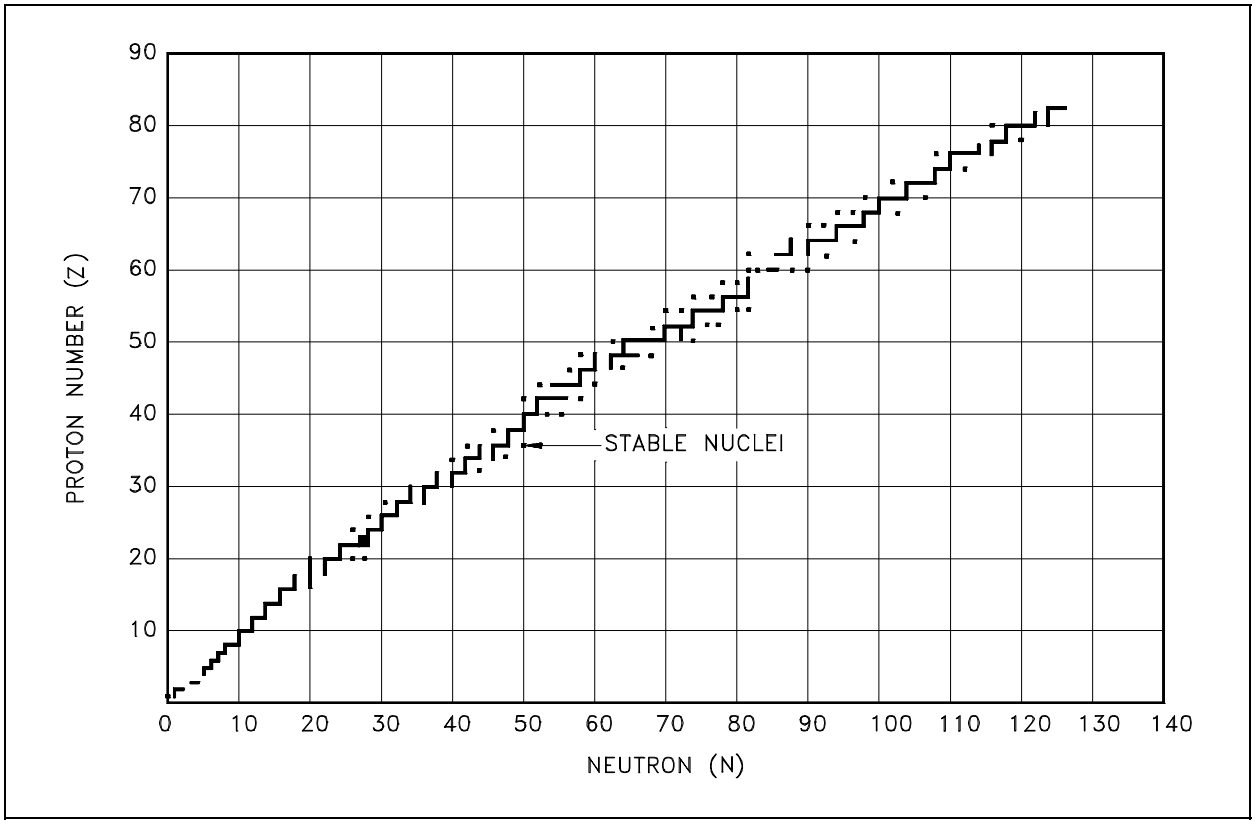


Figure 6 Neutron - Proton Plot of the Stable Nuclides

If a heavy nucleus were to split into two fragments, each fragment would form a nucleus that would have approximately the same neutron-to-proton ratio as the heavy nucleus. This high neutron-to-proton ratio places the fragments below and to the right of the stability curve displayed by Figure 6. The instability caused by this excess of neutrons is generally rectified by successive beta emissions, each of which converts a neutron to a proton and moves the nucleus toward a more stable neutron-to-proton ratio.

Natural Abundance of Isotopes

The relative abundance of an isotope in nature compared to other isotopes of the same element is relatively constant. The Chart of the Nuclides presents the relative abundance of the naturally occurring isotopes of an element in units of atom percent. Atom percent is the percentage of the atoms of an element that are of a particular isotope. Atom percent is abbreviated as a/o. For example, if a cup of water contains 8.23×10^{24} atoms of oxygen, and the isotopic abundance of oxygen-18 is 0.20%, then there are 1.65×10^{22} atoms of oxygen-18 in the cup.

The atomic weight for an element is defined as the average atomic weight of the isotopes of the element. The atomic weight for an element can be calculated by summing the products of the isotopic abundance of the isotope with the atomic mass of the isotope.

Example:

> Calculate the atomic weight for the element lithium. Lithium-6 has an atom percent abundance of 7.5% and an atomic mass of 6.015122 amu. Lithium-7 has an atomic abundance of 92.5% and an atomic mass of 7.016003 amu.

Solution:

$$\text{Atomic Mass Lithium} = (0.075)\ (6.015122\ \text{amu}) + (0.925)\ (7.016003\ \text{amu})$$
$$= 6.9409\ \text{amu}$$

The other common measurement of isotopic abundance is weight percent (w/o). Weight percent is the percent weight of an element that is a particular isotope. For example, if a sample of material contained 100 kg of uranium that was 28 w/o uranium-235, then 28 kg of uranium-235 was present in the sample.

Enriched and Depleted Uranium

Natural uranium mined from the earth contains the isotopes uranium-238, uranium-235 and uranium-234. The majority (99.2745%) of all the atoms in natural uranium are uranium-238. Most of the remaining atoms (0.72%) are uranium-235, and a slight trace (0.0055%) are uranium-234. Although all isotopes of uranium have similar chemical properties, each of the isotopes has significantly different nuclear properties. For reasons that will be discussed in later modules, the isotope uranium-235 is usually the desired material for use in reactors.

A vast amount of equipment and energy are expended in processes that separate the isotopes of uranium (and other elements). The details of these processes are beyond the scope of this module. These processes are called enrichment processes because they selectively increase the proportion of a particular isotope. The enrichment process typically starts with feed material that has the proportion of isotopes that occur naturally. The process results in two types of

In the case of uranium, the natural uranium ore is 0.72 a/o uranium-235. The desired outcome of the enrichment process is to produce enriched uranium. *Enriched uranium* is defined as uranium in which the isotope uranium-235 has a concentration greater than its natural value. The enrichment process will also result in the byproduct of depleted uranium. *Depleted uranium* is defined as uranium in which the isotope uranium-235 has a concentration less than its natural value. Although depleted uranium is referred to as a by-product of the enrichment process, it does have uses in the nuclear field and in commercial and defense industries.

Summary

The important information in this chapter is summarized below.

Chart of the Nuclides Summary

- Enriched uranium is uranium in which the isotope uranium-235 has a concentration greater than its natural value of 0.7%.
- Depleted uranium is uranium in which the isotope uranium-235 has a concentration less than its natural value of 0.7%.

MASS DEFECT AND BINDING ENERGY

The separate laws of Conservation of Mass and Conservation of Energy are not applied strictly on the nuclear level. It is possible to convert between mass and energy. Instead of two separate conservation laws, a single conservation law states that the sum of mass and energy is conserved. Mass does not magically appear and disappear at random. A decrease in mass will be accompanied by a corresponding increase in energy and vice versa.

EO 1.7 DEFINE the following terms:

a. Mass defect
b. Binding energy

EO 1.8 Given the atomic mass for a nuclide and the atomic masses of a neutron, proton, and electron, CALCULATE the mass defect and binding energy of the nuclide.

Mass Defect

Careful measurements have shown that the mass of a particular atom is always slightly less than the sum of the masses of the individual neutrons, protons, and electrons of which the atom consists. The difference between the mass of the atom and the sum of the masses of its parts is called the *mass defect* (Δm). The mass defect can be calculated using Equation (1-1). In calculating the mass defect it is important to use the full accuracy of mass measurements because the difference in mass is small compared to the mass of the atom. Rounding off the masses of atoms and particles to three or four significant digits prior to the calculation will result in a calculated mass defect of zero.

$$\Delta m = [\ Z(m_p + m_e) + (A-Z)m_n\ ] - m_{atom} \qquad (1\text{-}1)$$

where:

Δm = mass defect (amu)
m_p = mass of a proton (1.007277 amu)
m_n = mass of a neutron (1.008665 amu)
m_e = mass of an electron (0.000548597 amu)
m_{atom} = mass of nuclide ${}^{A}_{Z}X$ (amu)
Z = atomic number (number of protons)
A = mass number (number of nucleons)

Example:

Calculate the mass defect for lithium-7. The mass of lithium-7 is 7.016003 amu.

Solution:

$$\Delta m = \left[Z\left(m_p + m_e\right) + \left(A - Z\right) m_n\right] - m_{atom}$$

$$\Delta m = \left[3\left(1.007826\ amu\right) + \left(7 - 3\right) 1.008665\ amu\right] - 7.016003\ amu$$

$$\Delta m = 0.0421335\ amu$$

Binding Energy

The loss in mass, or mass defect, is due to the conversion of mass to binding energy when the nucleus is formed. *Binding energy* is defined as the amount of energy that must be supplied to a nucleus to completely separate its nuclear particles (nucleons). It can also be understood as the amount of energy that would be released if the nucleus was formed from the separate particles. Binding energy is the energy equivalent of the mass defect. Since the mass defect was converted to binding energy (BE) when the nucleus was formed, it is possible to calculate the binding energy using a conversion factor derived by the mass-energy relationship from Einstein's Theory of Relativity.

Einstein's famous equation relating mass and energy is $E = mc^2$ where c is the velocity of light ($c = 2.998 \times 10^8$ m/sec). The energy equivalent of 1 amu can be determined by inserting this quantity of mass into Einstein's equation and applying conversion factors.

$$E = m\,c^2$$

$$= 1\ amu \left(\frac{1.6606 \times 10^{-27}\ kg}{1\ amu}\right)\left(2.998 \times 10^8\ \frac{m}{sec}\right)^2\left(\frac{1\ N}{1\ \frac{kg\text{-}m}{sec^2}}\right)\left(\frac{1\ J}{1\ N\text{-}m}\right)$$

$$= 1.4924 \times 10^{-10}\ J\left(\frac{1\ MeV}{1.6022 \times 10^{-13}\ J}\right)$$

$$= 931.5\ MeV$$

Conversion Factors:

1 amu	=	1.6606×10^{-27} kg
1 newton	=	1 kg-m/sec^2
1 joule	=	1 newton-meter
1 MeV	=	1.6022×10^{-13} joules

Since 1 amu is equivalent to 931.5 MeV of energy, the binding energy can be calculated using Equation (1-2).

$$\text{B.E.} = \Delta m \left(\frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) \tag{1-2}$$

Example:

Calculate the mass defect and binding energy for uranium-235. One uranium-235 atom has a mass of 235.043924 amu.

Solution:

Step 1: Calculate the mass defect using Equation (1-1).

$$\Delta m = \left[Z \left(m_p + m_e \right) + \left(A - Z \right) m_n \right] - m_{atom}$$

$$\Delta m = \left[92 \left(1.007826 \text{ amu} \right) + \left(235 - 92 \right) 1.008665 \text{ amu} \right] - 235.043924 \text{ amu}$$

$$\Delta m = 1.91517 \text{ amu}$$

Step 2: Use the mass defect and Equation (1-2) to calculate the binding energy.

$$\text{B.E.} = \Delta m \left(\frac{931.5 \text{ MeV}}{1 \text{ amu}} \right)$$

$$= 1.91517 \text{ amu} \left(\frac{931.5 \text{ MeV}}{1 \text{ amu}} \right)$$

$$= 1784 \text{ MeV}$$

Energy Levels of Atoms

The electrons that circle the nucleus move in fairly well-defined orbits. Some of these electrons are more tightly bound in the atom than others. For example, only 7.38 eV is required to remove the outermost electron from a lead atom, while 88,000 eV is required to remove the innermost electron. The process of removing an electron from an atom is called ionization, and the energy required to remove the electron is called the ionization energy.

In a neutral atom (number of electrons = Z) it is possible for the electrons to be in a variety of different orbits, each with a different energy level. The state of lowest energy is the one in which the atom is normally found and is called the ground state. When the atom possesses more energy than its ground state energy, it is said to be in an excited state.

An atom cannot stay in the excited state for an indefinite period of time. An excited atom will eventually transition to either a lower-energy excited state, or directly to its ground state, by emitting a discrete bundle of electromagnetic energy called an x-ray. The energy of the x-ray will be equal to the difference between the energy levels of the atom and will typically range from several eV to 100,000 eV in magnitude.

Energy Levels of the Nucleus

The nucleons in the nucleus of an atom, like the electrons that circle the nucleus, exist in shells that correspond to energy states. The energy shells of the nucleus are less defined and less understood than those of the electrons. There is a state of lowest energy (the ground state) and discrete possible excited states for a nucleus. Where the discrete energy states for the electrons of an atom are measured in eV or keV, the energy levels of the nucleus are considerably greater and typically measured in MeV.

A nucleus that is in the excited state will not remain at that energy level for an indefinite period. Like the electrons in an excited atom, the nucleons in an excited nucleus will transition towards their lowest energy configuration and in doing so emit a discrete bundle of electromagnetic radiation called a gamma ray (γ-ray). The only differences between x-rays and γ-rays are their energy levels and whether they are emitted from the electron shell or from the nucleus.

The ground state and the excited states of a nucleus can be depicted in a nuclear energy-level diagram. The nuclear energy-level diagram consists of a stack of horizontal bars, one bar for each of the excited states of the nucleus. The vertical distance between the bar representing an excited state and the bar representing the ground state is proportional to the energy level of the excited state with respect to the ground state. This difference in energy between the ground state and the excited state is called the excitation energy of the excited state. The ground state of a nuclide has zero excitation energy. The bars for the excited states are labeled with their respective energy levels. Figure 7 is the energy level diagram for nickel-60.

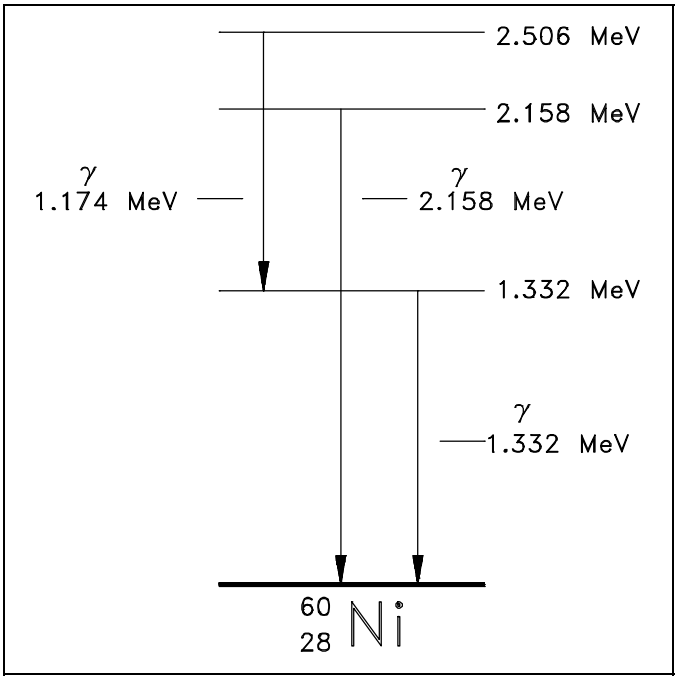


Figure 7 Energy Level Diagram - Nickel-60

Summary

The important information in this chapter is summarized below.

Mass Defect and Binding Energy Summary

- Mass defect is the difference between the mass of the atom and the sum of the masses of its constituent parts.
- Binding energy is the amount of energy that must be supplied to a nucleus to completely separate its nuclear particles. Binding energy is the energy equivalent of the mass defect.
- Mass defect can be calculated by using the equation below.

$$\Delta m = [\ Z(m_p + m_e) + (A\text{-}Z)m_n\ ] - m_{atom}$$

- Binding energy can be calculated by multiplying the mass defect by the factor of 931.5 MeV per amu.

MODES OF RADIOACTIVE DECAY

Most atoms found in nature are stable and do not emit particles or energy that change form over time. Some atoms, however, do not have stable nuclei. These atoms emit radiation in order to achieve a more stable configuration.

EO 2.1 DESCRIBE the following processes:

a. Alpha decay
b. Beta-minus decay
c. Beta-plus decay
d. Electron capture
e. Internal conversions
f. Isomeric transitions

EO 2.2 Given a Chart of the Nuclides, WRITE the radioactive decay chain for a nuclide.

EO 2.3 EXPLAIN why one or more gamma rays typically accompany particle emission.

EO 2.4 Given the stability curve on the Chart of the Nuclides, DETERMINE the type of radioactive decay that the nuclides in each region of the chart will typically undergo.

Stability of Nuclei

As mass numbers become larger, the ratio of neutrons to protons in the nucleus becomes larger for the stable nuclei. Non-stable nuclei may have an excess or deficiency of neutrons and undergo a transformation process known as beta (β) decay. Non-stable nuclei can also undergo a variety of other processes such as alpha (α) or neutron (n) decay. As a result of these decay processes, the final nucleus is in a more stable or more tightly bound configuration.

Natural Radioactivity

In 1896, the French physicist Becquerel discovered that crystals of a uranium salt emitted rays that were similar to x-rays in that they were highly penetrating, could affect a photographic plate, and induced electrical conductivity in gases. Becquerel's discovery was followed in 1898 by the identification of two other radioactive elements, polonium and radium, by Pierre and Marie Curie.

Heavy elements, such as uranium or thorium, and their unstable decay chain elements emit radiation in their naturally occurring state. Uranium and thorium, present since their creation at the beginning of geological time, have an extremely slow rate of decay. All naturally occurring nuclides with atomic numbers greater than 82 are radioactive.

Nuclear Decay

Whenever a nucleus can attain a more stable (i.e., more tightly bound) configuration by emitting radiation, a spontaneous disintegration process known as radioactive decay or nuclear decay may occur. In practice, this "radiation" may be electromagnetic radiation, particles, or both.

Detailed studies of radioactive decay and nuclear reaction processes have led to the formulation of useful conservation principles. The four principles of most interest in this module are discussed below.

Conservation of electric charge implies that charges are neither created nor destroyed. Single positive and negative charges may, however, neutralize each other. It is also possible for a neutral particle to produce one charge of each sign.

Conservation of mass number does not allow a net change in the number of nucleons. However, the conversion of a proton to a neutron and vice versa is allowed.

Conservation of mass and energy implies that the total of the kinetic energy and the energy equivalent of the mass in a system must be conserved in all decays and reactions. Mass can be converted to energy and energy can be converted to mass, but the sum of mass and energy must be constant.

Conservation of momentum is responsible for the distribution of the available kinetic energy among product nuclei, particles, and/or radiation. The total amount is the same before and after the reaction even though it may be distributed differently among entirely different nuclides and/or particles.

Alpha Decay (α)

Alpha decay is the emission of alpha particles (helium nuclei) which may be represented as either ${}^{4}_{2}\mathrm{He}$ or ${}^{4}_{2}\alpha$. When an unstable nucleus ejects an alpha particle, the atomic number is reduced by 2 and the mass number decreased by 4. An example is uranium-234 which decays by the ejection of an alpha particle accompanied by the emission of a 0.068 MeV gamma.

$$^{234}_{92}\mathrm{U} \rightarrow {}^{230}_{90}\mathrm{Th} + {}^{4}_{2}\alpha + \gamma + \mathrm{KE}$$

The combined kinetic energy of the daughter nucleus (Thorium-230) and the α particle is designated as KE. The sum of the KE and the gamma energy is equal to the difference in mass between the original nucleus (Uranium-234) and the final particles (equivalent to the binding energy released, since $\Delta m = BE$). The alpha particle will carry off as much as 98% of the kinetic energy and, in most cases, can be considered to carry off all the kinetic energy.

Beta Decay (β)

Beta decay is the emission of electrons of nuclear rather than orbital origin. These particles are electrons that have been expelled by excited nuclei and may have a charge of either sign.

If both energy and momentum are to be conserved, a third type of particle, the neutrino, ν, must be involved. The neutrino is associated with positive electron emission, and its antiparticle, the antineutrino, $\bar{\nu}$, is emitted with a negative electron. These uncharged particles have only the weakest interaction with matter, no mass, and travel at the speed of light. For all practical purposes, they pass through all materials with so few interactions that the energy they possess cannot be recovered. The neutrinos and antineutrinos are included here only because they carry a portion of the kinetic energy that would otherwise belong to the beta particle, and therefore, must be considered for energy and momentum to be conserved. They are normally ignored since they are not significant in the context of nuclear reactor applications.

Negative electron emission, represented as ${}^{0}_{-1}\mathrm{e}$, ${}^{0}_{-1}\beta$, or simply as e^- or β^-, effectively converts a neutron to a proton, thus increasing the atomic number by one and leaving the mass number unchanged. This is a common mode of decay for nuclei with an excess of neutrons, such as fission fragments below and to the right of the neutron-proton stability curve (refer to Figure 6). An example of a typical beta minus-decay reaction is shown below.

$$^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$$

Positively charged electrons (beta-plus) are known as positrons. Except for sign, they are nearly identical to their negatively charged cousins. When a positron, represented as ${}^{0}_{+1}e$, ${}^{0}_{+1}\beta$, or simply as e^{+} or β^{+}, is ejected from the nucleus, the atomic number is decreased by one and the mass number remains unchanged. A proton has been converted to a neutron. An example of a typical positron (beta-plus) decay is shown below.

$$ {}^{13}_{7}N \rightarrow {}^{13}_{6}C + {}^{0}_{+1}\beta + {}^{0}_{0}\nu $$

Electron Capture (EC, K-capture)

Nuclei having an excess of protons may capture an electron from one of the inner orbits which immediately combines with a proton in the nucleus to form a neutron. This process is called *electron capture* (EC). The electron is normally captured from the innermost orbit (the K-shell), and, consequently, this process is sometimes called K-capture. The following example depicts electron capture.

$$ {}^{7}_{4}Be + {}^{0}_{-1}e \rightarrow {}^{7}_{3}Li + {}^{0}_{0}\nu $$

A neutrino is formed at the same time that the neutron is formed, and energy carried off by it serves to conserve momentum. Any energy that is available due to the atomic mass of the product being appreciably less than that of the parent will appear as gamma radiation. Also, there will always be characteristic x-rays given off when an electron from one of the higher energy shells moves in to fill the vacancy in the K-shell. Electron capture is shown graphically in Figure 8.

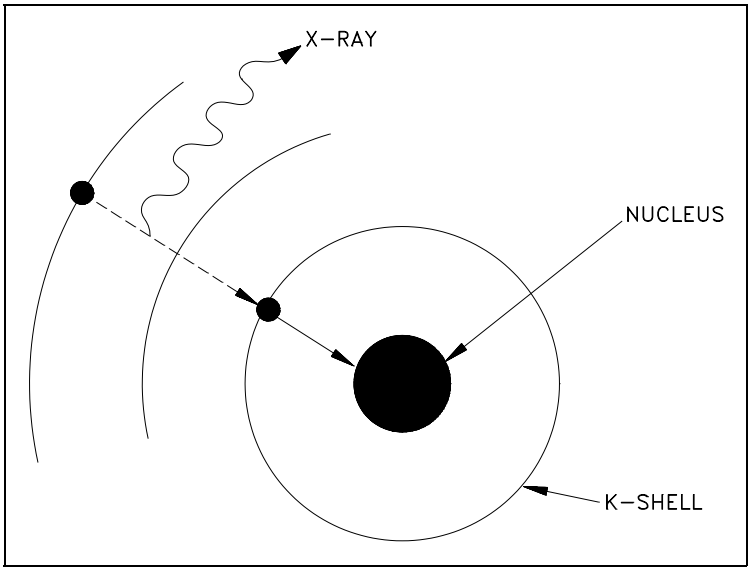


Figure 9 Orbital Electron Capture

Electron capture and positron emission result in the production of the same daughter product, and they exist as competing processes. For positron emission to occur, however, the mass of the daughter product must be less than the mass of the parent by an amount equal to at least twice the mass of an electron. This mass difference between the parent and daughter is necessary to account for two items present in the parent but not in the daughter. One item is the positron ejected from the nucleus of the parent. The other item is that the daughter product has one less orbital electron than the parent. If this requirement is not met, then orbital electron capture takes place exclusively.

Gamma Emission (γ)

Gamma radiation is a high-energy electromagnetic radiation that originates in the nucleus. It is emitted in the form of photons, discrete bundles of energy that have both wave and particle properties. Often a daughter nuclide is left in an excited state after a radioactive parent nucleus undergoes a transformation by alpha decay, beta decay, or electron capture. The nucleus will drop to the ground state by the emission of gamma radiation.

Internal Conversion

The usual method for an excited nucleus to go from the excited state to the ground state is by emission of gamma radiation. However, in some cases the gamma ray (photon) emerges from the nucleus only to interact with one of the innermost orbital electrons and, as a result, the energy of the photon is transferred to the electron. The gamma ray is then said to have undergone *internal conversion*. The conversion electron is ejected from the atom with kinetic energy equal to the gamma energy minus the binding energy of the orbital electron. An orbital electron then drops to a lower energy state to fill the vacancy, and this is accompanied by the emission of characteristic x-rays.

Isomers and Isomeric Transition

Isomeric transition commonly occurs immediately after particle emission; however, the nucleus may remain in an excited state for a measurable period of time before dropping to the ground state at its own characteristic rate. A nucleus that remains in such an excited state is known as a nuclear *isomer* because it differs in energy and behavior from other nuclei with the same atomic number and mass number. The decay of an excited nuclear isomer to a lower energy level is called an *isomeric transition*. It is also possible for the excited isomer to decay by some alternate means, for example, by beta emission.

An example of gamma emission accompanying particle emission is illustrated by the decay of nitrogen-16 below.

$$^{16}_{7}\mathrm{N} \rightarrow \left(^{16}_{8}\mathrm{O}\right)^{*} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$$

$$\left(^{16}_{8}\mathrm{O}\right)^{*} \rightarrow {}^{16}_{8}\mathrm{O} + {}^{0}_{0}\gamma$$

Decay Chains

When an unstable nucleus decays, the resulting daughter nucleus is not necessarily stable. The nucleus resulting from the decay of a parent is often itself unstable, and will undergo an additional decay. This is especially common among the larger nuclides.

It is possible to trace the steps of an unstable atom as it goes through multiple decays trying to achieve stability. The list of the original unstable nuclide, the nuclides that are involved as intermediate steps in the decay, and the final stable nuclide is known as the *decay chain*. One common method for stating the decay chain is to state each of the nuclides involved in the standard ${}^{A}_{Z}X$ format. Arrows are used between nuclides to indicate where decays occur, with the type of decay indicated above the arrow and the half-life below the arrow. The half-life for decay will be discussed in the next chapter.

Example:

> Write the decay chains for rubidium-91 and actinium-215. Continue the chains until a stable nuclide or a nuclide with a half-life greater than 1×10^6 years is reached.

Solution:

$$^{91}_{37}\text{Rb} \xrightarrow[58.0\ \text{s}]{\beta^-} {}^{91}_{38}\text{Sr} \xrightarrow[9.5\ \text{hrs}]{\beta^-} {}^{91}_{39}\text{Y} \xrightarrow[58.5\ \text{d}]{\beta^-} {}^{91}_{40}\text{Zr}$$

$$^{215}_{85}\text{At} \xrightarrow[0.10\ \text{ms}]{\alpha} {}^{211}_{83}\text{Bi} \xrightarrow[2.14\ \text{min}]{\alpha} {}^{207}_{81}\text{Tl} \xrightarrow[4.77\ \text{min}]{\beta^-} {}^{207}_{82}\text{Pb}$$

Predicting Type of Decay

Radioactive nuclides tend to decay in a way that results in a daughter nuclide that lies closer to the line of stability. Due to this, it is possible to predict the type of decay that a nuclide will undergo based on its location relative to the line of stability on the Chart of the Nuclides.

Figure 9 illustrates the type of decay nuclides in different regions of the chart will typically undergo. Nuclides that are below and to the right of the line of stability will usually undergo β^- decay. Nuclides that are above and to the left of the line of stability will usually undergo either β^+ decay or electron capture. Most nuclides that will undergo α decay are found in the upper right hand region of the chart. These are general rules that have many exceptions, especially in the region of the heavy nuclides.

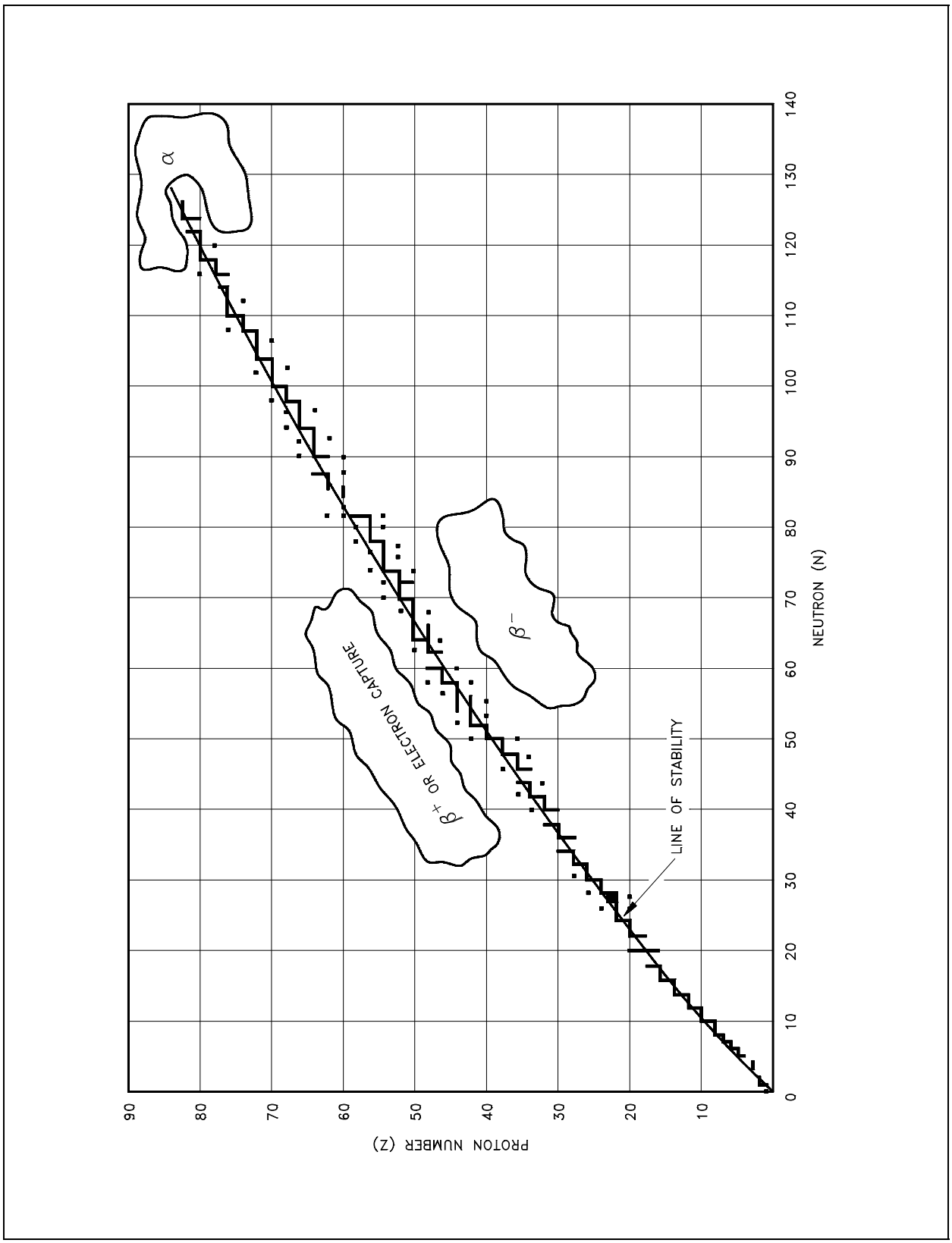


Figure 9 Types of Radioactive Decay Relative to the Line of Stability

Summary

The important information in this chapter is summarized below.

Modes of Radioactive Decay Summary

- Alpha decay is the emission of an alpha particle (2 protons and 2 neutrons) from an unstable nucleus. The daughter nuclide has an atomic number 2 less than the parent nuclide and a mass number 4 less than the parent nuclide. The daughter nucleus commonly releases its excitation energy by gamma emission.
- Beta-minus decay effectively converts a neutron to a proton and an electron, which is immediately ejected from the nucleus. The daughter nuclide has its atomic number increased by 1 and the same mass number compared to the parent.
- Beta-plus decay effectively converts a proton to a neutron and a positron, which is immediately ejected from the nucleus. The daughter nuclide has its atomic number decreased by 1 and the same mass number compared to the parent.
- In electron capture, the nucleus absorbs an electron from the innermost orbit. This electron combines with a proton to form a neutron.
- Internal conversion occurs when a gamma ray, emitted by the nucleus as it goes from the excited state to the ground state, interacts with one of the innermost electrons of the same atom. The electron is ejected from the atom.
- An isomeric transition is the decay of an excited nucleus to a lower-energy level by the emission of a gamma ray.
- Decay chains can be found by tracing the steps an unstable atom goes through as it tries to achieve stability.
- Many modes of radioactive decay result in a daughter nuclide that has an energy level above the ground state. This excitation energy is usually released immediately in the form of a gamma ray.
- The type of decay that a nuclide will typically undergo can be determined by its relationship to the line of stability on the Chart of the Nuclides. Nuclides that lie below and to the right of the line of stability will typically beta minus decay. Nuclides above and to the left of the line will typically either beta plus decay or electron capture. Most alpha emitters are found in the upper, right-hand corner of the chart.

RADIOACTIVITY

The rate at which a sample of radioactive material decays is not constant. As individual atoms of the material decay, there are fewer of those types of atoms remaining. Since the rate of decay is directly proportional to the number of atoms, the rate of decay will decrease as the number of atoms decreases.

EO 2.5 DEFINE the following terms:

a. Radioactivity
b. Curie
c. Becquerel
d. Radioactive decay constant
e. Radioactive half-life

EO 2.6 Given the number of atoms and either the half-life or decay constant of a nuclide, CALCULATE the activity.

EO 2.7 Given the initial activity and the decay constant of a nuclide, CALCULATE the activity at any later time.

EO 2.8 CONVERT between the half-life and decay constant for a nuclide.

EO 2.9 Given the Chart of the Nuclides and the original activity, PLOT the radioactive decay curve for a nuclide on either linear or semi-log coordinates.

EO 2.10 DEFINE the following terms:

a. Radioactive equilibrium
b. Transient radioactive equilibrium

Radioactive Decay Rates

Radioactivity is the property of certain nuclides of spontaneously emitting particles or gamma radiation. The decay of radioactive nuclides occurs in a random manner, and the precise time at which a single nucleus will decay cannot be determined. However, the average behavior of a very large sample can be predicted accurately by using statistical methods. These studies have revealed that there is a certain probability that in a given time interval a certain fraction of the nuclei within a sample of a particular nuclide will decay. This probability per unit time that an atom of a nuclide will decay is known as the *radioactive decay constant*, λ. The units for the decay constant are inverse time such as 1/second, 1/minute, 1/hour, or 1/year. These decay constant units can also be expressed as $second^{-1}$, $minute^{-1}$, $hour^{-1}$, and $year^{-1}$.

The *activity* (A) of a sample is the rate of decay of that sample. This rate of decay is usually measured in the number of disintegrations that occur per second. For a sample containing millions of atoms, the activity is the product of the decay constant and the number of atoms present in the sample.

The relationship between the activity, number of atoms, and decay constant is shown in Equation (1-3).

$$A = \lambda N \quad (1\text{-}3)$$

where:

A = Activity of the nuclide (disintegrations/second)
λ = decay constant of the nuclide ($second^{-1}$)
N = Number of atoms of the nuclide in the sample

Since λ is a constant, the activity and the number of atoms are always proportional.

Units of Measurement for Radioactivity

Two common units to measure the activity of a substance are the curie (Ci) and becquerel (Bq). A *curie* is a unit of measure of the rate of radioactive decay equal to 3.7×10^{10} disintegrations per second. This is approximately equivalent to the number of disintegrations that one gram of radium-226 will undergo in one second. A *becquerel* is a more fundamental unit of measure of radioactive decay that is equal to 1 disintegration per second. Currently, the curie is more widely used in the United States, but usage of the becquerel can be expected to broaden as the metric system slowly comes into wider use. The conversion between curies and becquerels is shown below.

1 curie = 3.7×10^{10} becquerels

Variation of Radioactivity Over Time

The rate at which a given radionuclide sample decays is stated in Equation (1-3) as being equal to the product of the number of atoms and the decay constant. From this basic relationship it is possible to use calculus to derive an expression which can be used to calculate how the number of atoms present will change over time. The derivation is beyond the scope of this text, but Equation (1-4) is the useful result.

$$N = N_o e^{-\lambda t} \quad (1\text{-}4)$$

where:

N = number of atoms present at time t
N_o = number of atoms initially present
λ = decay constant ($time^{-1}$)
t = time

Since the activity and the number of atoms are always proportional, they may be used interchangeably to describe any given radionuclide population. Therefore, the following is true.

$$A = A_o\, e^{-\lambda t} \tag{1-5}$$

where:

A = activity present at time t
A_o = activity initially present
λ = decay constant (time^{-1})
t = time

Radioactive Half-Life

One of the most useful terms for estimating how quickly a nuclide will decay is the radioactive half-life. The *radioactive half-life* is defined as the amount of time required for the activity to decrease to one-half of its original value. A relationship between the half-life and decay constant can be developed from Equation (1-5). The half-life can be calculated by solving Equation (1-5) for the time, t, when the current activity, A, equals one-half the initial activity A_o.

First, solve Equation (1-5) for t.

$$A = A_o\, e^{-\lambda t}$$

$$\frac{A}{A_o} = e^{-\lambda t}$$

$$\ln\left(\frac{A}{A_o}\right) = -\lambda t$$

$$t = \frac{-\ln\left(\frac{A}{A_o}\right)}{\lambda}$$

If A is equal to one-half of A_o, then A/A_o is equal to one-half. Substituting this in the equation above yields an expression for $t_{1/2}$.

$$t_{1/2} = \frac{-\ln\left(\frac{1}{2}\right)}{\lambda} \tag{1-6}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

The basic features of decay of a radionuclide sample are shown by the graph in Figure 10.

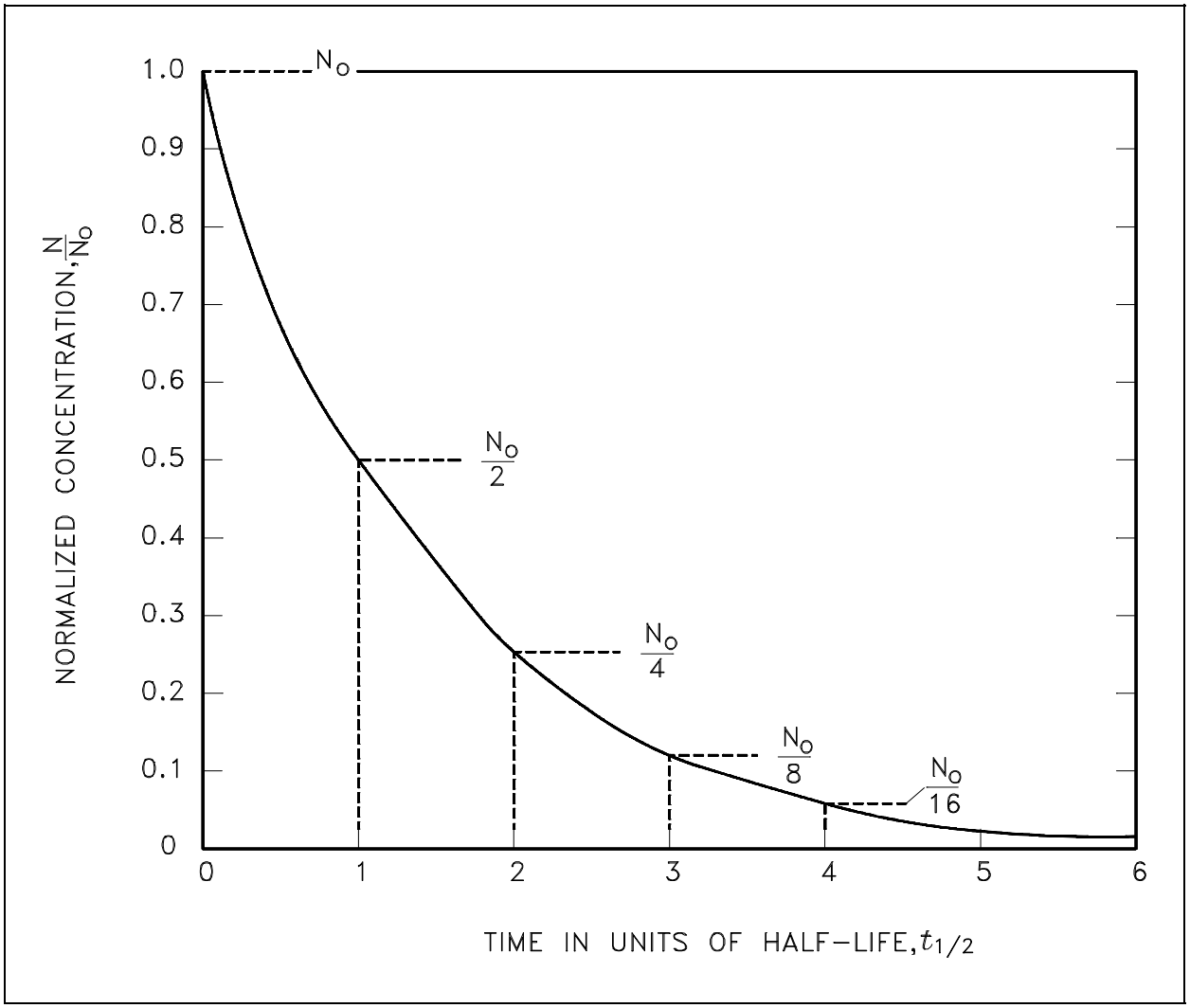


Figure 10 Radioactive Decay as a Function of Time in Units of Half-Life

Assuming an initial number of atoms N_o, the population, and consequently, the activity may be noted to decrease by one-half of this value in a time of one half-life. Additional decreases occur so that whenever one half-life elapses, the number of atoms drops to one-half of what its value was at the beginning of that time interval. After five half-lives have elapsed, only 1/32, or 3.1%, of the original number of atoms remains. After seven half-lives, only 1/128, or 0.78%, of the atoms remains. The number of atoms existing after 5 to 7 half-lives can usually be assumed to be negligible. The Chemistry Fundamentals Handbook contains additional information on calculating the number of atoms contained within a sample.

Example:

A sample of material contains 20 micrograms of californium-252.

Californium-252 has a half-life of 2.638 years.

Calculate:

(a) The number of californium-252 atoms initially present
(b) The activity of the californium-252 in curies
(c) The number of californium-252 atoms that will remain in 12 years
(d) The time it will take for the activity to reach 0.001 curies

Solution:

(a) The number of atoms of californium-252 can be determined as below.

$$N_{Cf\text{-}252} = \text{mass}\left(\frac{1\ \text{mole}}{\text{isotopic mass}}\right)\left(\frac{N_A}{1\ \text{mole}}\right)$$

$$= (20 \times 10^{-6}\ \text{g})\left(\frac{1\ \text{mole}}{252.08\ \text{g}}\right)\left(\frac{6.022 \times 10^{23}\ \text{atoms}}{1\ \text{mole}}\right)$$

$$= 4.78 \times 10^{16}\ \text{atoms}$$

(b) First, use Equation (1-6) to calculate the decay constant.

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$= \frac{0.693}{2.638\ \text{years}}$$

$$= 0.263\ \text{year}^{-1}$$

Use this value for the decay constant in Equation (1-3) to determine the activity.

$$A = \lambda N$$

$$= (0.263\ \text{year}^{-1})(4.78 \times 10^{16}\ \text{atoms})\left(\frac{1\ \text{year}}{365.25\ \text{days}}\right)\left(\frac{1\ \text{day}}{24\ \text{hours}}\right)\left(\frac{1\ \text{hour}}{3600\ \text{seconds}}\right)$$

$$= \left(3.98 \times 10^{8}\ \frac{\text{disintegrations}}{\text{second}}\right)\left(\frac{1\ \text{curie}}{3.7 \times 10^{10}\ \text{disintegrations/sec}}\right)$$

$$= 0.0108\ \text{curies}$$

(c) The number of californium atoms that will remain in 12 years can be calculated from Equation (1-4).

$$N = N_o\ e^{-\lambda t}$$

$$= \left(4.78 \times 10^{16}\right) e^{-(0.263/\text{yr})\ (12\ \text{yr})}$$

$$= 2.04 \times 10^{15}$$

(d) The time that it will take for the activity to reach 0.001 Ci can be determined from Equation (1-5). First, solve Equation (1-5) for time.

$$A = A_o\ e^{-\lambda t}$$

$$\frac{A}{A_o} = e^{-\lambda t}$$

$$\ln\left(\frac{A}{A_o}\right) = -\lambda t$$

$$t = \frac{-\ln\left(\frac{A}{A_o}\right)}{\lambda}$$

Inserting the appropriate values in the right side of this equation will result in the required time.

$$t = \frac{-\ln\left(\frac{0.001\ \text{Ci}}{0.0108\ \text{Ci}}\right)}{0.263\ \text{year}^{-1}}$$

$$t = 9.05\ \text{years}$$

Plotting Radioactive Decay

It is useful to plot the activity of a nuclide as it changes over time. Plots of this type can be used to determine when the activity will fall below a certain level. This plot is usually done showing activity on either a linear or a logarithmic scale. The decay of the activity of a single nuclide on a logarithmic scale will plot as a straight line because the decay is exponential.

Example:

Plot the radioactive decay curve for nitrogen-16 over a period of 100 seconds. The initial activity is 142 curies and the half-life of nitrogen-16 is 7.13 seconds. Plot the curve on both linear rectangular coordinates and on a semi-log scale.

Solution:

First, use Equation (1-6) to calculate the decay constant corresponding to a half-life of 7.13 seconds.

$$t_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$\lambda = \frac{0.693}{7.13 \text{ seconds}}$$

$$\lambda = 0.0972 \text{ second}^{-1}$$

Use the decay constant determined above to calculate the activity at various times using Equation (1-5).

$$A = A_o e^{-\lambda t}$$

Time	Activity
0 seconds	142 Ci
20 seconds	20.3 Ci
40 seconds	2.91 Ci
60 seconds	0.416 Ci
80 seconds	0.0596 Ci
100 seconds	0.00853 Ci

Plotting the data points calculated above on both linear and semilog scales results in the graphs shown in Figure 11.

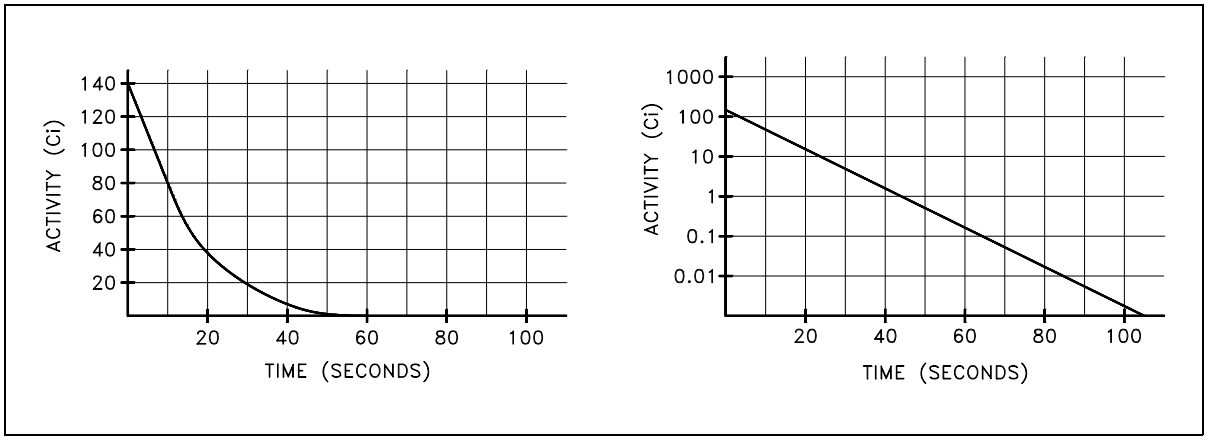


Figure 11 Linear and Semi-log Plots of Nitrogen-16 Decay

If a substance contains more than one radioactive nuclide, the total activity is the sum of the individual activities of each nuclide. As an example, consider a sample of material that contained 1×10^6 atoms of iron-59 that has a half-life of 44.51 days ($\lambda = 1.80 \times 10^{7}$ sec^{1}), 1×10^6 atoms of manganese-54 that has a half-life of 312.2 days ($\lambda = 2.57 \times 10^{-8}$ sec^{-1}), and 1×10^6 atoms of cobalt-60 that has a half-life of 1925 days ($\lambda = 4.17 \times 10^{-9}$ sec^{-1}).

The initial activity of each of the nuclides would be the product of the number of atoms and the decay constant.

$$A_{Fe-59} = N_{Fe-59}\ \lambda_{Fe-59}$$
$$= (1 \times 10^6 \text{ atoms})\ (1.80 \times 10^{-7} \text{ sec}^{-1})$$
$$= 0.180 \text{ Ci}$$

$$A_{Mn-54} = N_{Mn-54}\ \lambda_{Mn-54}$$
$$= (1 \times 10^6 \text{ atoms})\ (2.57 \times 10^{-8} \text{ sec}^{-1})$$
$$= 0.0257 \text{ Ci}$$

$$A_{Co-60} = N_{Co-60}\ \lambda_{Co-60}$$
$$= (1 \times 10^6 \text{ atoms})\ (4.17 \times 10^{-9} \text{ sec}^{-1})$$
$$= 0.00417 \text{ Ci}$$

Plotting the manner in which the activities of each of the three nuclides decay over time demonstrates that initially the activity of the shortest-lived nuclide (iron-59) dominates the total activity, then manganese-54 dominates. After almost all of the iron and manganese have decayed away, the only contributor to activity will be the cobalt-60. A plot of this combined decay is shown in Figure 12.

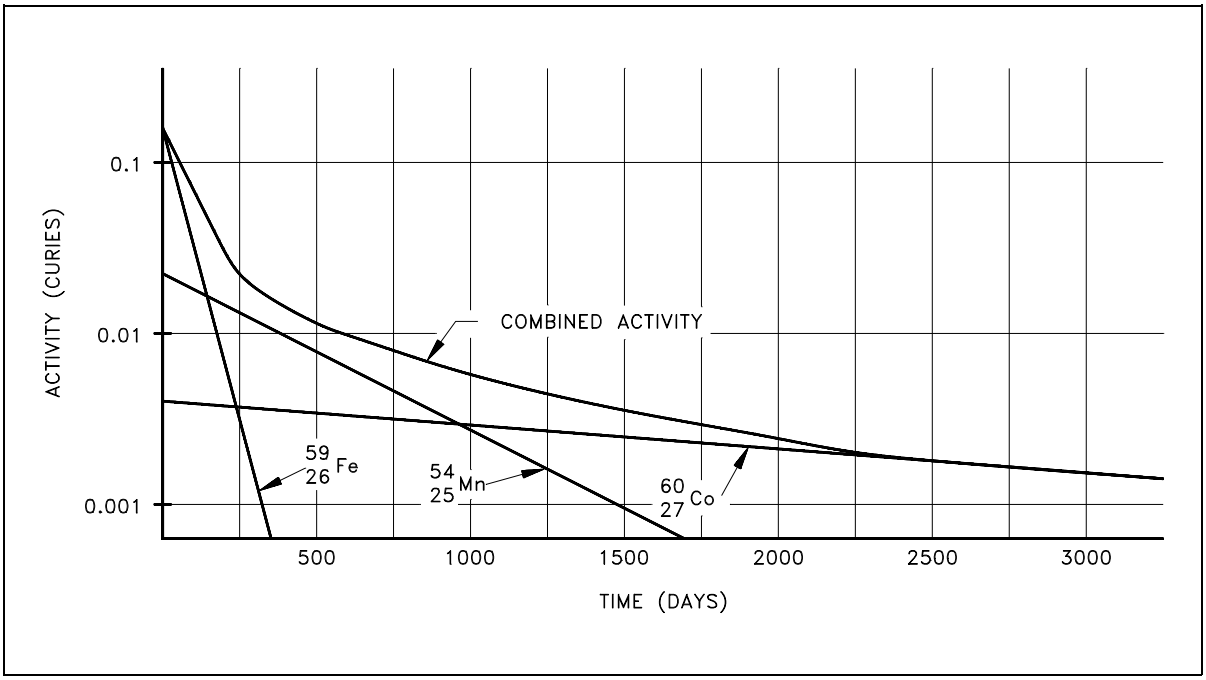


Figure 12 Combined Decay of Iron-56, Manganese-54, and Cobalt-60

Radioactive Equilibrium

Radioactive equilibrium exists when a radioactive nuclide is decaying at the same rate at which it is being produced. Since the production rate and decay rate are equal, the number of atoms present remains constant over time.

An example of radioactive equilibrium is the concentration of sodium-24 in the coolant circulating through a sodium-cooled nuclear reactor. Assume that the sodium-24 is being produced at a rate of 1×10^6 atoms per second. If the sodium-24 were stable and did not decay, the amount of sodium-24 present after some period of time could be calculated by multiplying the production rate by the amount of time. Plotting the amount of material present would result in the graph in Figure 13.

However, sodium-24 is not stable, and it decays with a half-life of 14.96 hours. If no sodium-24 is present initially and production starts at a rate of 1×10^6 atoms per second, the rate of decay will initially be zero because there is no sodium-24 present to decay. The rate of decay of sodium-24 will increase as the amount of sodium-24 increases.

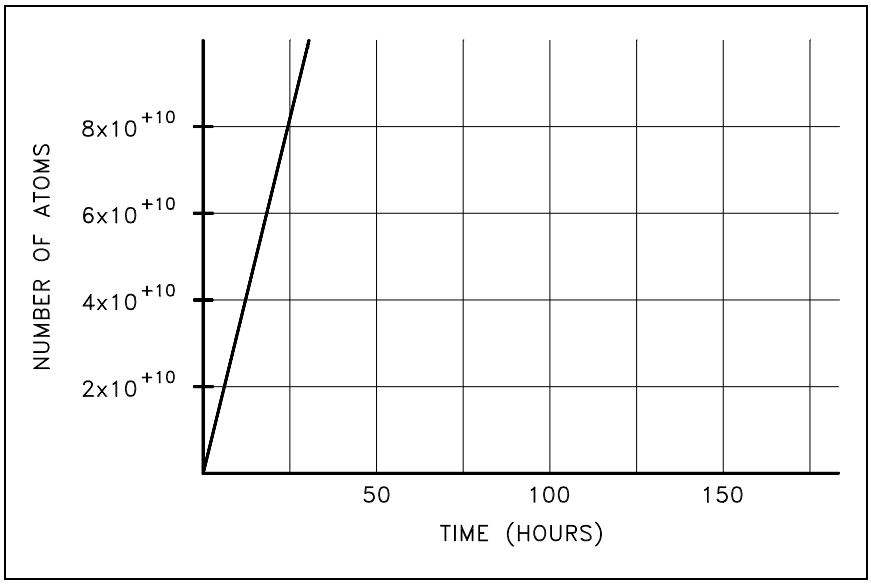


Figure 13 Cumulative Production of Sodium-24 Over Time

The amount of sodium-24 present will initially increase rapidly, then it will increase at a continually decreasing rate until the rate of decay is equal to the rate of production. It is possible to calculate how much sodium-24 will be present at equilibrium by setting the production rate (R) equal to the decay rate (λ N).

$$\mathbf{R} = \lambda \mathbf{N}$$

$$\mathbf{N} = \frac{\mathbf{R}}{\lambda}$$

where:

R = production rate (atoms/second)
λ = decay constant (second^{-1})
N = number of atoms

It is possible to calculate the equilibrium value for sodium-24 being produced at a rate of 1 x 10^6 atoms/second.

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$= \frac{0.693}{14.96 \text{ hours}} \left(\frac{1 \text{ hour}}{3600 \text{ seconds}} \right)$$

$$= 1.287 \text{ x } 10^{-5} \text{ second}^{-1}$$

$$\mathbf{N} = \frac{\mathbf{R}}{\lambda}$$

$$= \frac{1 \text{ x } 10^{6} \frac{\text{atoms}}{\text{second}}}{1.287 \text{ x } 10^{-5} \text{ second}^{-1}}$$

$$= 7.77 \text{ x } 10^{10} \text{ atoms}$$

The development of the equation to calculate how the amount of sodium-24 changes over time as it approaches the equilibrium value is beyond the scope of this handbook. However, the equation is presented below.

$$N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$$

This equation can be used to calculate the values of the amount of sodium-24 present at different times. As the time increases, the exponential term approaches zero, and the number of atoms present will approach R/λ. A plot of the approach of sodium-24 to equilibrium is shown in Figure 14.

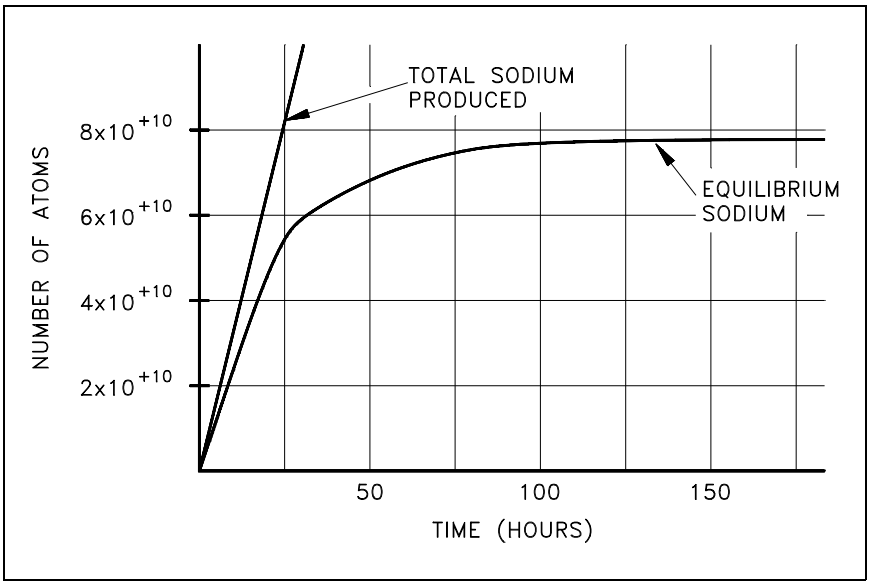


Figure 14 Approach of Sodium-24 to Equilibrium

Transient Radioactive Equilibrium

Transient radioactive equilibrium occurs when the parent nuclide and the daughter nuclide decay at essentially the same rate.

For transient equilibrium to occur, the parent must have a long half-life when compared to the daughter. An example of this type of compound decay process is barium-140, which decays by beta emission to lanthanum-140, which in turn decays by beta emission to stable cerium-140.

$$^{140}_{56}\mathrm{Ba} \xrightarrow[12.75\ \text{days}]{\beta^-} {}^{140}_{57}\mathrm{La} \xrightarrow[1.678\ \text{days}]{\beta^-} {}^{140}_{58}\mathrm{Ce}$$

The decay constant for barium-140 is considerably smaller than the decay constant for lanthanum-140. Remember that the rate of decay of both the parent and daughter can be represented as λN. Although the decay constant for barium-140 is smaller, the actual rate of decay (λN) is initially larger than that of lanthanum-140 because of the great difference in their initial concentrations. As the concentration of the daughter increases, the rate of decay of the daughter will approach and eventually match the decay rate of the parent. When this occurs, they are said to be in transient equilibrium. A plot of the barium-lanthanum-cerium decay chain reaching transient equilibrium is shown in Figure 15.

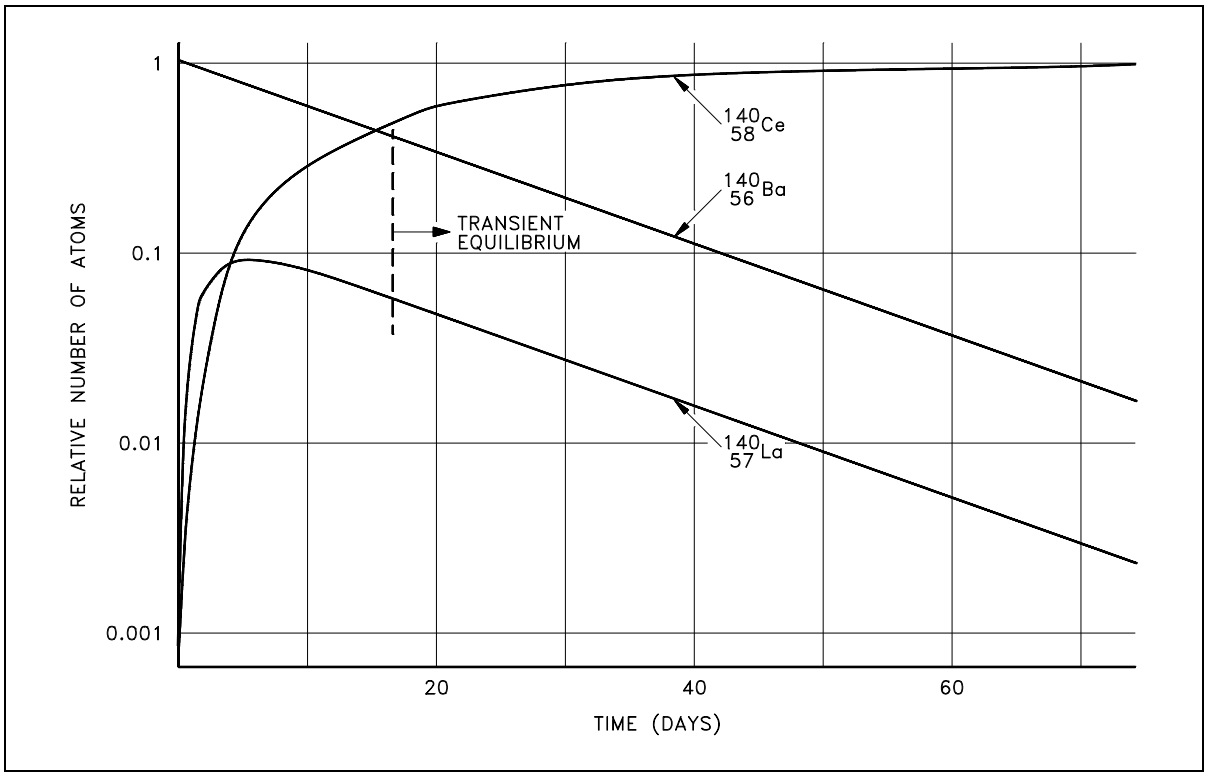


Figure 15 Transient Equilibrium in the Decay of Barium-140

Secular equilibrium occurs when the parent has an extremely long half-life. In the long decay chain for a naturally radioactive element, such as thorium-232, where all of the elements in the chain are in secular equilibrium, each of the descendants has built up to an equilibrium amount and all decay at the rate set by the original parent. The only exception is the final stable element on the end of the chain. Its number of atoms is constantly increasing.

Summary

The important information in this chapter is summarized on the following page.

Radioactivity Summary

- Radioactivity is the decay of unstable atoms by the emission of particles and electromagnetic radiation.
- A curie (Ci) is a unit of radioactivity equal to 3.7×10^{10} disintegrations per second.
- A becquerel (Bq) is a unit of radioactivity equal to 1 disintegration per second.
- The radioactive decay constant (λ) is the probability per unit time that an atom will decay.
- The radioactive half-life is the amount of time required for the activity to decrease to one-half its original value.
- The activity of a substance can be calculated from the number of atoms and the decay constant based on the equation below.

$$A = \lambda N$$

- The amount of activity remaining after a particular time can be calculated from the equation below.

$$A = A_o e^{-\lambda t}$$

- The relationship between the decay constant and the half-life is shown below.

$$t_{1/2} = \frac{0.693}{\lambda}$$

- Plots of radioactive decay can be useful to describe the variation of activity over time. If decay is plotted using semi-log scale the plot results in a straight line.
- Radioactive equilibrium exists when the production rate of a material is equal to the removal rate.
- Transient radioactive equilibrium exists when the parent nuclide and the daughter nuclide decay at essentially the same rate. This occurs only when the parent has a long half-life compared to the daughter.

NEUTRON INTERACTIONS

Neutrons can cause many different types of interactions. The neutron may simply scatter off the nucleus in two different ways, or it may actually be absorbed into the nucleus. If a neutron is absorbed into the nucleus, it may result in the emission of a gamma ray or a subatomic particle, or it may cause the nucleus to fission.

EO 3.1 DESCRIBE the following scattering interactions between a neutron and a nucleus:

a. Elastic scattering
b. Inelastic scattering

EO 3.2 STATE the conservation laws that apply to an elastic collision between a neutron and a nucleus.

EO 3.3 DESCRIBE the following reactions where a neutron is absorbed in a nucleus:

a. Radiative capture
b. Particle ejection

Scattering

A neutron *scattering* reaction occurs when a nucleus, after having been struck by a neutron, emits a single neutron. Despite the fact that the initial and final neutrons do not need to be (and often are not) the same, the net effect of the reaction is as if the projectile neutron had merely "bounced off," or scattered from, the nucleus. The two categories of scattering reactions, elastic and inelastic scattering, are described in the following paragraphs.

Elastic Scattering

In an *elastic scattering* reaction between a neutron and a target nucleus, there is no energy transferred into nuclear excitation. Momentum and kinetic energy of the "system" are conserved although there is usually some transfer of kinetic energy from the neutron to the target nucleus. The target nucleus gains the amount of kinetic energy that the neutron loses.

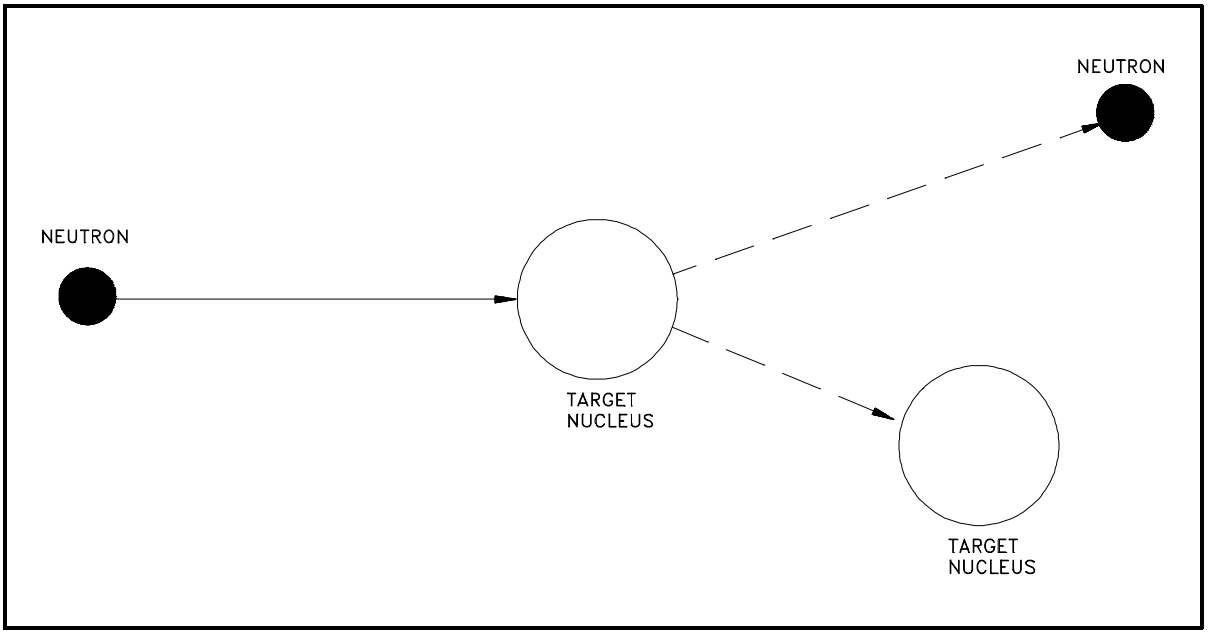


Figure 16 Elastic Scattering

Figure 16 illustrates the process of elastic scattering of a neutron off a target nucleus. In the elastic scattering reaction, the conservation of momentum and kinetic energy is represented by the equations below.

Conservation of momentum (mv)

$$(m_n\ v_{n,i}) + (m_T\ v_{T,i}) = (m_n\ v_{n,f}) + (m_T\ v_{T,f})$$

Conservation of kinetic energy $\left(\frac{1}{2}\ m\ v^2\right)$

$$\left(\frac{1}{2}\ m_n\ v_{n,i}^2\right) + \left(\frac{1}{2}\ m_T\ v_{T,i}^2\right) = \left(\frac{1}{2}\ m_n\ v_{n,f}^2\right) + \left(\frac{1}{2}\ m_T\ v_{T,f}^2\right)$$

where:

m_n = mass of the neutron
m_T = mass of the target nucleus
$v_{n,i}$ = initial neutron velocity
$v_{n,f}$ = final neutron velocity
$v_{T,i}$ = initial target velocity
$v_{T,f}$ = final target velocity

Elastic scattering of neutrons by nuclei can occur in two ways. The more unusual of the two interactions is the absorption of the neutron, forming a compound nucleus, followed by the re-emission of a neutron in such a way that the total kinetic energy is conserved and the nucleus returns to its ground state. This is known as *resonance elastic scattering* and is very dependent upon the initial kinetic energy possessed by the neutron. Due to formation of the compound nucleus, it is also referred to as compound elastic scattering. The second, more usual method, is termed *potential elastic scattering* and can be understood by visualizing the neutrons and nuclei to be much like billiard balls with impenetrable surfaces. Potential scattering takes place with incident neutrons that have an energy of up to about 1 MeV. In potential scattering, the neutron does not actually touch the nucleus and a compound nucleus is not formed. Instead, the neutron is acted on and scattered by the short range nuclear forces when it approaches close enough to the nucleus.

Inelastic Scattering

In *inelastic scattering*, the incident neutron is absorbed by the target nucleus, forming a compound nucleus. The compound nucleus will then emit a neutron of lower kinetic energy which leaves the original nucleus in an excited state. The nucleus will usually, by one or more gamma emissions, emit this excess energy to reach its ground state. Figure 17 shows the process of inelastic scattering.

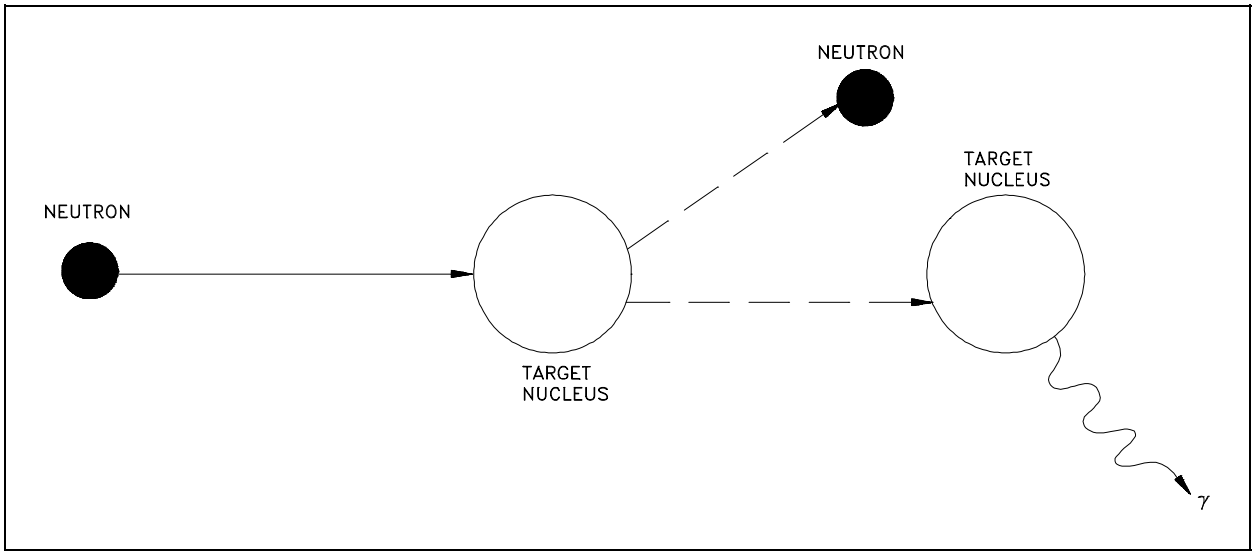


Figure 17 Inelastic Scattering

For the nucleus that has reached its ground state, the sum of the kinetic energy of the exit neutron, the target nucleus, and the total gamma energy emitted is equal to the initial kinetic energy of the incident neutron.

Absorption Reactions

Most *absorption reactions* result in the loss of a neutron coupled with the production of a charged particle or gamma ray. When the product nucleus is radioactive, additional radiation is emitted at some later time. Radiative capture, particle ejection, and fission are all categorized as absorption reactions and are briefly described below.

Radiative Capture

In *radiative capture* the incident neutron enters the target nucleus forming a compound nucleus. The compound nucleus then decays to its ground state by gamma emission. An example of a radiative capture reaction is shown below.

$$ {}^{1}_{0}n + {}^{238}_{92}U \rightarrow \left({}^{239}_{92}U \right)^{*} \rightarrow {}^{239}_{92}U + {}^{0}_{0}\gamma $$

Particle Ejection

In a *particle ejection* reaction the incident particle enters the target nucleus forming a compound nucleus. The newly formed compound nucleus has been excited to a high enough energy level to cause it to eject a new particle while the incident neutron remains in the nucleus. After the new particle is ejected, the remaining nucleus may or may not exist in an excited state depending upon the mass-energy balance of the reaction. An example of a particle ejection reaction is shown below.

$$ {}^{1}_{0}n + {}^{10}_{5}B \rightarrow \left({}^{11}_{5}B \right)^{*} \rightarrow {}^{7}_{3}Li + {}^{4}_{2}\alpha $$

Fission

One of the most important interactions that neutrons can cause is fission, in which the nucleus that absorbs the neutron actually splits into two similarly sized parts. Fission will be discussed in detail in the next chapter.

Summary

The important information in this chapter is summarized below.

Neutron Interactions Summary

- Interactions where a neutron scatters off a target nucleus are either elastic or inelastic. In elastic scattering, kinetic energy and momentum are conserved and no energy is transferred into excitation energy of the target nucleus. In inelastic scattering, some amount of kinetic energy is transferred into excitation energy of the target nucleus.
- The conservation principles that apply to an elastic collision are conservation of kinetic energy and conservation of momentum.
- Radiative capture is the absorption of a neutron by the target nucleus, resulting in an excited nucleus which subsequently (typically within a small fraction of a second) releases its excitation energy in the form of a gamma ray.
- Particle ejection occurs when a neutron is absorbed by a target nucleus, resulting in the formation of a compound nucleus. The compound nucleus immediately ejects a particle (for example, alpha or proton).

NUCLEAR FISSION

Nuclear fission is a process in which an atom splits and releases energy, fission products, and neutrons. The neutrons released by fission can, in turn, cause the fission of other atoms.

EO 4.1 **EXPLAIN the fission process using the liquid drop model of a nucleus.**

EO 4.2 **DEFINE the following terms:**

a. Excitation energy (E_{exc})
b. Critical energy (E_{crit})

EO 4.3 **DEFINE the following terms:**

a. Fissile material
b. Fissionable material
c. Fertile material

EO 4.4 **DESCRIBE the processes of transmutation, conversion, and breeding.**

EO 4.5 **DESCRIBE the curve of Binding Energy per Nucleon versus mass number and give a qualitative description of the reasons for its shape.**

EO 4.6 **EXPLAIN why only the heaviest nuclei are easily fissioned.**

EO 4.7 **EXPLAIN why uranium-235 fissions with thermal neutrons and uranium-238 fissions only with fast neutrons.**

Fission

In the fission reaction the incident neutron enters the heavy target nucleus, forming a compound nucleus that is excited to such a high energy level ($E_{exc} > E_{crit}$) that the nucleus "splits" (fissions) into two large fragments plus some neutrons. An example of a typical fission reaction is shown below.

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow \left({}^{236}_{92}U \right)^{*} \rightarrow {}^{140}_{55}Cs + {}^{93}_{37}Rb + 3\left({}^{1}_{0}n \right) $$

A large amount of energy is released in the form of radiation and fragment kinetic energy.

Liquid Drop Model of a Nucleus

The nucleus is held together by the attractive nuclear force between nucleons, which was discussed in a previous chapter. The characteristics of the nuclear force are listed below.

(a) very short range, with essentially no effect beyond nuclear dimensions ($\sim 10^{-13}$ cm)

(b) stronger than the repulsive electrostatic forces within the nucleus

(c) independent of nucleon pairing, in that the attractive forces between pairs of neutrons are no different than those between pairs of protons or a neutron and a proton

(d) saturable, that is, a nucleon can attract only a few of its nearest neighbors

One theory of fission considers the fissioning of a nucleus similar in some respects to the splitting of a liquid drop. This analogy is justifiable to some extent by the fact that a liquid drop is held together by molecular forces that tend to make the drop spherical in shape and that try to resist any deformation in the same manner as nuclear forces are assumed to hold the nucleus together. By considering the nucleus as a liquid drop, the fission process can be described.

Referring to Figure 18(A), the nucleus in the ground state is undistorted, and its attractive nuclear forces are greater than the repulsive electrostatic forces between the protons within the nucleus. When an incident particle (in this instance a neutron) is absorbed by the target nucleus, a compound nucleus is formed. The compound nucleus temporarily contains all the charge and mass involved in the reaction and exists in an excited state. The excitation energy added to the compound nucleus is equal to the binding energy contributed by the incident particle plus the kinetic energy possessed by that particle. Figure 18(B) illustrates the excitation energy thus imparted to the compound nucleus, which may cause it to oscillate and become distorted. If the excitation energy is greater than a certain critical energy, the oscillations may cause the compound nucleus to become dumbbell-shaped. When this happens, the attractive nuclear forces (short-range) in the neck area are small due to saturation, while the repulsive electrostatic forces (long-range) are only slightly less than before. When the repulsive electrostatic forces exceed the attractive nuclear forces, nuclear fission occurs, as illustrated in Figure 18(C).

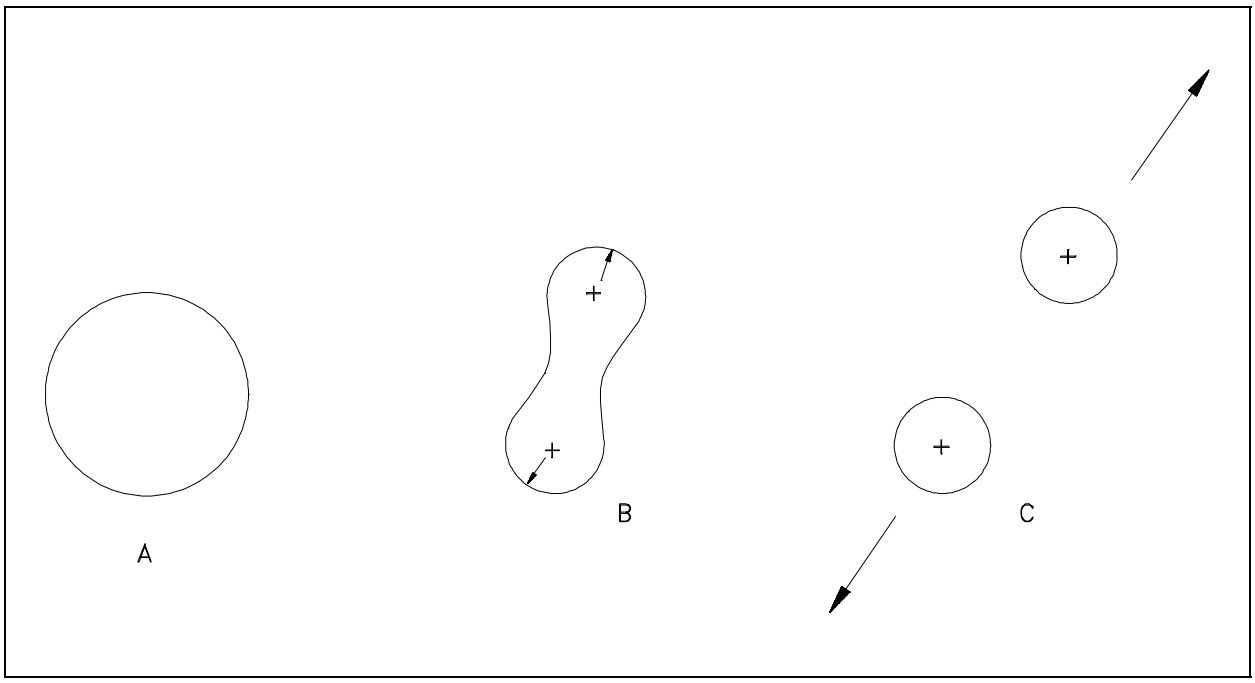


Figure 18 Liquid Drop Model of Fission

Critical Energy

The measure of how far the energy level of a nucleus is above its ground state is called the *excitation energy* (E_{exc}). For fission to occur, the excitation energy must be above a particular value for that nuclide. The *critical energy* (E_{crit}) is the minimum excitation energy required for fission to occur.

Fissile Material

A *fissile material* is composed of nuclides for which fission is possible with neutrons of any energy level. What is especially significant about these nuclides is their ability to be fissioned with zero kinetic energy neutrons (thermal neutrons). Thermal neutrons have very low kinetic energy levels (essentially zero) because they are roughly in equilibrium with the thermal motion of surrounding materials. Therefore, in order to be classified as fissile, a material must be capable of fissioning after absorbing a thermal neutron. Consequently, they impart essentially no kinetic energy to the reaction. Fission is possible in these materials with thermal neutrons, since the change in binding energy supplied by the neutron addition alone is high enough to exceed the critical energy. Some examples of fissile nuclides are uranium-235, uranium-233, and plutonium-239.

Fissionable Material

A *fissionable material* is composed of nuclides for which fission with neutrons is possible. All fissile nuclides fall into this category. However, also included are those nuclides that can be fissioned only with high energy neutrons. The change in binding energy that occurs as the result of neutron absorption results in a nuclear excitation energy level that is less than the required critical energy. Therefore, the additional excitation energy must be supplied by the kinetic energy of the incident neutron. The reason for this difference between fissile and fissionable materials is the so-called odd-even effect for nuclei. It has been observed that nuclei with even numbers of neutrons and/or protons are more stable than those with odd numbers. Therefore, adding a neutron to change a nucleus with an odd number of neutrons to a nucleus with an even number of neutrons produces an appreciably higher binding energy than adding a neutron to a nucleus already possessing an even number of neutrons. Some examples of nuclides requiring high energy neutrons to cause fission are thorium-232, uranium-238, and plutonium-240. Table 4 indicates the critical energy (E_{crit}) and the binding energy change for an added neutron (BE_n) to target nuclei of interest. For fission to be possible, the change in binding energy plus the kinetic energy must equal or exceed the critical energy ($\Delta BE + KE \geq E_{crit}$).

TABLE 4
Critical Energies Compared to Binding Energy of Last Neutron

Target Nucleus	Critical Energy E_{crit}	Binding Energy of Last Neutron BE_n	$BE_n - E_{crit}$
$^{232}_{90}Th$	7.5 MeV	5.4 MeV	-2.1 MeV
$^{238}_{92}U$	7.0 MeV	5.5 MeV	-1.5 MeV
$^{235}_{92}U$	6.5 MeV	6.8 MeV	+0.3 MeV
$^{233}_{92}U$	6.0 MeV	7.0 MeV	+1.0 MeV
$^{239}_{94}Pu$	5.0 MeV	6.6 MeV	+1.6 MeV

Uranium-235 fissions with thermal neutrons because the binding energy released by the absorption of a neutron is greater than the critical energy for fission; therefore uranium-235 is a fissile material. The binding energy released by uranium-238 absorbing a thermal neutron is less than the critical energy, so additional energy must be possessed by the neutron for fission to be possible. Consequently, uranium-238 is a fissionable material.

Fertile Material

All of the neutron absorption reactions that do not result in fission lead to the production of new nuclides through the process known as *transmutation*. These nuclides can, in turn, be transmuted again or may undergo radioactive decay to produce still different nuclides. The nuclides that are produced by this process are referred to as transmutation products. Because several of the fissile nuclides do not exist in nature, they can only be produced by nuclear reactions (transmutation). The target nuclei for such reactions are said to be fertile. *Fertile materials* are materials that can undergo transmutation to become fissile materials. Figure 19 traces the transmutation mechanism by which two fertile nuclides, thorium-232 and uranium-238, produce uranium-233 and plutonium-239, respectively.

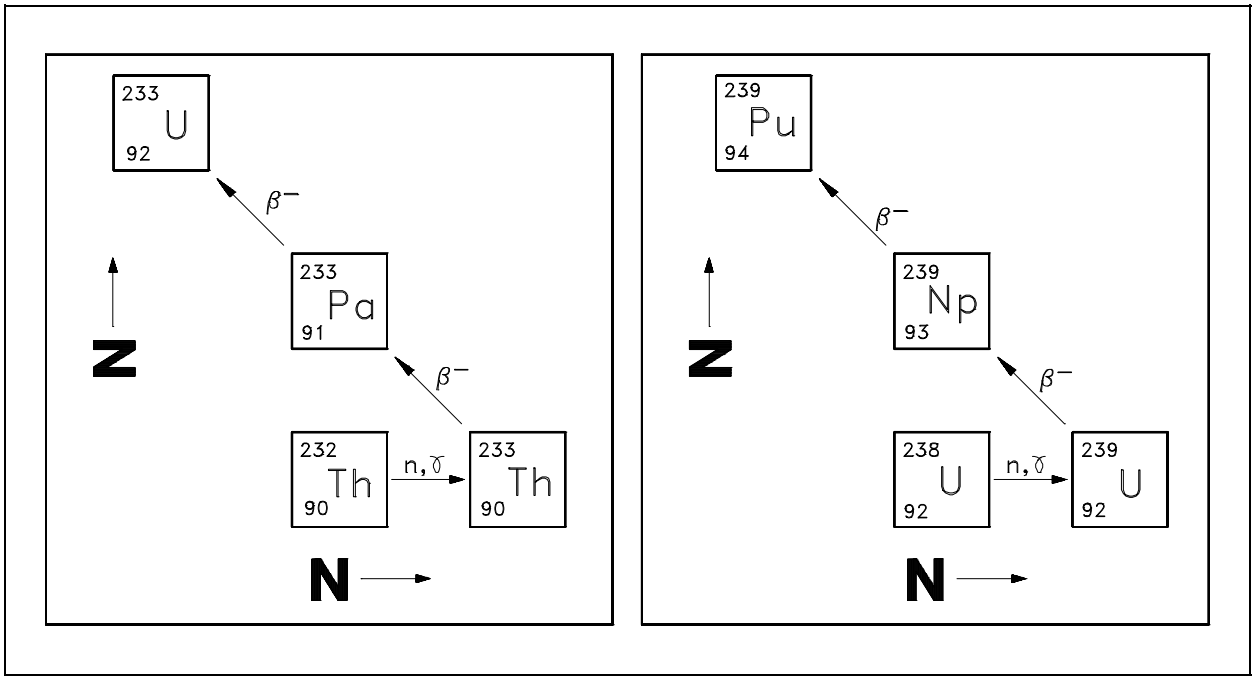


Figure 19 Conversion of Fertile Nuclides to Fissile Nuclides

If a reactor contains fertile material in addition to its fissile fuel, some new fuel will be produced as the original fuel is burned up. This is called *conversion*. Reactors that are specifically designed to produce fissionable fuel are called "breeder" reactors. In such reactors, the amount of fissionable fuel produced is greater than the amount of fuel burnup. If less fuel is produced than used, the process is called conversion, and the reactor is termed a "converter."

Binding Energy Per Nucleon (BE/A)

As the number of particles in a nucleus increases, the total binding energy also increases. The rate of increase, however, is not uniform. This lack of uniformity results in a variation in the amount of binding energy associated with each nucleon within the nucleus. This variation in the binding energy per nucleon (BE/A) is easily seen when the average BE/A is plotted versus atomic mass number (A), as shown in Figure 20.

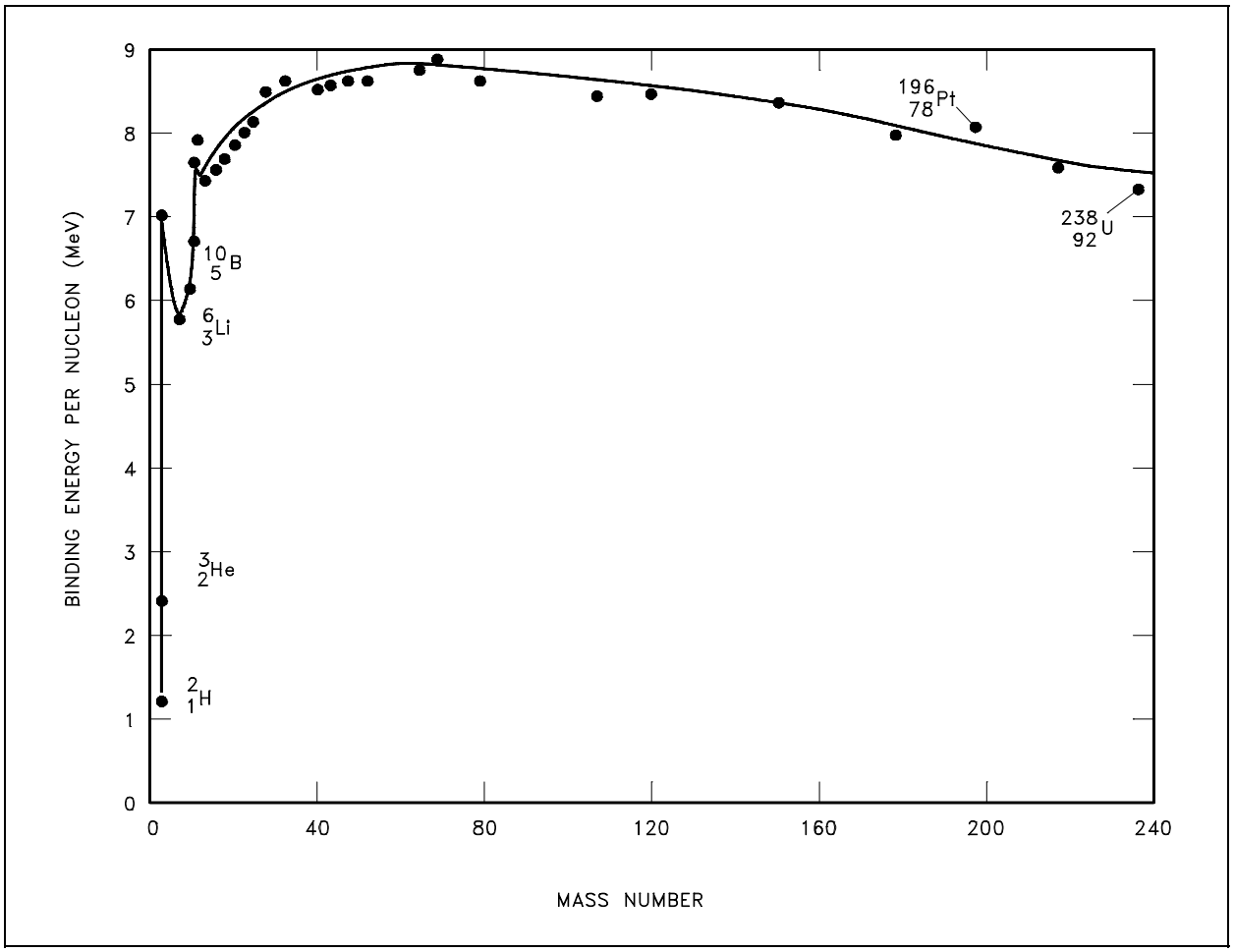


Figure 20 Binding Energy per Nucleon vs. Mass Number

Figure 20 illustrates that as the atomic mass number increases, the binding energy per nucleon decreases for A > 60. The BE/A curve reaches a maximum value of 8.79 MeV at A = 56 and decreases to about 7.6 MeV for A = 238. The general shape of the BE/A curve can be explained using the general properties of nuclear forces. The nucleus is held together by very short-range attractive forces that exist between nucleons. On the other hand, the nucleus is being forced apart by long range repulsive electrostatic (coulomb) forces that exist between all the protons in the nucleus.

As the atomic number and the atomic mass number increase, the repulsive electrostatic forces within the nucleus increase due to the greater number of protons in the heavy elements. To overcome this increased repulsion, the proportion of neutrons in the nucleus must increase to maintain stability. This increase in the neutron-to-proton ratio only partially compensates for the growing proton-proton repulsive force in the heavier, naturally occurring elements. Because the repulsive forces are increasing, less energy must be supplied, on the average, to remove a nucleon from the nucleus. The BE/A has decreased. The BE/A of a nucleus is an indication of its degree of stability. Generally, the more stable nuclides have higher BE/A than the less stable ones. The increase in the BE/A as the atomic mass number decreases from 260 to 60 is the primary reason for the energy liberation in the fission process. In addition, the increase in the BE/A as the atomic mass number increases from 1 to 60 is the reason for the energy liberation in the fusion process, which is the opposite reaction of fission.

The heaviest nuclei require only a small distortion from a spherical shape (small energy addition) for the relatively large coulomb forces forcing the two halves of the nucleus apart to overcome the attractive nuclear forces holding the two halves together. Consequently, the heaviest nuclei are easily fissionable compared to lighter nuclei.

Summary

The important information in this chapter is summarized on the following page.

Nuclear Fission Summary

- The fission process can be explained using the liquid drop model of a nucleus. In the ground state the nucleus is nearly spherical in shape. After the absorption of a neutron, the nucleus will be in an excited state and start to oscillate and become distorted. If the oscillations cause the nucleus to become shaped like a dumbbell, the repulsive electrostatic forces will overcome the short-range attractive nuclear forces, and the nucleus will split in two.

- Excitation energy is the amount of energy a nucleus has above its ground state.

- Critical energy is the minimum excitation energy that a nucleus must have before it can fission.

- Fissile material is material for which fission is possible with neutrons that have zero kinetic energy. Fissionable material is material for which fission caused by neutron absorption is possible provided the kinetic energy added with the binding energy is greater than the critical energy. Fertile material is material that can undergo transmutation to become fissile material.

- Transmutation is the process of neutron absorption and subsequent decay, which changes one nuclide to another nuclide. Conversion is the process of transmuting fertile material into fissile material in a reactor, where the amount of fissile material produced is less than the amount of fissile material consumed. Breeding is the same as conversion, except the amount of fissile material produced is more than the amount of fissile material consumed.

- The curve of binding energy per nucleon increases quickly through the light nuclides and reaches a maximum at a mass number of about 56. The curve decreases slowly for mass numbers greater than 60.

- The heaviest nuclei are easily fissionable because they require only a small distortion from the spherical shape to allow the coulomb forces to overcoming the attractive nuclear force, forcing the two halves of the nucleus apart.

- Uranium-235 fissions with thermal neutrons because the binding energy released by the absorption of a neutron is greater than the critical energy for fission. The binding energy released by uranium-238 absorbing a neutron is less than the critical energy, so additional kinetic energy must be possessed by the neutron for fission to be possible.

ENERGY RELEASE FROM FISSION

Fission of heavy nuclides converts a small amount of mass into an enormous amount of energy. The amount of energy released by fission can be determined based on either the change in mass that occurs during the reaction or by the difference in binding energy per nucleon between the fissile nuclide and the fission products.

EO 4.8 CHARACTERIZE the fission products in terms of mass groupings and radioactivity.

EO 4.9 Given the nuclides involved and their masses, CALCULATE the energy released from fission.

EO 4.10 Given the curve of Binding Energy per nucleon versus mass number, CALCULATE the energy released from fission.

Calculation of Fission Energy

Nuclear fission results in the release of enormous quantities of energy. It is necessary to be able to calculate the amount of energy that will be produced. The logical manner in which to pursue this is to first investigate a typical fission reaction such as the one listed below.

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow \left({}^{236}_{92}U\right)^{*} \rightarrow {}^{140}_{55}Cs + {}^{93}_{37}Rb + 3\left({}^{1}_{0}n\right) $$

It can be seen that when the compound nucleus splits, it breaks into two fission fragments, rubidium-93, cesium-140, and some neutrons. Both fission products then decay by multiple β^- emissions as a result of the high neutron-to-proton ratio possessed by these nuclides.

In most cases, the resultant fission fragments have masses that vary widely. Figure 21 gives the percent yield for atomic mass numbers. The most probable pair of fission fragments for the thermal fission of the fuel uranium-235 have masses of about 95 and 140. Note that the vertical axis of the fission yield curve is on a logarithmic scale. Therefore, the formation of fission fragments of mass numbers of about 95 and 140 is highly likely.

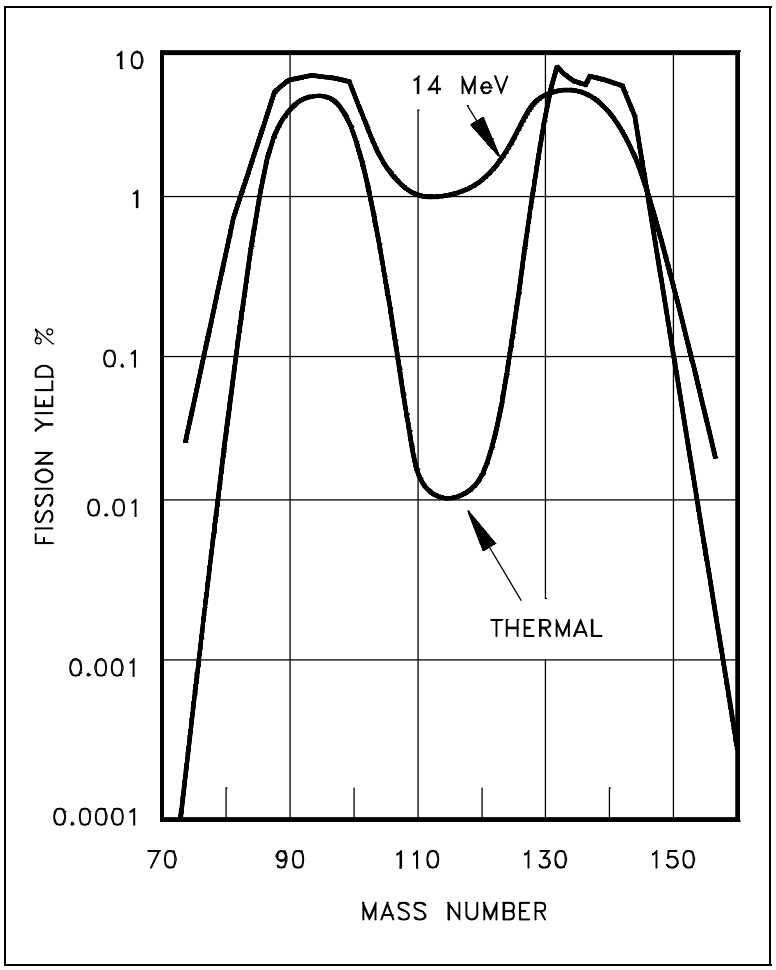


Figure 21 Uranium-235 Fission Yield vs. Mass Number

Referring now to the binding energy per nucleon curve (Figure 20), we can estimate the amount of energy released by our "typical" fission by plotting this reaction on the curve and calculating the change in binding energy (ΔBE) between the reactants on the left-hand side of the fission equation and the products on the right-hand side. Plotting the reactant and product nuclides on the curve shows that the total binding energy of the system after fission is greater than the total binding energy of the system before fission. When there is an increase in the total binding energy of a system, the system has become more stable by releasing an amount of energy equal to the increase in total binding energy of the system. Therefore, in the fission process, the energy liberated is equal to the increase in the total binding energy of the system.

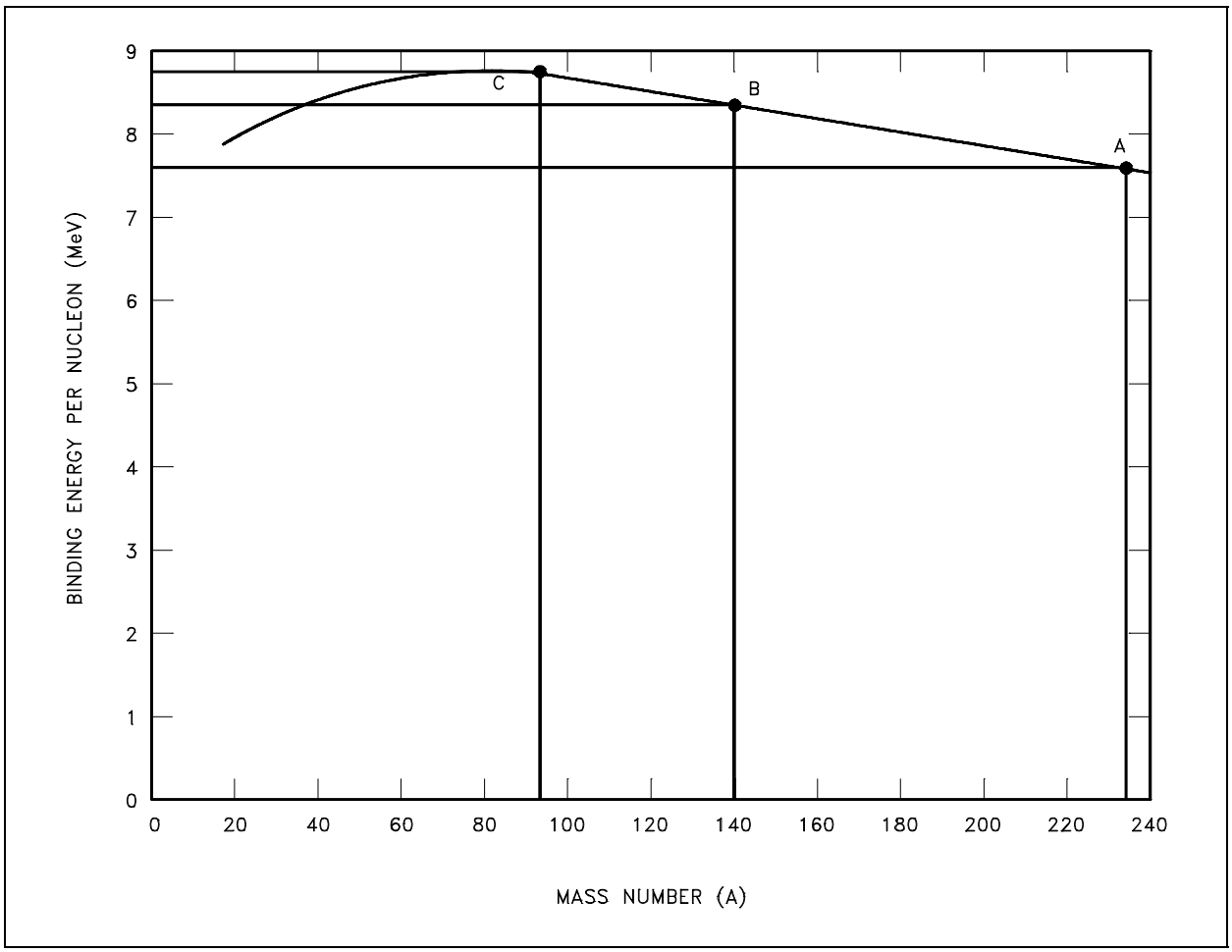


Figure 22 Change in Binding Energy for Typical Fission

Figure 22 graphically depicts that the binding energy per nucleon for the products (C, rubidium-93 and B, cesium-140) is greater than that for the reactant (A, uranium-235). The total binding energy for a nucleus can be found by multiplying the binding energy per nucleon by the number of nucleons.

TABLE 5
Binding Energies Calculated from Binding Energy per Nucleon Curve

Nuclide	B.E. per Nucleon (BE/A)	Mass Number (A)	Binding Energy (BE/A) x (A)
$^{93}_{37}Rb$	8.7 MeV	93	809 MeV
$^{140}_{55}Cs$	8.4 MeV	140	1176 MeV
$^{235}_{92}U$	7.6 MeV	235	1786 MeV

The energy released will be equivalent to the difference in binding energy (ΔBE) between the reactants and the products.

$$\begin{aligned}\Delta BE &= BE_{products} - BE_{reactants} \\ &= \left(BE_{Rb-93} + BE_{Cs-140}\right) - \left(BE_{U-235}\right) \\ &= (809\ MeV + 1176\ MeV) - 1786\ MeV \\ &= 199\ MeV\end{aligned}$$

The energy liberation during the fission process can also be explained from the standpoint of the conservation of mass-energy. During the fission process, there is a decrease in the mass of the system. There must, therefore, be energy liberated equal to the energy equivalent of the mass lost in the process. This method is more accurate than the previously illustrated method and is used when actually calculating the energy liberated during the fission process.

Again, referring to the "typical" fission reaction.

$$^{1}_{0}n + {}^{235}_{92}U \rightarrow \left({}^{236}_{92}U\right)^{*} \rightarrow {}^{140}_{55}Cs + {}^{93}_{37}Rb + 3\left({}^{1}_{0}n\right)$$

E_{Inst}, the instantaneous energy, is the energy released immediately after the fission process. It is equal to the energy equivalent of the mass lost in the fission process. It can be calculated as shown below.

Mass of the Reactants		Mass of the Products	
$^{235}_{92}U$	235.043924 amu	$^{93}_{37}Rb$	92.91699 amu
$^{1}_{0}n$	1.008665 amu	$^{140}_{55}Cs$	139.90910 amu
		$3\ (^{1}_{0}n)$	3.02599 amu
	236.052589 amu		235.85208 amu

$$\begin{aligned}\text{Mass difference} &= \text{Mass of Reactants} - \text{Mass of Products} \\ &= 236.052589\ amu - 235.85208\ amu \\ &= 0.200509\ amu\end{aligned}$$

This mass difference can be converted to an energy equivalent.

$$\begin{aligned}E_{Inst} &= 0.020059\ amu\left(\frac{931.5\ MeV}{amu}\right) \\ &= 186.8\ MeV\end{aligned}$$

The total energy released per fission will vary from the fission to the next depending on what fission products are formed, but the average total energy released per fission of uranium-235 with a thermal neutron is 200 MeV.

As illustrated in the preceding example, the majority of the energy liberated in the fission process is released immediately after the fission occurs and appears as the kinetic energy of the fission fragments, kinetic energy of the fission neutrons, and instantaneous gamma rays. The remaining energy is released over a period of time after the fission occurs and appears as kinetic energy of the beta, neutrino, and decay gamma rays.

Estimation of Decay Energy

In addition to this instantaneous energy release during the actual fission reaction, there is additional energy released when the fission fragments decay by β^- emission. This additional energy is called decay energy, E_{Decay}. The decay chains for rubidium-93 and cesium-140 are shown below.

$$^{93}_{37}\mathrm{Rb} \xrightarrow{\beta^-} {}^{93}_{38}\mathrm{Sr} \xrightarrow{\beta^-} {}^{93}_{39}\mathrm{Y} \xrightarrow{\beta^-} {}^{93}_{40}\mathrm{Zr} \xrightarrow{\beta^-} {}^{93}_{41}\mathrm{Nb}$$

$$^{140}_{55}\mathrm{Cs} \xrightarrow{\beta^-} {}^{140}_{56}\mathrm{Ba} \xrightarrow{\beta^-} {}^{140}_{57}\mathrm{La} \xrightarrow{\beta^-} {}^{140}_{58}\mathrm{Ce}$$

The energy released during the decay for each chain will be equivalent to the mass difference between the original fission product and the sum of the final stable nuclide and the beta particles emitted.

The energy released in the decay chain of rubidium-93 is calculated below.

$$E_{Decay} = \left[m_{Rb-93} - \left(m_{Nb-93} + 4\ m_{electron}\right)\right]\left(\frac{931.5\text{MeV}}{\text{amu}}\right)$$

$$= \left[92.91699\ \text{amu} - \left(92.90638\ \text{amu} + 4\ \left(0.0005486\ \text{amu}\right)\right)\right]\left(\frac{931.5\ \text{MeV}}{\text{amu}}\right)$$

$$= 0.008416\ \text{amu}\left(\frac{931.5\ \text{MeV}}{\text{amu}}\right)$$

$$= 7.84\ \text{MeV}$$

The energy released in the decay chain of cesium-140 is calculated below.

$$E_{Decay} = \left[m_{Rb-93} - \left(m_{Nb-93} + 3\ m_{electron}\right)\right]\left(\frac{931.5MeV}{amu}\right)$$

$$= \left[139.90910\ amu - \left(139.90543\ amu + 3\ \left(0.0005486\ amu\right)\right)\right]\left(\frac{931.5\ MeV}{amu}\right)$$

$$= 0.000202\ amu\left(\frac{931.5\ MeV}{amu}\right)$$

$$= 1.89\ MeV$$

The total decay energy is the sum of the energies of the two chains, or 9.73 MeV.

Distribution of Fission Energy

The average energy distribution for the energy released per fission with a thermal neutron in uranium-235 is shown in Tables 6 and 7.

TABLE 6
Instantaneous Energy from Fission

Kinetic Energy of Fission Products	167 Mev
Energy of Fission Neutrons	5 MeV
Instantaneous Gamma-ray Energy	5 MeV
Capture Gamma-ray Energy	10 MeV
Total Instantaneous Energy	187 MeV

TABLE 7
Delayed Energy from Fission

Beta Particles From Fission Products	7 Mev
Gamma-rays from Fission Products	6 MeV
Neutrinos	10 MeV
Total Delayed Energy	23 MeV

Because the 10 MeV of neutrino energy shown in Table 7 is not absorbed in the reactor, the average value of 200 MeV per fission is still accurate. Note in Table 6 that some fission neutrons undergo radiative capture and the resultant gamma ray emission provides an additional 10 MeV of instantaneous energy, which contributes to the total of 187 MeV instantaneous energy.

All of the energy released, with the exception of the neutrino energy, is ultimately transformed into heat through a number of processes. The fission fragments, with their high positive charge and kinetic energy, cause ionization directly as they rip orbital electrons from the surrounding atoms. In this ionization process, kinetic energy is transferred to the surrounding atoms of the fuel material, resulting in an increase in temperature. The beta particles and gamma rays also give up their energy through ionization, and the fission neutrons interact and lose their energy through elastic scattering. Of the 200 MeV released per fission, about seven percent (13 MeV) is released at some time after the instant of fission. When a reactor is shut down, fissions essentially cease, but energy is still being released from the decay of fission products. The heat produced by this decay energy is referred to as "decay heat." Although decay energy represents about seven percent of reactor heat production during reactor operation, once the reactor is shut down the decay heat production drops off quickly to a small fraction of its value while operating. The decay heat produced is significant, however, and systems must be provided to keep the reactor cool even after shutdown.

Summary

The important information in this chapter is summarized below.

Energy Release From Fission Summary

- Fission products have some general characteristics in common.
  They generally decay by β^- emission.
  The most common mass numbers are grouped near 95 and 140.
- The energy released by fission can be calculated based on the difference in mass between the masses of the reactants before fission and the fission fragments and fission neutrons after fission.
- Another method to determine the energy released by fission is based on the change in binding energy per nucleon between the fissile nuclide and the fission products.

INTERACTION OF RADIATION WITH MATTER

Different types of radiation interact with matter in widely different ways. A large, massive, charged alpha particle cannot penetrate a piece of paper and even has a limited range in dry air. A neutrino, at the other extreme, has a low probability of interacting with any matter, even if it passed through the diameter of the earth.

EO 5.1 DESCRIBE interactions of the following with matter:

a. Alpha particle
b. Beta particle
c. Positron
d. Neutron

EO 5.2 DESCRIBE the following ways that gamma radiation interacts with matter:

a. Photoelectric effect
b. Compton scattering
c. Pair production

Interaction of Radiation With Matter

Radiation can be classified into two general groups, charged and uncharged; therefore, it may be expected that interactions with matter fall into two general types. Charged particles directly ionize the media through which they pass, while uncharged particles and photons can cause ionization only indirectly or by secondary radiation.

A moving charged particle has an electrical field surrounding it, which interacts with the atomic structure of the medium through which it is passing. This interaction decelerates the particle and accelerates electrons in the atoms of the medium. The accelerated electrons may acquire enough energy to escape from the parent atom. This process, whereby radiation "strips" off orbital electrons, is called ionization. Uncharged moving particles have no electrical field, so they can only lose energy and cause ionization by such means as collisions or scattering. A photon can lose energy by the photoelectric effect, Compton effect, or pair production.

Because ionizing radiation creates ions in pairs, the intensity of ionization or the specific ionization is defined as the number of ion-pairs formed per centimeter of travel in a given material. The amount of ionization produced by a charged particle per unit path length, which is a measure of its ionizing power, is roughly proportional to the particle's mass and the square of its charge as illustrated in the equation below.

$$I = \frac{m\ z^2}{K.E.}$$

where:

I is the ionizing power
m is the mass of the particle
z is the number of unit charges it carries
K.E. is its kinetic energy

Since m for an alpha particle is about 7300 times as large as m for a beta particle, and z is twice as great, an alpha will produce much more ionization per unit path length than a beta particle of the same energy. This phenomenon occurs because the larger alpha particle moves slower for a given energy and thus acts on a given electron for a longer time.

Alpha Radiation

Alpha radiation is normally produced from the radioactive decay of heavy nuclides and from certain nuclear reactions. The alpha particle consists of 2 neutrons and 2 protons, so it is essentially the same as the nucleus of a helium atom. Because it has no electrons, the alpha particle has a charge of +2. This positive charge causes the alpha particle to strip electrons from the orbits of atoms in its vicinity. As the alpha particle passes through material, it removes electrons from the orbits of atoms it passes near. Energy is required to remove electrons and the energy of the alpha particle is reduced by each reaction. Eventually the particle will expend its kinetic energy, gain 2 electrons in orbit, and become a helium atom. Because of its strong positive charge and large mass, the alpha particle deposits a large amount of energy in a short distance of travel. This rapid, large deposition of energy limits the penetration of alpha particles. The most energetic alpha particles are stopped by a few centimeters of air or a sheet of paper.

Beta-Minus Radiation

A beta-minus particle is an electron that has been ejected at a high velocity from an unstable nucleus. An electron has a small mass and an electrical charge of -1. Beta particles cause ionization by displacing electrons from atom orbits. The ionization occurs from collisions with orbiting electrons. Each collision removes kinetic energy from the beta particle, causing it to slow down. Eventually the beta particle will be slowed enough to allow it to be captured as an

orbiting electron in an atom. Although more penetrating than the alpha, the beta is relatively easy to stop and has a low power of penetration. Even the most energetic beta radiation can be stopped by a few millimeters of metal.

Positron Radiation

Positively charged electrons are called positrons. Except for the positive charge, they are identical to beta-minus particles and interact with matter in a similar manner. Positrons are very short-lived, however, and quickly are annihilated by interaction with a negatively charged electron, producing two gammas with a combined energy (calculated below) equal to the rest mass of the positive and negative electrons.

$$2 \text{ electrons} \left(\frac{0.000549 \text{ amu}}{\text{electron}} \right) \left(\frac{931.5 \text{ MeV}}{\text{amu}} \right) = 1.02 \text{ MeV}$$

Neutron Radiation

Neutrons have no electrical charge. They have nearly the same mass as a proton (a hydrogen atom nucleus). A neutron has hundreds of times more mass than an electron, but 1/4 the mass of an alpha particle. The source of neutrons is primarily nuclear reactions, such as fission, but they may also be produced from the decay of radioactive nuclides. Because of its lack of charge, the neutron is difficult to stop and has a high penetrating power.

Neutrons are attenuated (reduced in energy and numbers) by three major interactions, elastic scatter, inelastic scatter, and absorption. In elastic scatter, a neutron collides with a nucleus and bounces off. This reaction transmits some of the kinetic energy of the neutron to the nucleus of the atom, resulting in the neutron being slowed, and the atom receives some kinetic energy (motion). This process is sometimes referred to as "the billiard ball effect."

As the mass of the nucleus approaches the mass of the neutron, this reaction becomes more effective in slowing the neutron. Hydrogenous material attenuates neutrons most effectively.

In the inelastic scatter reaction, the same neutron/nucleus collision occurs as in elastic scatter. However, in this reaction, the nucleus receives some internal energy as well as kinetic energy. This slows the neutron, but leaves the nucleus in an excited state. When the nucleus decays to its original energy level, it normally emits a gamma ray.

In the absorption reaction, the neutron is actually absorbed into the nucleus of an atom. The neutron is captured, but the atom is left in an excited state. If the nucleus emits one or more gamma rays to reach a stable level, the process is called radiative capture. This reaction occurs at most neutron energy levels, but is more probable at lower energy levels.

Gamma Radiation

Gamma radiation is electromagnetic radiation. It is commonly referred to as a gamma ray and is very similar to an x-ray. The difference is that gamma rays are emitted from the nucleus of an atom, and x-rays are produced by orbiting electrons. The x-ray is produced when orbiting electrons move to a lower energy orbit or when fast-moving electrons approaching an atom are deflected and decelerated as they react with the atom's electrical field (called Bremsstrahlung). The gamma ray is produced by the decay of excited nuclei and by nuclear reactions. Because the gamma ray has no mass and no charge, it is difficult to stop and has a very high penetrating power. A small fraction of the original gamma stream will pass through several feet of concrete or several meters of water.

There are three methods of attenuating gamma rays. The first method is referred to as the photo-electric effect. When a low energy gamma strikes an atom, the total energy of the gamma is expended in ejecting an electron from orbit. The result is ionization of the atom and expulsion of a high energy electron. This reaction is most predominant with low energy gammas interacting in materials with high atomic weight and rarely occurs with gammas having an energy above 1 MeV. Annihilation of the gamma results. Any gamma energy in excess of the binding energy of the electron is carried off by the electron in the form of kinetic energy.

The second method of attenuation of gammas is called Compton scattering. The gamma interacts with an orbital or free electron; however, in this case, the photon loses only a fraction of its energy. The actual energy loss depending on the scattering angle of the gamma. The gamma continues on at lower energy, and the energy difference is absorbed by the electron. This reaction becomes important for gamma energies of about 0.1 MeV and higher.

At higher energy levels, a third method of attenuation is predominant. This method is pair-production. When a high energy gamma passes close enough to a heavy nucleus, the gamma completely disappears, and an electron and a positron are formed. For this reaction to take place, the original gamma must have at least 1.02 MeV energy. Any energy greater than 1.02 MeV becomes kinetic energy shared between the electron and positron. The probability of pair-production increases significantly for higher energy gammas.

Summary

The important information in this chapter is summarized below.

Interaction of Radiation with Matter Summary

- An alpha particle deposits a large amount of energy in a short distance of travel due to its large mass and charge.
- Beta-minus particles interact with the electrons orbiting the nucleus of atoms, causing ionization by displacing the electrons. The beta particle loses energy with each interaction. After the beta particle loses enough energy, it is captured in the orbital shells of an atom.
- Positrons interact with matter much the same way as beta minus particles. After the positron has lost most of its energy by ionizing atoms, it is annihilated by interaction with an electron. The electron-positron pair disappear and are replaced by two gammas, each with the energy equivalent of the mass of an electron (0.51 MeV).
- Neutrons interact with matter by elastic scattering, inelastic scattering, or absorption.
- Photoelectric effect is where a gamma interacts with an electron orbiting an atom. The entire energy of the gamma is transferred to the electron, and the electron is ejected from its orbit.
- In Compton scattering a gamma interacts with an orbital electron, but only part of the gamma energy is transferred to the electron. The electron is ejected from its orbit, and the gamma is scattered off at a lower energy.
- In pair-production, a gamma interacts with the electric field of a nucleus and is converted into an electron-positron pair. The gamma must have an energy greater than 1.02 MeV for this to occur.

Intentionally Left Blank

Department of Energy
Fundamentals Handbook

NUCLEAR PHYSICS AND REACTOR THEORY
Module 2
Reactor Theory (Neutron Characteristics)

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont.)

LIST OF FIGURES

LIST OF TABLES

REFERENCES

- Foster, Arthur R. and Wright, Robert L. Jr., Basic Nuclear Engineering, 3rd Edition, Allyn and Bacon, Inc., 1977.
- Jacobs, A.M., Kline, D.E., and Remick, F. J., Basic Principles of Nuclear Science and Reactors, Van Nostrand Company, Inc., 1960.
- Kaplan, Irving, Nuclear Physics, 2nd Edition, Addison-Wesley Company, 1962.
- Knief, Ronald Allen, Nuclear Energy Technology: Theory and Practice of Commercial Nuclear Power, McGraw-Hill, 1981.
- Lamarsh, John R., Introduction to Nuclear Engineering, Addison-Wesley Company, 1977.
- Lamarsh, John R., Introduction to Nuclear Reactor Theory, Addison-Wesley Company, 1972.
- General Electric Company, Nuclides and Isotopes: Chart of the Nuclides, 14th Edition, General Electric Company, 1989.
- Academic Program for Nuclear Power Plant Personnel, Volume III, Columbia, MD, General Physics Corporation, Library of Congress Card #A 326517, 1982.
- Glasstone, Samuel, Sourcebook on Atomic Energy, Robert F. Krieger Publishing Company, Inc., 1979.
- Glasstone, Samuel and Sesonske, Alexander, Nuclear Reactor Engineering, 3rd Edition, Van Nostrand Reinhold Company, 1981.

TERMINAL OBJECTIVE

1.0 Without references, **EXPLAIN** how neutron sources produce neutrons.

ENABLING OBJECTIVES

1.1 **DEFINE** the following terms:

a. Intrinsic neutron source
b. Installed neutron source

1.2 **LIST** three examples of reactions that produce neutrons in intrinsic neutron sources.

1.3 **LIST** three examples of reactions that produce neutrons in installed neutron sources.

TERMINAL OBJECTIVE

2.0 Given the necessary information for calculations, **EXPLAIN** basic concepts in reactor physics and perform calculations.

ENABLING OBJECTIVES

2.1 **DEFINE** the following terms:

a.	Atom density	d.	Barn
b.	Neutron flux	e.	Macroscopic cross section
c.	Microscopic cross section	f.	Mean free path

2.2 **EXPRESS** macroscopic cross section in terms of microscopic cross section.

2.3 **DESCRIBE** how the absorption cross section of typical nuclides varies with neutron energy at energies below the resonance absorption region.

2.4 **DESCRIBE** the cause of resonance absorption in terms of nuclear energy levels.

2.5 **DESCRIBE** the energy dependence of resonance absorption peaks for typical light and heavy nuclei.

2.6 **EXPRESS** mean free path in terms of macroscopic cross section.

2.7 Given the number densities (or total density and component fractions) and microscopic cross sections of components, **CALCULATE** the macroscopic cross section for a mixture.

2.8 **CALCULATE** a macroscopic cross section given a material density, atomic mass, and microscopic cross section.

2.9 **EXPLAIN** neutron shadowing or self-shielding.

2.10 Given the neutron flux and macroscopic cross section, **CALCULATE** the reaction rate.

2.11 **DESCRIBE** the relationship between neutron flux and reactor power.

ENABLING OBJECTIVES (Cont.)

2.12 **DEFINE** the following concepts:

a. Thermalization
b. Moderator
c. Moderating ratio
d. Average logarithmic energy decrement
e. Macroscopic slowing down power

2.13 **LIST** three desirable characteristics of a moderator.

2.14 Given an average fractional energy loss per collision, **CALCULATE** the energy loss after a specified number of collisions.

TERMINAL OBJECTIVE

3.0 Without references, **EXPLAIN** the production process and effects on fission of prompt and delayed neutrons.

ENABLING OBJECTIVES

3.1 **STATE** the origin of prompt neutrons and delayed neutrons.

3.2 **STATE** the approximate fraction of neutrons that are born as delayed neutrons from the fission of the following nuclear fuels:

a. Uranium-235
b. Plutonium-239

3.3 **EXPLAIN** the mechanism for production of delayed neutrons.

3.4 **EXPLAIN** prompt and delayed neutron generation times.

3.5 Given prompt and delayed neutron generation times and delayed neutron fraction, **CALCULATE** the average generation time.

3.6 **EXPLAIN** the effect of delayed neutrons on reactor control.

TERMINAL OBJECTIVE

4.0 Without references, **DESCRIBE** the neutron energy spectrum for the type of reactor presented in this module.

ENABLING OBJECTIVES

4.1 **STATE** the average energy at which prompt neutrons are produced.

4.2 **DESCRIBE** the neutron energy spectrum in the following reactors:

a. Fast reactor
b. Thermal reactor

4.3 **EXPLAIN** the reason for the particular shape of the fast, intermediate, and slow energy regions of the neutron flux spectrum for a thermal reactor.

NEUTRON SOURCES

Neutrons from a variety of sources are always present in a reactor core. This is true even when the reactor is shut down. Some of these neutrons are produced by naturally occurring (intrinsic) neutron sources, while others may be the result of fabricated (installed) neutron sources that are incorporated into the design of the reactor. The neutrons produced by sources other than neutron-induced fission are often grouped together and classified as source neutrons.

EO 1.1 DEFINE the following terms:

a. Intrinsic neutron source
b. Installed neutron source

EO 1.2 LIST three examples of reactions that produce neutrons in intrinsic neutron sources.

EO 1.3 LIST three examples of reactions that produce neutrons in installed neutron sources.

Neutron Sources

In addition to neutron-induced fission, neutrons are produced by other reactions. The neutrons produced by reactions other than neutron-induced fission are called *source* neutrons. Source neutrons are important because they ensure that the neutron population remains high enough to allow a visible indication of neutron level on the most sensitive monitoring instruments while the reactor is shutdown and during the startup sequence. This verifies instrument operability and allows monitoring of neutron population changes. Source neutrons can be classified as either intrinsic or installed neutron sources.

Intrinsic Neutron Sources

Some neutrons will be produced in the materials present in the reactor due to a variety of unavoidable reactions that occur because of the nature of these materials. *Intrinsic neutron sources* are those neutron-producing reactions that always occur in reactor materials.

A limited number of neutrons will always be present, even in a reactor core that has never been operated, due to spontaneous fission of some heavy nuclides that are present in the fuel. Uranium-238, uranium-235, and plutonium-239 undergo spontaneous fission to a limited extent. Uranium-238, for example, yields almost 60 neutrons per hour per gram. Table 1 illustrates a comparison of the rate at which different heavy nuclides produce neutrons by spontaneous fission. Californium-252 is not an intrinsic neutron source, but will be discussed in the section on installed neutron sources.

TABLE 1
Neutron Production by Spontaneous Fission

Nuclide	$T_{1/2}$ (Fission)	$T_{1/2}$ (α-decay)	neutrons/sec/gram
$^{235}_{92}U$	1.8×10^{17} years	6.8×10^{8} years	8.0×10^{-4}
$^{238}_{92}U$	8.0×10^{15} years	4.5×10^{9} years	1.6×10^{-2}
$^{239}_{94}Pu$	5.5×10^{5} years	2.4×10^{4} years	3.0×10^{-2}
$^{240}_{94}Pu$	1.2×10^{11} years	6.6×10^{3} years	1.0×10^{3}
$^{252}_{98}Cf$	66.0 years	2.65 years	2.3×10^{12}

Another intrinsic neutron source is a reaction involving natural boron and fuel. In some reactors, natural boron is loaded into the reactor core as a neutron absorber to improve reactor control or increase core life-time. Boron-11 (80.1% of natural boron) undergoes a reaction with the alpha particle emitted by the radioactive decay of heavy nuclides in the fuel to yield a neutron as shown below.

$$^{11}_{5}B + ^{4}_{2}\alpha \rightarrow ^{14}_{7}N + ^{1}_{0}n$$

The boron-11 must be mixed with, or in very close proximity to, the fuel for this reaction because of the short path length of the alpha particle. For a reactor core with this configuration, this (α,n) reaction is an important source of neutrons for reactor startup.

In a reactor that has been operated, another source of neutrons becomes significant. Neutrons may be produced by the interaction of a gamma ray and a deuterium nucleus. This reaction is commonly referred to as a photoneutron reaction because it is initiated by electromagnetic radiation and results in the production of a neutron. The photoneutron reaction is shown below.

$$^{2}_{1}H + ^{0}_{0}\gamma \rightarrow \left(^{2}_{1}H\right)^{*} \rightarrow ^{1}_{1}H + ^{1}_{0}n$$

There is an abundant supply of high energy gammas in a reactor that has been operated because many of the fission products are gamma emitters. All water-cooled reactors have some deuterium present in the coolant in the reactor core because a small fraction of natural hydrogen is the isotope deuterium. The atom percentage of deuterium in the water ranges from close to the naturally occurring value (0.015%) for light water reactors to above 90% deuterium for heavy water reactors. Therefore, the required conditions for production of photoneutrons exist. The supply of gamma rays decreases with time after shutdown as the gamma emitters decay; therefore, the photoneutron production rate also decreases. In a few particular reactors, additional D_2O (heavy water) may be added to the reactor to increase the production of photoneutrons following a long shutdown period.

Installed Neutron Sources

Because intrinsic neutron sources can be relatively weak or dependent upon the recent power history of the reactor, many reactors have artificial sources of neutrons installed. These neutron sources ensure that shutdown neutron levels are high enough to be detected by the nuclear instruments at all times. This provides a true picture of reactor conditions and any change in these conditions. An *installed neutron source* is an assembly placed in or near the reactor for the sole purpose of producing source neutrons.

One strong source of neutrons is the artificial nuclide californium-252, which emits neutrons at the rate of about 2×10^{12} neutrons per second per gram as the result of spontaneous fission. Important drawbacks for some applications may be its high cost and its short half-life (2.65 years).

Many installed neutron sources use the (α,n) reaction with beryllium. These sources are composed of a mixture of metallic beryllium (100% beryllium-9) with a small quantity of an alpha particle emitter, such as a compound of radium, polonium, or plutonium. The reaction that occurs is shown below.

$$ {}^{9}_{4}\mathrm{Be} + {}^{4}_{2}\alpha \rightarrow \left({}^{13}_{6}\mathrm{C}\right)^{*} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n} $$

The beryllium is intimately (homogeneously) mixed with the alpha emitter and is usually enclosed in a stainless steel capsule.

Another type of installed neutron source that is widely used is a photoneutron source that employs the (γ,n) reaction with beryllium. Beryllium is used for photoneutron sources because its stable isotope beryllium-9 has a weakly attached last neutron with a binding energy of only 1.66 MeV. Thus, a gamma ray with greater energy than 1.66 MeV can cause neutrons to be ejected by the (γ,n) reaction as shown below.

$$ {}^{9}_{4}\mathrm{Be} + {}^{0}_{0}\gamma \rightarrow \left({}^{9}_{4}\mathrm{Be}\right)^{*} \rightarrow {}^{8}_{4}\mathrm{Be} + {}^{1}_{0}\mathrm{n} $$

Many startup sources of this type use antimony and beryllium because after activation with neutrons the radioactive antimony becomes an emitter of high energy gammas. The photoneutron sources of this type are constructed somewhat differently from the (α,n) types. One design incorporates a capsule of irradiated antimony enclosed in a beryllium sleeve. The entire assembly is then encased in a stainless steel cladding. A large reactor may have several neutron sources of this type installed within the core.

Summary

The important information in this chapter is summarized below.

Neutron Sources Summary

- Intrinsic neutron sources are sources of neutrons from materials that are in the reactor for other purposes such as fuel, burnable poison, or moderator.
- Installed neutron sources are materials or components placed in the reactor specifically for the purpose of producing source neutrons.
- Examples of intrinsic neutron sources are listed below.

  Spontaneous fission of heavy nuclides in fuel, such as uranium-238, uranium-235, and plutonium-239, results in fission fragments and free neutrons.

  Boron-11 mixed with the fuel undergoes an alpha-neutron reaction and becomes nitrogen-14.

  Deuterium present in the reactor coolant undergoes a gamma-neutron reaction and becomes hydrogen-1.
- Examples of installed neutron sources are listed below.

  Spontaneous fission of californium-252 results in fission fragments and free neutrons.

  Beryllium-9 undergoes an alpha-neutron reaction (alpha from the decay of plutonium, polonium, or radium) and becomes carbon-12.

  Beryllium-9 undergoes a gamma-neutron reaction (high energy gamma from decay of antimony-124) and becomes beryllium-8.

NUCLEAR CROSS SECTIONS AND NEUTRON FLUX

To determine the frequency of neutron interactions, it is necessary to describe the availability of neutrons to cause interaction and the probability of a neutron interacting with material. The availability of neutrons and the probability of interaction are quantified by the neutron flux and nuclear cross section.

EO 2.1 DEFINE the following terms:

a. Atom density
b. Neutron flux
c. Microscopic cross section
d. Barn
e. Macroscopic cross section
f. Mean free path

EO 2.2 EXPRESS macroscopic cross section in terms of microscopic cross section.

EO 2.3 DESCRIBE how the absorption cross section of typical nuclides varies with neutron energy at energies below the resonance absorption region.

EO 2.4 DESCRIBE the cause of resonance absorption in terms of nuclear energy levels.

EO 2.5 DESCRIBE the energy dependence of resonance absorption peaks for typical light and heavy nuclei.

EO 2.6 EXPRESS mean free path in terms of macroscopic cross section.

EO 2.7 Given the number densities (or total density and component fractions) and microscopic cross sections of components, CALCULATE the macroscopic cross section for a mixture.

EO 2.8 CALCULATE a macroscopic cross section given a material density, atomic mass, and microscopic cross section.

EO 2.9 EXPLAIN neutron shadowing or self-shielding.

Introduction

Fission neutrons are born with an average energy of about 2 MeV. These fast neutrons interact with the reactor core materials in various absorption and scattering reactions. Collisions that result in scattering are useful in slowing neutrons to thermal energies. Thermal neutrons may be absorbed by fissile nuclei to produce more fissions or be absorbed in fertile material for conversion to fissionable fuel. Absorption of neutrons in structural components, coolant, and other non-fuel material results in the removal of neutrons without fulfilling any useful purpose.

To safely and efficiently operate a nuclear reactor it is necessary to predict the probability that a particular absorption or scattering reaction will occur. Once these probabilities are known, if the availability of neutrons can be determined, then the rate at which these nuclear reactions take place can be predicted.

Atom Density

One important property of a material is the atom density. The *atom density* is the number of atoms of a given type per unit volume of the material. To calculate the atom density of a substance use Equation (2-1).

$$\mathbf{N} = \frac{\rho \, \mathbf{N_A}}{\mathbf{M}} \qquad (2\text{-}1)$$

where:

N = atom density (atoms/cm^3)
ρ = density (g/cm^3)
N_A = Avogadro's number (6.022×10^{23} atoms/mole)
M = gram atomic weight

Example:

A block of aluminum has a density of 2.699 g/cm^3. If the gram atomic weight of aluminum is 26.9815 g, calculate the atom density of the aluminum.

Solution:

$$N = \frac{\rho N_A}{M}$$

$$= \frac{2.699 \ \frac{g}{cm^3} \left(6.022 \times 10^{23} \ \frac{atoms}{mole} \right)}{26.9815 \ \frac{g}{mole}}$$

$$= 6.024 \times 10^{22} \ \frac{atoms}{cm^3}$$

Cross Sections

The probability of a neutron interacting with a nucleus for a particular reaction is dependent upon not only the kind of nucleus involved, but also the energy of the neutron. Accordingly, the absorption of a thermal neutron in most materials is much more probable than the absorption of a fast neutron. Also, the probability of interaction will vary depending upon the type of reaction involved.

The probability of a particular reaction occurring between a neutron and a nucleus is called the *microscopic cross section* (σ) of the nucleus for the particular reaction. This cross section will vary with the energy of the neutron. The microscopic cross section may also be regarded as the effective area the nucleus presents to the neutron for the particular reaction. The larger the effective area, the greater the probability for reaction.

Because the microscopic cross section is an area, it is expressed in units of area, or square centimeters. A square centimeter is tremendously large in comparison to the effective area of a nucleus, and it has been suggested that a physicist once referred to the measure of a square centimeter as being "as big as a barn" when applied to nuclear processes. The name has persisted and microscopic cross sections are expressed in terms of *barns*. The relationship between barns and cm^2 is shown below.

1 barn = 10^{-24} cm^2

Whether a neutron will interact with a certain volume of material depends not only on the microscopic cross section of the individual nuclei but also on the number of nuclei within that volume. Therefore, it is necessary to define another kind of cross section known as the macroscopic cross section (Σ). The *macroscopic cross section* is the probability of a given reaction occurring per unit travel of the neutron. Σ is related to the microscopic cross section (σ) by the relationship shown below.

$$\Sigma = N \sigma \tag{2-2}$$

where:

Σ = macroscopic cross section (cm^{-1})
N = atom density of material ($atoms/cm^3$)
σ = microscopic cross-section (cm^2)

The difference between the microscopic and macroscopic cross sections is extremely important and is restated for clarity. The microscopic cross section (σ) represents the effective target area that a single nucleus presents to a bombarding particle. The units are given in barns or cm^2. The macroscopic cross section (Σ) represents the effective target area that is presented by all of the nuclei contained in 1 cm^3 of the material. The units are given as 1/cm or cm^{-1}.

A neutron interacts with an atom of the material it enters in two basic ways. It will either interact through a scattering interaction or through an absorption reaction. The probability of a neutron being absorbed by a particular atom is the microscopic cross section for absorption, σ_a. The probability of a neutron scattering off of a particular nucleus is the microscopic cross section for scattering, σ_s. The sum of the microscopic cross section for absorption and the microscopic cross section for scattering is the total microscopic cross section, σ_T.

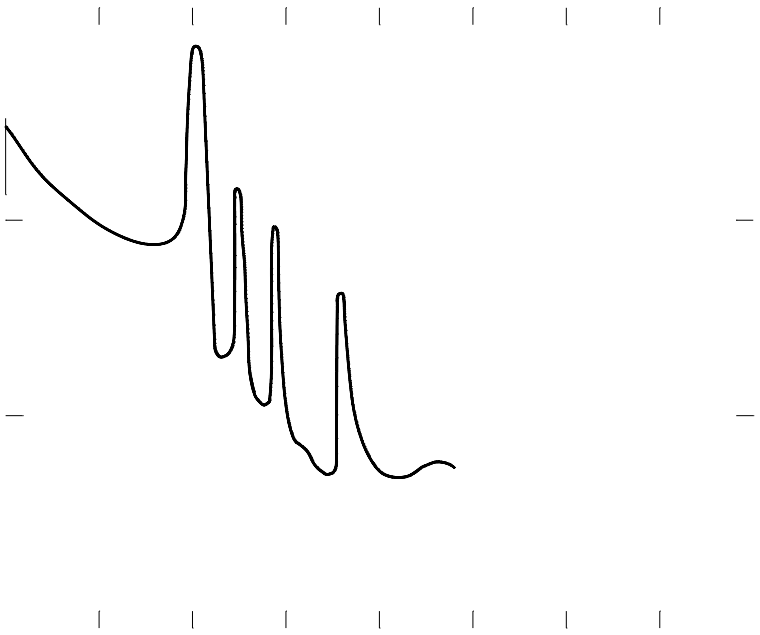
$$\sigma_T = \sigma_a + \sigma_s$$

Both the absorption and the scattering microscopic cross sections can be further divided. For instance, the scattering cross section is the sum of the elastic scattering cross section (σ_{se}) and the inelastic scattering cross section (σ_{si}).

$$\sigma_s = \sigma_{se} + \sigma_{si}$$

The microscopic absorption cross section (σ_a) includes all reactions except scattering. However, for most purposes it is sufficient to merely separate it into two categories, fission (σ_f) and capture (σ_c). Radiative capture of neutrons was described in the Neutron Interactions chapter of Module 1.

$$\sigma_a = \sigma_f + \sigma_c$$



Assuming that uranium-236 has a nuclear quantum energy level at 6.8 MeV above its ground state, calculate the kinetic energy a neutron must possess to undergo resonant absorption in uranium-235 at this resonance energy level.

BE = [Mass(^{235}U) + Mass(neutron) - Mass(^{236}U)] x 931 MeV/amu

BE = (235.043925 + 1.008665 - 236.045563) x 931 MeV/amu

BE = (0.007025 amu) x 931 MeV/amu = 6.54 MeV

6.8 MeV - 6.54 MeV = 0.26 MeV

The difference between the binding energy and the quantum energy level equals the amount of kinetic energy the neutron must possess. The typical heavy nucleus will have many closely-spaced resonances starting in the low energy (eV) range. This is because heavy nuclei are complex and have more possible configurations and corresponding energy states. Light nuclei, being less complex, have fewer possible energy states and fewer resonances that are sparsely distributed at higher energy levels.

For higher neutron energies, the absorption cross section steadily decreases as the energy of the neutron increases. This is called the "fast neutron region." In this region the absorption cross sections are usually less than 10 barns.

With the exception of hydrogen, for which the value is fairly large, the elastic scattering cross sections are generally small, for example, 5 barns to 10 barns. This is close to the magnitude of the actual geometric cross sectional area expected for atomic nuclei. In potential scattering, the cross section is essentially constant and independent of neutron energy. Resonance elastic scattering and inelastic scattering exhibit resonance peaks similar to those associated with absorption cross sections. The resonances occur at lower energies for heavy nuclei than for light nuclei. In general, the variations in scattering cross sections are very small when compared to the variations that occur in absorption cross sections.

Mean Free Path

If a neutron has a certain probability of undergoing a particular interaction in one centimeter of travel, then the inverse of this value describes how far the neutron will travel (in the average case) before undergoing an interaction. This average distance traveled by a neutron before interaction is known as the *mean free path* for that interaction and is represented by the symbol λ. The relationship between the mean free path (λ) and the macroscopic cross section (Σ) is shown below.

$$\lambda = \frac{1}{\Sigma} \qquad (2\text{-}3)$$

Calculation of Macroscopic Cross Section and Mean Free Path

Most materials are composed of several elements, and because most elements are composed of several isotopes, most materials involve many cross sections, one for each isotope involved. Therefore, to include all the isotopes within a given material, it is necessary to determine the macroscopic cross section for each isotope and then sum all the individual macroscopic cross sections. Equation (2-4) can be used to determine the macroscopic cross section for a composite material.

$$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + N_3 \sigma_3 + ... N_n \sigma_n \quad (2\text{-}4)$$

where:

N_n = the number of nuclei per cm^3 of the n^{th} element
σ_n = the microscopic cross section of the n^{th} element

The following example problems illustrate the calculation of the macroscopic cross section for a single element and for combinations of materials.

Example 1:

Find the macroscopic thermal neutron absorption cross section for iron, which has a density of 7.86 g/cm^3. The microscopic cross section for absorption of iron is 2.56 barns and the gram atomic weight is 55.847 g.

Solution:

Step 1: Using Equation (2-1), calculate the atom density of iron.

$$N = \frac{\rho N_A}{M}$$

$$= \frac{7.86 \frac{g}{cm^3} \left(6.022 \times 10^{23} \frac{atoms}{mole} \right)}{55.847 \frac{g}{mole}}$$

$$= 8.48 \times 10^{22} \frac{atoms}{cm^3}$$

Step 2: Use this atom density in Equation (2-2) to calculate the macroscopic cross section.

$$\Sigma_a = N \sigma_a$$

$$= 8.48 \times 10^{22} \frac{atoms}{cm^3} (2.56 \text{ barns}) \left(\frac{1 \times 10^{-24} cm^2}{1 \text{ barn}} \right)$$

$$= 0.217 \ cm^{-1}$$

Example 2:

An alloy is composed of 95% aluminum and 5% silicon (by weight). The density of the alloy is 2.66 g/cm^3. Properties of aluminum and silicon are shown below.

Element	Gram Atomic Weight	$\sigma_{a\ (barns)}$	$\sigma_{s\ (barns)}$
Aluminum	26.9815	0.23	1.49
Silicon	28.0855	0.16	2.20

1. Calculate the atom densities for the aluminum and silicon.

2. Determine the absorption and scattering macroscopic cross sections for thermal neutrons.

3. Calculate the mean free paths for absorption and scattering.

Solution:

Step 1: The density of the aluminum will be 95% of the total density. Using Equation (2-1) yields the atom densities.

$$N_{Al} = \frac{\rho_{Al}\, N_A}{M_{Al}}$$

$$= \frac{0.95 \left(2.66\ \frac{g}{cm^3}\right) \left(6.022 \times 10^{23}\ \frac{atoms}{mole}\right)}{26.9815\ \frac{g}{mole}}$$

$$= 5.64 \times 10^{22}\ \frac{atoms}{cm^3}$$

$$N_{Si} = \frac{\rho_{Si}\, N_A}{M_{Si}}$$

$$= \frac{0.05 \left(2.66\ \frac{g}{cm^3}\right) \left(6.022 \times 10^{23}\ \frac{atoms}{mole}\right)}{28.0855\ \frac{g}{mole}}$$

$$= 2.85 \times 10^{21}\ \frac{atoms}{cm^3}$$

Step 2: The macroscopic cross sections for absorption and scattering are calculated using Equation (2-4).

$$\Sigma_a = N_{Al}\,\sigma_{a,Al} + N_{Si}\,\sigma_{a,Si}$$

$$= \left(5.64 \times 10^{22}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(0.23 \times 10^{-24}\ \text{cm}^2\right) + \left(2.85 \times 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(0.16 \times 10^{-24}\ \text{cm}^2\right)$$

$$= 0.0134\ \text{cm}^{-1}$$

$$\Sigma_s = N_{Al}\,\sigma_{s,Al} + N_{Si}\,\sigma_{s,Si}$$

$$= \left(5.64 \times 10^{22}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(1.49 \times 10^{-24}\ \text{cm}^2\right) + \left(2.85 \times 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(2.20 \times 10^{-24}\ \text{cm}^2\right)$$

$$= 0.0903\ \text{cm}^{-1}$$

Step 3: The mean free paths are calculated by inserting the macroscopic cross sections calculated above into Equation (2-3).

$$\lambda_a = \frac{1}{\Sigma_a}$$

$$= \frac{1}{0.01345\ \text{cm}^{-1}}$$

$$= 74.3\ \text{cm}$$

$$\lambda_s = \frac{1}{\Sigma_s}$$

$$= \frac{1}{0.0903\ \text{cm}^{-1}}$$

$$= 11.1\ \text{cm}$$

Thus, a neutron must travel an average of 74.3 cm to interact by absorption in this alloy, but it must travel only 11.1 cm to interact by scattering.

Effects of Temperature on Cross Section

As discussed, the microscopic absorption cross section varies significantly as neutron energy varies. The microscopic cross sections provided on most charts and tables are measured for a standard neutron velocity of 2200 meters/second, which corresponds to an ambient temperature of 68°F. Therefore, if our material is at a higher temperature, the absorption cross section will be lower than the value for 68°F, and any cross sections which involve absorption (for example, σ_a, σ_c, σ_f) must be corrected for the existing temperature.

The following formula is used to correct microscopic cross sections for temperature. Although the example illustrates absorption cross section, the same formula may be used to correct capture and fission cross sections.

$$\sigma = \sigma_o \left(\frac{T_o}{T} \right)^{1/2}$$

where:

σ = microscopic cross section corrected for temperature
σ_o = microscopic cross section at reference temperature (68°F or 20°C)
T_o = reference temperature (68°F) in degrees Rankine (°R) or Kelvin (°K)
T = temperature for which corrected value is being calculated

NOTE: When using this formula, all temperatures must be converted to °R or °K.

°R = °F + 460
°K = °C + 273

Example:

What is the value of σ_f for uranium-235 for thermal neutrons at 500°F? Uranium-235 has a σ_f of 583 barns at 68°F.

Solution:

$$\sigma_f = \sigma_{f,o} \left(\frac{T_o}{T} \right)^{1/2}$$

$$= 583 \text{ barns} \left(\frac{68°F + 460}{500°F + 460} \right)^{1/2}$$

$$= 432 \text{ barns}$$

Neutron Flux

Macroscopic cross sections for neutron reactions with materials determine the probability of one neutron undergoing a specific reaction per centimeter of travel through that material. If one wants to determine how many reactions will actually occur, it is necessary to know how many neutrons are traveling through the material and how many centimeters they travel each second.

It is convenient to consider the number of neutrons existing in one cubic centimeter at any one instant and the total distance they travel each second while in that cubic centimeter. The number of neutrons existing in a cm^3 of material at any instant is called *neutron density* and is represented by the symbol n with units of neutrons/cm^3. The total distance these neutrons can travel each second will be determined by their velocity.

A good way of defining *neutron flux* (ϕ) is to consider it to be the total path length covered by all neutrons in one cubic centimeter during one second. Mathematically, this is the equation below.

$$\phi = n v \qquad (2\text{-}5)$$

where:

ϕ = neutron flux (neutrons/cm^2-sec)
n = neutron density (neutrons/cm^3)
v = neutron velocity (cm/sec)

The term neutron flux in some applications (for example, cross section measurement) is used as parallel beams of neutrons traveling in a single direction. The *intensity* (I) of a neutron beam is the product of the neutron density times the average neutron velocity. The directional beam intensity is equal to the number of neutrons per unit area and time (neutrons/cm^2-sec) falling on a surface perpendicular to the direction of the beam.

One can think of the neutron flux in a reactor as being comprised of many neutron beams traveling in various directions. Then, the neutron flux becomes the scalar sum of these directional flux intensities (added as numbers and not vectors), that is, $\phi = I_1 + I_2 + I_3 + ... I_n$. Since the atoms in a reactor do not interact preferentially with neutrons from any particular direction, all of these directional beams contribute to the total rate of reaction. In reality, at a given point within a reactor, neutrons will be traveling in all directions.

Self-Shielding

In some locations within the reactor, the flux level may be significantly lower than in other areas due to a phenomenon referred to as *neutron shadowing* or *self-shielding*. For example, the interior of a fuel pin or pellet will "see" a lower average flux level than the outer surfaces since an appreciable fraction of the neutrons will have been absorbed and therefore cannot reach the interior of the fuel pin. This is especially important at resonance energies, where the absorption cross sections are large.

Summary

The important information in this chapter is summarized below.

Nuclear Cross Section and Neutron Flux Summary

- Atom density (N) is the number of atoms of a given type per unit volume of material.
- Microscopic cross section (σ) is the probability of a given reaction occurring between a neutron and a nucleus.
- Microscopic cross sections are measured in units of barns, where 1 barn = 10^{-24} cm^2.
- Macroscopic cross section (Σ) is the probability of a given reaction occurring per unit length of travel of the neutron. The units for macroscopic cross section are cm^{-1}.
- The mean free path (λ) is the average distance that a neutron travels in a material between interactions.
- Neutron flux (ϕ) is the total path length traveled by all neutrons in one cubic centimeter of material during one second.
- The macroscopic cross section for a material can be calculated using the equation below.

  $$\Sigma = N \sigma$$

- The absorption cross section for a material usually has three distinct regions. At low neutron energies (<1 eV) the cross section is inversely proportional to the neutron velocity.
- Resonance absorption occurs when the sum of the kinetic energy of the neutron and its binding energy is equal to an allowed nuclear energy level of the nucleus.
- Resonance peaks exist at intermediate energy levels. For higher neutron energies, the absorption cross section steadily decreases as the neutron energy increases.
- The mean free path equals $1/\Sigma$.
- The macroscopic cross section for a mixture of materials can be calculated using the equation below.

  $$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + N_3 \sigma_3 + ... N_n \sigma_n$$

- Self-shielding is where the local neutron flux is depressed within a material due to neutron absorption near the surface of the material.

REACTION RATES

It is possible to determine the rate at which a nuclear reaction will take place based on the neutron flux, cross section for the interaction, and atom density of the target. This relationship illustrates how a change in one of these items affects the reaction rate.

EO 2.10 Given the neutron flux and macroscopic cross section, CALCULATE the reaction rate.

EO 2.11 DESCRIBE the relationship between neutron flux and reactor power.

Reaction Rates

If the total path length of all the neutrons in a cubic centimeter in a second is known, (neutron flux (ϕ)), and if the probability of having an interaction per centimeter path length is also known (macroscopic cross section (Σ)), multiply them together to get the number of interactions taking place in that cubic centimeter in one second. This value is known as the reaction rate and is denoted by the symbol R. The reaction rate can be calculated by the equation shown below.

$$R = \phi \Sigma \quad (2\text{-}6)$$

where:

R = reaction rate (reactions/sec)
ϕ = neutron flux (neutrons/cm^2-sec)
Σ = macroscopic cross section (cm^{-1})

Substituting the fact that $\Sigma = N \sigma$ into Equation (2-6) yields the equation below.

$$\mathbf{R} = \phi \mathbf{N} \sigma$$

where:

σ = microscopic cross section (cm^2)
N = atom density (atoms/cm^3)

The reaction rate calculated will depend on which macroscopic cross section is used in the calculation. Normally, the reaction rate of greatest interest is the fission reaction rate.

Example:

If a one cubic centimeter section of a reactor has a macroscopic fission cross section of 0.1 cm^{-1}, and if the thermal neutron flux is 10^{13} neutrons/cm^2-sec, what is the fission rate in that cubic centimeter?

Solution:

$$R_f = \phi \Sigma_f$$

$$= \left(1 \times 10^{13} \frac{\text{neutrons}}{\text{cm}^2\text{-sec}}\right)\left(0.1 \text{ cm}^{-1}\right)$$

$$= 1 \times 10^{12} \frac{\text{fissions}}{\text{cm}^3\text{-sec}}$$

In addition to using Equation (2-6) to determine the reaction rate based on the physical properties of material, it is also possible to algebraically manipulate the equation to determine physical properties if the reaction rate is known.

Example:

A reactor operating at a flux level of 3×10^{13} neutrons/cm^2-sec contains 10^{20} atoms of uranium-235 per cm^3. The reaction rate is 1.29×10^{12} fission/cm^3. Calculate Σ_f and σ_f.

Solution:

Step 1: The macroscopic cross section can be determined by solving Equation (2-6) for Σ_f and substituting the appropriate values.

$$R_f = \phi \Sigma_f$$

$$\Sigma_f = \frac{R_f}{\phi}$$

$$= \frac{1.29 \times 10^{12} \frac{\text{fissions}}{\text{cm}^3\text{-sec}}}{3 \times 10^{13} \frac{\text{neutrons}}{\text{cm}^2\text{-sec}}}$$

$$= 0.043 \text{ cm}^{-1}$$

Step 2: To find the microscopic cross section, replace Σ_f with (N x σ_f) and solve for σ_f.

$$R_f = \phi\ N\ \sigma_f$$

$$\sigma_f = \frac{R_f}{N\ \phi}$$

$$= \frac{1.29\ x\ 10^{12}\ \frac{\text{fissions}}{\text{cm}^3\text{-sec}}}{\left(1\ x\ 10^{20}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(3\ x\ 10^{13}\ \frac{\text{neutrons}}{\text{cm}^2\text{-sec}}\right)}$$

$$= 4.3\ x\ 10^{-22}\ \text{cm}^2\left(\frac{1\ \text{barn}}{1\ x\ 10^{-24}\ \text{cm}^2}\right)$$

$$= 430\ \text{barns}$$

Reactor Power Calculation

Multiplying the reaction rate per unit volume by the total volume of the core results in the total number of reactions occurring in the core per unit time. If the amount of energy involved in each reaction were known, it would be possible to determine the rate of energy release (power) due to a certain reaction.

In a reactor where the average energy per fission is 200 MeV, it is possible to determine the number of fissions per second that are necessary to produce one watt of power using the following conversion factors.

1 fission = 200 MeV
1 MeV = 1.602×10^{-6} ergs
1 erg = 1×10^{-7} watt-sec

$$1\ \text{watt}\left(\frac{1\ \text{erg}}{1\ x\ 10^{-7}\ \text{watt-sec}}\right)\left(\frac{1\ \text{MeV}}{1.602\ x\ 10^{-6}\text{erg}}\right)\left(\frac{1\ \text{fission}}{200\ \text{MeV}}\right) = 3.12\ x\ 10^{10}\ \frac{\text{fissions}}{\text{second}}$$

This is equivalent to stating that 3.12×10^{10} fissions release 1 watt-second of energy.

The power released in a reactor can be calculated based on Equation (2-6). Multiplying the reaction rate by the volume of the reactor results in the total fission rate for the entire reactor. Dividing by the number of fissions per watt-sec results in the power released by fission in the reactor in units of watts. This relationship is shown mathematically in Equation (2-7) below.

$$P = \frac{\phi_{th}\ \Sigma_f\ V}{3.12\ x\ 10^{10}\ \frac{\text{fissions}}{\text{watt-sec}}} \qquad (2\text{-}7)$$

where:

P = power (watts)
ϕ_{th} = thermal neutron flux (neutrons/cm^2-sec)
Σ_f = macroscopic cross section for fission (cm^{-1})
V = volume of core (cm^3)

Relationship Between Neutron Flux and Reactor Power

In an operating reactor the volume of the reactor is constant. Over a relatively short period of time (days or weeks), the number density of the fuel atoms is also relatively constant. Since the atom density and microscopic cross section are constant, the macroscopic cross section must also be constant. Examining Equation (2-7), it is apparent that if the reactor volume and macroscopic cross section are constant, then the reactor power and the neutron flux are directly proportional. This is true for day-to-day operation. The neutron flux for a given power level will increase very slowly over a period of months due to the burnup of the fuel and resulting decrease in atom density and macroscopic cross section.

Summary

The important information in this chapter is summarized below.

Reaction Rates Summary

- The reaction rate is the number of interactions of a particular type occurring in a cubic centimeter of material in a second.
- The reaction rate can be calculated by the equation below.

$$R = \phi \Sigma$$

- Over a period of several days, while the atom density of the fuel can be considered constant, the neutron flux is directly proportional to reactor power.

NEUTRON MODERATION

In thermal reactors, the neutrons that cause fission are at a much lower energy than the energy level at which they were born from fission. In this type of reactor, specific materials must be included in the reactor design to reduce the energy level of the neutrons in an efficient manner.

EO 2.12 DEFINE the following concepts:

a. Thermalization
b. Moderator
c. Moderating ratio
d. Average logarithmic energy decrement
e. Macroscopic slowing down power

EO 2.13 LIST three desirable characteristics of a moderator.

EO 2.14 Given an average fractional energy loss per collision, CALCULATE the energy loss after a specified number of collisions.

Neutron Slowing Down and Thermalization

Fission neutrons are produced at an average energy level of 2 MeV and immediately begin to slow down as the result of numerous scattering reactions with a variety of target nuclei. After a number of collisions with nuclei, the speed of a neutron is reduced to such an extent that it has approximately the same average kinetic energy as the atoms (or molecules) of the medium in which the neutron is undergoing elastic scattering. This energy, which is only a small fraction of an electron volt at ordinary temperatures (0.025 eV at 20°C), is frequently referred to as the thermal energy, since it depends upon the temperature. Neutrons whose energies have been reduced to values in this region (< 1 eV) are designated thermal neutrons. The process of reducing the energy of a neutron to the thermal region by elastic scattering is referred to as *thermalization*, slowing down, or moderation. The material used for the purpose of thermalizing neutrons is called a *moderator*. A good moderator reduces the speed of neutrons in a small number of collisions, but does not absorb them to any great extent. Slowing the neutrons in as few collisions as possible is desirable in order to reduce the amount of neutron leakage from the core and also to reduce the number of resonance absorptions in non-fuel materials. Neutron leakage and resonance absorption will be discussed in the next module.

The ideal moderating material (moderator) should have the following nuclear properties.

- large scattering cross section
- small absorption cross section
- large energy loss per collision

A convenient measure of energy loss per collision is the logarithmic energy decrement. The *average logarithmic energy decrement* is the average decrease per collision in the logarithm of the neutron energy. This quantity is represented by the symbol ξ (Greek letter xi).

$$\xi = \ln E_i - \ln E_f$$

$$\xi = \ln \left(\frac{E_i}{E_f} \right) \quad (2\text{-}8)$$

where:

ξ = average logarithmic energy decrement
E_i = average initial neutron energy
E_f = average final neutron energy

The symbol ξ is commonly called the average logarithmic energy decrement because of the fact that a neutron loses, on the average, a fixed fraction of its energy per scattering collision. Since the fraction of energy retained by a neutron in a single elastic collision is a constant for a given material, ξ is also a constant. Because it is a constant for each type of material and does not depend upon the initial neutron energy, ξ is a convenient quantity for assessing the moderating ability of a material.

The values for the lighter nuclei are tabulated in a variety of sources. The following commonly used approximation may be used when a tabulated value is not available.

$$\xi = \frac{2}{A + \frac{2}{3}}$$

This approximation is relatively accurate for mass numbers (A) greater than 10, but for some low values of A it may be in error by over three percent.

Since ξ represents the average logarithmic energy loss per collision, the total number of collisions necessary for a neutron to lose a given amount of energy may be determined by dividing ξ into the difference of the natural logarithms of the energy range in question. The number of collisions (N) to travel from any energy, E_{high}, to any lower energy, E_{low}, can be calculated as shown below.

$$N = \frac{\ln E_{high} - \ln E_{low}}{\xi}$$

$$= \frac{\ln \left(\frac{E_{high}}{E_{low}} \right)}{\xi}$$

Example:

How many collisions are required to slow a neutron from an energy of 2 MeV to a thermal energy of 0.025 eV, using water as the moderator? Water has a value of 0.948 for ξ.

Solution:

$$N = \frac{\ln \left(\frac{E_{high}}{E_{low}} \right)}{\xi}$$

$$= \frac{\ln \left(\frac{2 \times 10^6 \text{ eV}}{0.025 \text{ eV}} \right)}{0.948}$$

$$= 19.2 \text{ collisions}$$

Sometimes it is convenient, based upon information known, to work with an average fractional energy loss per collision as opposed to an average logarithmic fraction. If the initial neutron energy level and the average fractional energy loss per collision are known, the final energy level for a given number of collisions may be computed using the following formula.

$$E_N = E_o (1 - x)^N \quad (2\text{-}9)$$

where:

E_o = initial neutron energy
E_N = neutron energy after N collisions
x = average fractional energy loss per collision
N = number of collisions

Example:

If the average fractional energy loss per collision in hydrogen is 0.63, what will be the energy of a 2 MeV neutron after (a) 5 collisions? (b) 10 collisions?

Solution:

a)

$$E_N = E_o (1 - x)^N$$
$$E_5 = (2 \times 10^6 \text{ eV}) (1 - 0.63)^5$$
$$= 13.9 \text{ keV}$$

b)

$$E_N = E_o (1 - x)^N$$
$$E_{10} = (2 \times 10^6 \text{ eV}) (1 - 0.63)^{10}$$
$$= 96.2 \text{ eV}$$

Macroscopic Slowing Down Power

Although the logarithmic energy decrement is a convenient measure of the ability of a material to slow neutrons, it does not measure all necessary properties of a moderator. A better measure of the capabilities of a material is the macroscopic slowing down power. The *macroscopic slowing down power* (MSDP) is the product of the logarithmic energy decrement and the macroscopic cross section for scattering in the material. Equation (2-10) illustrates how to calculate the macroscopic slowing down power.

$$MSDP = \xi \Sigma_s \quad (2\text{-}10)$$

Moderating Ratio

Macroscopic slowing down power indicates how rapidly a neutron will slow down in the material in question, but it still does not fully explain the effectiveness of the material as a moderator. An element such as boron has a high logarithmic energy decrement and a good slowing down power, but it is a poor moderator because of its high probability of absorbing neutrons.

The most complete measure of the effectiveness of a moderator is the moderating ratio. The *moderating ratio* is the ratio of the macroscopic slowing down power to the macroscopic cross section for absorption. The higher the moderating ratio, the more effectively the material performs as a moderator. Equation (2-11) shows how to calculate the moderating ratio of a material.

$$MR = \frac{\xi \, \Sigma_s}{\Sigma_a} \tag{2-11}$$

Moderating properties of different materials are compared in Table 2.

TABLE 2
Moderating Properties of Materials

Material	ξ	Number of Collisions to Thermalize	Macroscopic Slowing Down Power	Moderating Ratio
H_2O	0.927	19	1.425	62
D_2O	0.510	35	0.177	4830
Helium	0.427	42	9×10^{-6}	51
Beryllium	0.207	86	0.154	126
Boron	0.171	105	0.092	0.00086
Carbon	0.158	114	0.083	216

Summary

The important information in this chapter is summarized below.

Neutron Moderation Summary

- Thermalization is the process of reducing the energy level of a neutron from the energy level at which it is produced to an energy level in the thermal range.
- The moderator is the reactor material that is present for the purpose of thermalizing neutrons.
- Moderating ratio is the ratio of the macroscopic slowing down power to the macroscopic cross section for absorption.
- The average logarithmic energy decrement (ξ) is the average change in the logarithm of neutron energy per collision.
- Macroscopic slowing down power is the product of the average logarithmic energy decrement and the macroscopic cross section for scattering.
- There are three desirable characteristics of a moderator.

  1. large scattering cross section
  2. small absorption cross section
  3. large energy loss per collision

- The energy loss after a specified number of collisions can be calculated using the equation below.

$$E_N = E_o (1 - x)^N$$

PROMPT AND DELAYED NEUTRONS

Not all neutrons are released at the same time following fission. Most neutrons are released virtually instantaneously and are called prompt neutrons. A very small fraction of neutrons are released after the decay of fission products and are called delayed neutrons. Although delayed neutrons are a very small fraction of the total number of neutrons, they play an extremely important role in the control of the reactor.

EO 3.1 STATE the origin of prompt neutrons and delayed neutrons.

EO 3.2 STATE the approximate fraction of neutrons that are born as delayed neutrons from the fission of the following nuclear fuels:

a. Uranium-235
b. Plutonium-239

EO 3.3 EXPLAIN the mechanism for production of delayed neutrons.

EO 3.4 EXPLAIN prompt and delayed neutron generation times.

EO 3.5 Given prompt and delayed neutron generation times and delayed neutron fraction, CALCULATE the average generation time.

EO 3.6 EXPLAIN the effect of delayed neutrons on reactor control.

Neutron Classification

The great majority (over 99%) of the neutrons produced in fission are released within about 10^{-13} seconds of the actual fission event. These are called *prompt neutrons*. A small portion of fission neutrons are *delayed neutrons*, which are produced for some time after the fission process has taken place. The delayed neutrons are emitted immediately following the first beta decay of a fission fragment known as a delayed neutron precursor. An example of a delayed neutron precursor is bromine-87, shown below.

$$^{87}_{35}\mathrm{Br} \xrightarrow[55.9\ \mathrm{sec}]{\beta^-} {}^{87}_{36}\mathrm{Kr} \xrightarrow[\text{instantaneous}]{n} {}^{86}_{36}\mathrm{Kr}_{\text{stable}}$$

For most applications, it is convenient to combine the known precursors into groups with appropriately averaged properties. These groups vary somewhat depending on the fissile material in use. Table 3 lists the characteristics for the six precursor groups resulting from thermal fission of uranium-235. The fraction of all neutrons that are produced by each of these precursors is called the delayed neutron fraction for that precursor. The total fraction of all neutrons born as delayed neutrons is called the *delayed neutron fraction* (β). The fraction of delayed neutrons produced varies depending on the predominant fissile nuclide in use. The delayed neutron fractions (β) for the fissile nuclides of most interest are as follows: uranium-233 (0.0026), uranium-235 (0.0065), uranium-238 (0.0148), and plutonium-239 (0.0021).

TABLE 3
Delayed Neutron Precursor Groups for Thermal Fission in Uranium-235

Group	Half-Life (sec)	Delayed Neutron Fraction	Average Energy (MeV)
1	55.7	0.00021	0.25
2	22.7	0.00142	0.46
3	6.2	0.00127	0.41
4	2.3	0.00257	0.45
5	0.61	0.00075	0.41
6	0.23	0.00027	-
Total	-	0.0065	-

Neutron Generation Time

The neutron generation time is the time required for neutrons from one generation to cause the fissions that produce the next generation of neutrons. The generation time for prompt neutrons (ℓ^* - pronounced "ell-star") is the total time from birth to rebirth. Three time intervals are involved: (a) the time it takes a fast neutron to slow down to thermal energy, (b) the time the now thermal neutron exists prior to absorption in fuel, and (c) the time required for a fissionable nucleus to emit a fast neutron after neutron absorption.

Fast neutrons slow to thermal energies or leak out of the reactor in 10^{-4} seconds to 10^{-6} seconds, depending on the moderator. In water moderated reactors, thermal neutrons tend to exist for about 10^{-4} seconds before they are absorbed. Fission and fast neutron production following neutron absorption in a fissionable nucleus occurs in about 10^{-13} seconds. Thus, fast reactors have an ℓ^* of about 10^{-6} seconds, while thermal reactors have an ℓ^* of about 10^{-6} seconds + 10^{-4} seconds, which is about 10^{-4} seconds to 10^{-5} seconds.

On the other hand, the average generation time for the six delayed neutron groups is the total time from the birth of the fast neutron to the emission of the delayed neutron. Again, three time intervals are involved: (a) the time it takes a fast neutron to slow down to thermal energy, (b) the time the thermal neutron exists prior to absorption, and (c) the average time from neutron absorption to neutron emission by the six precursor groups. The average time for decay of precursors from uranium-235 is 12.5 seconds. The other terms in the delayed neutron generation time are insignificant when compared to this value, and the average delayed neutron generation time becomes ~ 12.5 seconds.

A neutron generation time in the range of 10^{-4} seconds to 10^{-5} seconds or faster could result in very rapid power excursions, and control would not be possible without the dependence upon delayed neutrons to slow down the rate of the reaction. The average generation time, and hence the rate that power can rise, is determined largely by the delayed neutron generation time. The following equation shows this mathematically.

$$\text{Time}_{\text{average}} = \text{Time}_{\text{prompt}} (1 - \beta) + \text{Time}_{\text{delayed}} (\beta) \qquad (2\text{-}12)$$

Example:

> Assume a prompt neutron generation time for a particular reactor of 5×10^{-5} seconds and a delayed neutron generation time of 12.5 seconds. If β is 0.0065, calculate the average generation time.

Solution:

$$\begin{aligned} \text{Time}_{\text{average}} &= \text{Time}_{\text{prompt}} (1 - \beta) + \text{Time}_{\text{delayed}} (\beta) \\ &= (5 \times 10^{-5} \text{ seconds}) (0.9935) + (12.5 \text{ seconds}) (0.0065) \\ &= 0.0813 \text{ seconds} \end{aligned}$$

This example demonstrates the effect delayed neutrons have on the neutron generation time and thus reactor control. If a reactor were to be operated in a sustained chain reaction using only prompt neutrons ($\beta = 0$), the generation time from the previous example would be about 5×10^{-5} seconds. However, by operating the reactor such that a 0.0065 fraction of neutrons are delayed, the generation life time is extended to 0.0813 seconds, providing time for adequate operator control. Therefore, although only a small fraction of the total neutron population, delayed neutrons are extremely important to the control and maintenance of a sustained fission chain reaction.

Summary

The important information in this chapter is summarized on the following page.

Prompt and Delayed Neutrons Summary

- Prompt neutrons are released directly from fission within 10^{-13} seconds of the fission event.

- Delayed neutrons are released from the decay of fission products that are called delayed neutron precursors. Delayed neutron precursors are grouped according to half-life. Half-lives vary from fractions of a second to almost a minute.

- The fraction of neutrons born as delayed neutrons is different for different fuel materials. Following are values for some common fuel materials.

Uranium-235	0.0065
Plutonium-239	0.0021

- Delayed neutrons are produced by a classification of fission products known as delayed neutron precursors. When a delayed neutron precursor undergoes a β^- decay, it results in an excited daughter nucleus which immediately ejects a neutron. Therefore, these delayed neutrons appear with a half-life of the delayed neutron precursor.

- The delayed neutron generation time is the total time from the birth of the fast neutron to the emission of the delayed neutron in the next generation. Delayed neutron generation times are dominated by the half-life of the delayed neutron precursor. The average delayed neutron generation time is about 12.5 seconds.

- A prompt neutron generation time is the sum of the amount of time it takes a fast neutron to thermalize, the amount of time the neutron exists as a thermal neutron before it is absorbed, and the amount of time between a fissionable nuclide absorbing a neutron and fission neutrons being released. Prompt neutron generation time is about 5×10^{-5} seconds.

- The average neutron generation time can be calculated from the prompt and delayed neutron generation times and the delayed neutron fraction using Equation (2-12).

$$\text{Time}_{\text{average}} = \text{Time}_{\text{prompt}} (1 - \beta) + \text{Time}_{\text{delayed}} (\beta)$$

- Delayed neutrons are responsible for the ability to control the rate at which power can rise in a reactor. If only prompt neutrons existed, reactor control would not be possible due to the rapid power changes.

NEUTRON FLUX SPECTRUM

The number of neutrons that exist at a given energy level varies. A plot of either the fraction of neutrons or the neutron flux at a given energy versus the energy level is called a neutron energy spectrum. The neutron energy spectrum varies widely for different types of reactors.

EO 4.1 STATE the average energy at which prompt neutrons are produced.

EO 4.2 DESCRIBE the neutron energy spectrum in the following reactors:

a. Fast reactor
b. Thermal reactor

EO 4.3 EXPLAIN the reason for the particular shape of the fast, intermediate, and slow energy regions of the neutron flux spectrum for a thermal reactor.

Prompt Neutron Energies

The neutrons produced by fission are high energy neutrons, and almost all fission neutrons have energies between 0.1 MeV and 10 MeV. The neutron energy distribution, or spectrum, may best be described by plotting the fraction of neutrons per MeV as a function of neutron energy, as shown in Figure 2. From this figure it can be seen that the most probable neutron energy is about 0.7 MeV. Also, from this data it can be shown that the average energy of fission neutrons is about 2 MeV. Figure 2 is the neutron energy spectrum for thermal fission in uranium-235. The values will vary slightly for other nuclides.

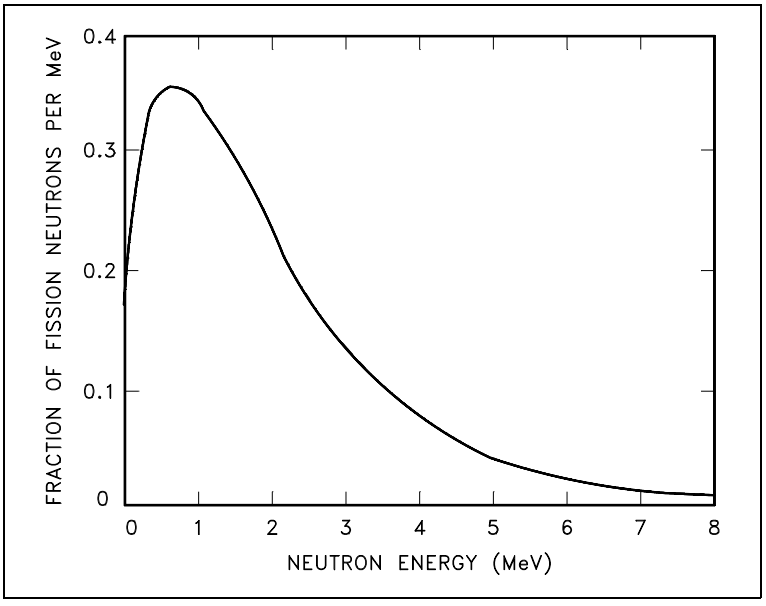


Figure 2 Prompt Fission Neutron Energy Spectrum for Thermal Fission of Uranium-235

Thermal and Fast Breeder Reactor Neutron Spectra

The spectrum of neutron energies produced by fission varies significantly from the energy spectrum, or flux, existing in a reactor at a given time. Figure 3 illustrates the difference in neutron flux spectra between a thermal reactor and a fast breeder reactor. The energy distribution of neutrons from fission is essentially the same for both reactors, so the differences in the curve shapes may be attributed to the neutron moderation or slowing down effects.

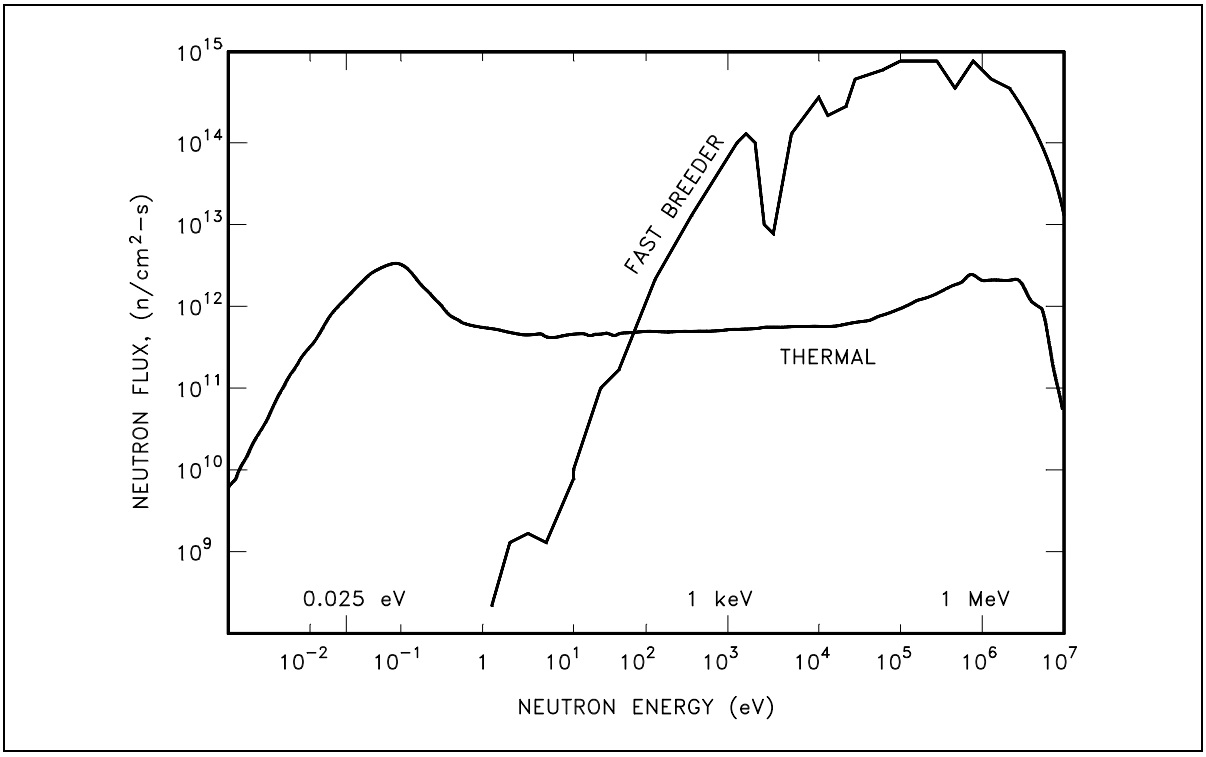


Figure 3 Comparison of Neutron Flux Spectra for Thermal and Fast Breeder Reactor

No attempt is made to thermalize or slow down neutrons in the fast breeder reactor (liquid metal cooled); therefore, an insignificant number of neutrons exist in the thermal range. For the thermal reactor (water moderated), the spectrum of neutrons in the fast region (> 0.1 MeV) has a shape similar to that for the spectrum of neutrons emitted by the fission process.

In the thermal reactor, the flux in the intermediate energy region (1 eV to 0.1 MeV) has approximately a 1/E dependence. That is, if the energy (E) is halved, the flux doubles. This 1/E dependence is caused by the slowing down process, where elastic collisions remove a constant fraction of the neutron energy per collision (on the average), independent of energy; thus, the neutron loses larger amounts of energy per collision at higher energies than at lower energies. The fact that the neutrons lose a constant fraction of energy per collision causes the neutrons to tend to "pile up" at lower energies, that is, a greater number of neutrons exist at the lower energies as a result of this behavior.

In the thermal region the neutrons achieve a thermal equilibrium with the atoms of the moderator material. In any given collision they may gain or lose energy, and over successive collisions will gain as much energy as they lose. These thermal neutrons, even at a specific temperature, do not all have the same energy or velocity; there is a distribution of energies, usually referred to as the Maxwell distribution (e.g., Figure 2). The energies of most thermal neutrons lie close to the most probable energy, but there is a spread of neutrons above and below this value.

Most Probable Neutron Velocities

The *most probable velocity* (v_p) of a thermal neutron is determined by the temperature of the medium and can be determined by Equation (2-13) .

$$v_p = \sqrt{\frac{2\ k\ T}{m}} \tag{2-13}$$

where:

v_p = most probable velocity of neutron (cm/sec)
k = Boltzman's constant (1.38×10^{-16} erg/°K)
T = absolute temperature in degrees Kelvin (°K)
m = mass of neutron (1.66×10^{-24} grams)

Example:

Calculate the most probable velocities for neutrons in thermal equilibrium with their surroundings at the following temperatures. a) 20°C, b) 260°C.

Solution:

a) Calculate the most probable velocity for 20°C using Equation (2-13).

$$v_p = \sqrt{\frac{2\ k\ T}{m}}$$

$$= \sqrt{\frac{2\left(1.38 \times 10^{-16}\ \frac{\text{erg}}{\text{°K}}\right)(293\text{°K})}{1.66 \times 10^{-24}\ \text{g}}}$$

$$= 2.2 \times 10^{5}\ \frac{\text{cm}}{\text{sec}}\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)$$

$$= 2200\ \frac{\text{m}}{\text{sec}}$$

b) Calculate the most probable velocity for 260°C using Equation (2-13).

$$v_p = \sqrt{\frac{2\ k\ T}{m}}$$

$$= \sqrt{\frac{2\left(1.38\ x\ 10^{-16}\ \frac{erg}{°K}\right)(533°K)}{1.66\ x\ 10^{-24}\ g}}$$

$$= 2.977\ x\ 10^{5}\ \frac{cm}{sec}\left(\frac{1\ m}{100\ cm}\right)$$

$$= 2977\ \frac{m}{sec}$$

From these calculations it is evident that the most probable velocity of a thermal neutron increases as temperature increases. The most probable velocity at 20°C is of particular importance since reference data, such as nuclear cross sections, are tabulated for a neutron velocity of 2200 meters per second.

Summary

The important information in this chapter is summarized below.

Neutron Flux Spectrum Summary

- Prompt neutrons are born at energies between 0.1 MeV and 10 MeV. The average prompt neutron energy is about 2 MeV.
- Fast reactors have a neutron energy spectrum that has the same shape as the prompt neutron energy spectrum.
- Thermal reactors have a neutron energy spectrum that has two pronounced peaks, one in the thermal energy region where the neutrons are in thermal equilibrium with the core materials and another in the fast region at energies where neutrons are produced. The flux in the intermediate region (1 eV to 0.1 MeV) has a roughly 1/E dependence.
- The neutron flux spectrum for the fast energy region of a thermal reactor has a shape similar to that of the spectrum of neutrons emitted by the fission process.
- The reason for the 1/E flux dependence at intermediate energy levels in a thermal reactor is due to the neutrons' tendency to lose a constant fraction of energy per collision. Since the neutrons lose a greater amount at the higher energies, the neutrons tend to "pile up" at lower energies where they lose less energy per collision.
- The neutron flux spectrum for the slow region of a thermal reactor contains a peak at the energy where the neutrons are in thermal equilibrium with the atoms of the surrounding material.

Intentionally Left Blank

Department of Energy
Fundamentals Handbook

NUCLEAR PHYSICS AND REACTOR THEORY
Module 3
Reactor Theory (Nuclear Parameters)

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont.)

LIST OF FIGURES

LIST OF TABLES

REFERENCES

- Foster, Arthur R. and Wright, Robert L. Jr., Basic Nuclear Engineering, 3rd Edition, Allyn and Bacon, Inc., 1977.
- Jacobs, A.M., Kline, D.E., and Remick, F.J., Basic Principles of Nuclear Science and Reactors, Van Nostrand Company, Inc., 1960.
- Kaplan, Irving, Nuclear Physics, 2nd Edition, Addison-Wesley Company, 1962.
- Knief, Ronald Allen, Nuclear Energy Technology: Theory and Practice of Commercial Nuclear Power, McGraw-Hill, 1981.
- Lamarsh, John R., Introduction to Nuclear Engineering, Addison-Wesley Company, 1977.
- Lamarsh, John R., Introduction to Nuclear Reactor Theory, Addison-Wesley Company, 1972.
- General Electric Company, Nuclides and Isotopes: Chart of the Nuclides, 14th Edition, General Electric Company, 1989.
- Academic Program for Nuclear Power Plant Personnel, Volume III, Columbia, MD, General Physics Corporation, Library of Congress Card #A 326517, 1982.
- Glasstone, Samuel, Sourcebook on Atomic Energy, Robert F. Krieger Publishing Company, Inc., 1979.
- Glasstone, Samuel and Sesonske, Alexander, Nuclear Reactor Engineering, 3rd Edition, Van Nostrand Reinhold Company, 1981.

TERMINAL OBJECTIVE

1.0 Using appropriate references, **DESCRIBE** the neutron life cycle discussed in this module.

ENABLING OBJECTIVES

1.1 **DEFINE** the following terms:

a. Infinite multiplication factor, k_{∞}
b. Effective multiplication factor, k_{eff}
c. Subcritical
d. Critical
e. Supercritical

1.2 **DEFINE** each term in the six factor formula using the ratio of the number of neutrons present at different points in the neutron life cycle.

1.3 Given the macroscopic cross sections for various materials, **CALCULATE** the thermal utilization factor.

1.4 Given microscopic cross sections for absorption and fission, atom density, and ν, **CALCULATE** the reproduction factor.

1.5 Given the numbers of neutrons present at the start of a generation and values for each factor in the six factor formula, **CALCULATE** the number of neutrons that will be present at any point in the life cycle.

1.6 **LIST** physical changes in the reactor core that will have an effect on the thermal utilization factor, reproduction factor, or resonance escape probability.

1.7 **EXPLAIN** the effect that temperature changes will have on the following factors:

a. Thermal utilization factor
b. Resonance escape probability
c. Fast non-leakage probability
d. Thermal non-leakage probability

1.8 Given the number of neutrons in a reactor core and the effective multiplication factor, **CALCULATE** the number of neutrons present after any number of generations.

ENABLING OBJECTIVES (Cont.)

1.9 **DEFINE** the term reactivity.

1.10 **CONVERT** between reactivity and the associated value of k_{eff}.

1.11 **CONVERT** measures of reactivity between the following units:

a. $\Delta k/k$
b. $\%\Delta k/k$
c. $10^{-4}\ \Delta k/k$
d. Percent millirho (pcm)

1.12 **EXPLAIN** the relationship between reactivity coefficients and reactivity defects.

TERMINAL OBJECTIVE

2.0 From memory, **EXPLAIN** how reactivity varies with the thermodynamic properties of the moderator and the fuel.

ENABLING OBJECTIVES

2.1 **EXPLAIN** the conditions of over moderation and under moderation.

2.2 **EXPLAIN** why many reactors are designed to be operated in an under moderated condition.

2.3 **STATE** the effect that a change in moderator temperature will have on the moderator to fuel ratio.

2.4 **DEFINE** the temperature coefficient of reactivity.

2.5 **EXPLAIN** why a negative temperature coefficient of reactivity is desirable.

2.6 **EXPLAIN** why the fuel temperature coefficient is more effective than the moderator temperature coefficient in terminating a rapid power rise.

2.7 **EXPLAIN** the concept of Doppler broadening of resonance absorption peaks.

2.8 **LIST** two nuclides that are present in some types of reactor fuel assemblies that have significant resonance absorption peaks.

2.9 **DEFINE** the pressure coefficient of reactivity.

2.10 **EXPLAIN** why the pressure coefficient of reactivity is usually negligible in a reactor cooled and moderated by a subcooled liquid.

2.11 **DEFINE** the void coefficient of reactivity.

2.12 **IDENTIFY** the moderator conditions under which the void coefficient of reactivity becomes significant.

TERMINAL OBJECTIVE

3.0 Without references, **DESCRIBE** the use of neutron poisons.

ENABLING OBJECTIVES

3.1 **DEFINE** the following terms:

a. Burnable poison
b. Non-burnable poison
c. Chemical shim

3.2 **EXPLAIN** the use of burnable neutron poisons in a reactor core.

3.3 **LIST** the advantages and disadvantages of chemical shim over fixed burnable poisons.

3.4 **STATE** two reasons why fixed non-burnable neutron poisons are used in reactor cores.

3.5 **STATE** an example of a material used as a fixed non-burnable neutron poison.

TERMINAL OBJECTIVE

4.0 Without references, **DESCRIBE** the effects of fission product poisons on a reactor.

ENABLING OBJECTIVES

4.1 **LIST** two methods of production and two methods of removal for xenon-135 during reactor operation.

4.2 **STATE** the equation for equilibrium xenon-135 concentration.

4.3 **DESCRIBE** how equilibrium xenon-135 concentration varies with reactor power level.

4.4 **DESCRIBE** the causes and effects of a xenon oscillation.

4.5 **DESCRIBE** how xenon-135 concentration changes following a reactor shutdown from steady-state conditions.

4.6 **EXPLAIN** the effect that pre-shutdown power levels have on the xenon-135 concentration after shutdown.

4.7 **STATE** the approximate time following a reactor shutdown at which the reactor can be considered "xenon free."

4.8 **EXPLAIN** what is meant by the following terms:

a. Xenon precluded startup
b. Xenon dead time

4.9 **DESCRIBE** how xenon-135 concentration changes following an increase or a decrease in the power level of a reactor.

4.10 **DESCRIBE** how samarium-149 is produced and removed from the reactor core during reactor operation.

4.11 **STATE** the equation for equilibrium samarium-149 concentration.

4.12 **DESCRIBE** how equilibrium samarium-149 concentration varies with reactor power level.

ENABLING OBJECTIVES (Cont.)

4.13 **DESCRIBE** how samarium-149 concentration changes following a reactor shutdown from steady-state conditions.

4.14 **DESCRIBE** how samarium-149 concentration changes following a reactor startup.

4.15 **STATE** the conditions under which helium-3 will have a significant effect on the reactivity of a reactor.

TERMINAL OBJECTIVE

5.0 Without references, **DESCRIBE** how control rods affect the reactor core.

ENABLING OBJECTIVES

5.1 **DESCRIBE** the difference between a "grey" neutron absorbing material and a "black" neutron absorbing material.

5.2 **EXPLAIN** why a "grey" neutron absorbing material may be preferable to a "black" neutron absorbing material for use in control rods.

5.3 **EXPLAIN** why resonance absorbers are sometimes preferred over thermal absorbers as a control rod material.

5.4 **DEFINE** the following terms:

a. Integral control rod worth
b. Differential control rod worth

5.5 **DESCRIBE** the shape of a typical differential control rod worth curve and explain the reason for the shape.

5.6 **DESCRIBE** the shape of a typical integral control rod worth curve and explain the reason for the shape.

5.7 Given an integral or differential control rod worth curve, **CALCULATE** the reactivity change due to a control rod movement between two positions.

5.8 Given differential control rod worth data, **PLOT** differential and integral control rod worth curves.

NEUTRON LIFE CYCLE

Some number of the fast neutrons produced by fission in one generation will eventually cause fission in the next generation. The series of steps that fission neutrons go through as they slow to thermal energies and are absorbed in the reactor is referred to as the neutron life cycle. The neutron life cycle is markedly different between fast reactors and thermal reactors. This chapter presents the neutron life cycle for thermal reactors.

EO 1.1 DEFINE the following terms:

a. Infinite multiplication factor, k_∞
b. Effective multiplication factor, k_{eff}
c. Subcritical
d. Critical
e. Supercritical

EO 1.2 DEFINE each term in the six factor formula using the ratio of the number of neutrons present at different points in the neutron life cycle.

EO 1.3 Given the macroscopic cross sections for various materials, CALCULATE the thermal utilization factor.

EO 1.4 Given microscopic cross sections for absorption and fission, atom density, and ν, CALCULATE the reproduction factor.

EO 1.5 Given the numbers of neutrons present at the start of a generation and values for each factor in the six factor formula, CALCULATE the number of neutrons that will be present at any point in the life cycle.

EO 1.6 LIST physical changes in the reactor core that will have an effect on the thermal utilization factor, reproduction factor, or resonance escape probability.

EO 1.7 EXPLAIN the effect that temperature changes will have on the following factors:

a. Thermal utilization factor
b. Resonance escape probability
c. Fast non-leakage probability
d. Thermal non-leakage probability

Infinite Multiplication Factor, k_∞

Not all of the neutrons produced by fission will have the opportunity to cause new fissions because some neutrons will be absorbed by non-fissionable material. Some will be absorbed parasitically in fissionable material and will not cause fission, and others will leak out of the reactor. For the maintenance of a self-sustaining chain reaction, however, it is not necessary that every neutron produced in fission initiate another fission. The minimum condition is for each nucleus undergoing fission to produce, on the average, at least one neutron that causes fission of another nucleus. This condition is conveniently expressed in terms of a multiplication factor.

The number of neutrons absorbed or leaking out of the reactor will determine the value of this multiplication factor, and will also determine whether a new generation of neutrons is larger, smaller, or the same size as the preceding generation. Any reactor of a finite size will have neutrons leak out of it. Generally, the larger the reactor, the lower the fraction of neutron leakage. For simplicity, we will first consider a reactor that is infinitely large, and therefore has no neutron leakage. A measure of the increase or decrease in neutron flux in an infinite reactor is the infinite multiplication factor, k_∞. The *infinite multiplication factor* is the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation. This can be expressed mathematically as shown below.

$$k_\infty = \frac{\text{neutron production from fission in one generation}}{\text{neutron absorption in the preceding generation}}$$

Four Factor Formula

A group of fast neutrons produced by fission can enter into several reactions. Some of these reactions reduce the size of the neutron group while other reactions allow the group to increase in size or produce a second generation. There are four factors that are completely independent of the size and shape of the reactor that give the inherent multiplication ability of the fuel and moderator materials without regard to leakage. This *four factor formula* accurately represents the infinite multiplication factor as shown in the equation below.

$$k_\infty = \varepsilon\ p\ f\ \eta$$

where:

ε = fast fission factor
p = resonance escape probability
f = thermal utilization factor
η = reproduction factor

Each of these four factors, which are explained in the following subsections, represents a process that adds to or subtracts from the initial neutron group produced in a generation by fission.

Fast Fission Factor, (ε)

The first process that the neutrons of one generation may undergo is fast fission. Fast fission is fission caused by neutrons that are in the fast energy range. Fast fission results in the net increase in the fast neutron population of the reactor core. The cross section for fast fission in uranium-235 or uranium-238 is small; therefore, only a small number of fast neutrons cause fission. The fast neutron population in one generation is therefore increased by a factor called the fast fission factor. The *fast fission factor* (ε) is defined as the ratio of the net number of fast neutrons produced by all fissions to the number of fast neutrons produced by thermal fissions. The mathematical expression of this ratio is shown below.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

In order for a neutron to be absorbed by a fuel nucleus as a fast neutron, it must pass close enough to a fuel nucleus while it is a fast neutron. The value of ε will be affected by the arrangement and concentrations of the fuel and the moderator. The value of ε is essentially 1.00 for a homogenous reactor where the fuel atoms are surrounded by moderator atoms. However, in a heterogeneous reactor, all the fuel atoms are packed closely together in elements such as pins, rods, or pellets. Neutrons emitted from the fission of one fuel atom have a very good chance of passing near another fuel atom before slowing down significantly. The arrangement of the core elements results in a value of about 1.03 for ε in most heterogeneous reactors. The value of ε is not significantly affected by variables such as temperature, pressure, enrichment, or neutron poison concentrations. Poisons are non-fuel materials that easily absorb neutrons and will be discussed in more detail later.

Resonance Escape Probability, (p)

After increasing in number as a result of some fast fissions, the neutrons continue to diffuse through the reactor. As the neutrons move they collide with nuclei of fuel and non-fuel material and moderator in the reactor losing part of their energy in each collision and slowing down. While they are slowing down through the resonance region of uranium-238, which extends from about 6 eV to 200 eV, there is a chance that some neutrons will be captured. The probability that a neutron will not be absorbed by a resonance peak is called the resonance escape probability. The *resonance escape probability* (p) is defined as the ratio of the number of neutrons that reach thermal energies to the number of fast neutrons that start to slow down. This ratio is shown below.

$$p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

The value of the resonance escape probability is determined largely by the fuel-moderator arrangement and the amount of enrichment of uranium-235 (if any is used). To undergo resonance absorption, a neutron must pass close enough to a uranium-238 nucleus to be absorbed while slowing down. In a homogeneous reactor the neutron does its slowing down in the region of the fuel nuclei, and this condition is easily met. This means that a neutron has a high probability of being absorbed by uranium-238 while slowing down; therefore, its escape probability is lower. In a heterogeneous reactor, however, the neutron slows down in the moderator where there are no atoms of uranium-238 present. Therefore, it has a low probability of undergoing resonance absorption, and its escape probability is higher.

The value of the resonance escape probability is not significantly affected by pressure or poison concentration. In water moderated, low uranium-235 enrichment reactors, raising the temperature of the fuel will raise the resonance absorption in uranium-238 due to the doppler effect (an apparent broadening of the normally narrow resonance peaks due to thermal motion of nuclei). The increase in resonance absorption lowers the resonance escape probability, and the fuel temperature coefficient for resonance escape is negative (explained in detail later). The temperature coefficient of resonance escape probability for the moderator temperature is also negative. As water temperature increases, water density decreases. The decrease in water density allows more resonance energy neutrons to enter the fuel and be absorbed. The value of the resonance escape probability is always slightly less than one (normally 0.95 to 0.99).

The product of the fast fission factor and the resonance escape probability (ε p) is the ratio of the number of fast neutrons that survive slowing down (thermalization) compared to the number of fast neutrons originally starting the generation.

Thermal Utilization Factor, (f)

Once thermalized, the neutrons continue to diffuse throughout the reactor and are subject to absorption by other materials in the reactor as well as the fuel. The thermal utilization factor describes how effectively thermal neutrons are absorbed by the fuel, or how well they are utilized within the reactor. The *thermal utilization factor* (f) is defined as the ratio of the number of thermal neutrons absorbed in the fuel to the number of thermal neutrons absorbed in any reactor material. This ratio is shown below.

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}}$$

The thermal utilization factor will always be less than one because some of the thermal neutrons absorbed within the reactor will be absorbed by atoms of non-fuel materials.

An equation can be developed for the thermal utilization factor in terms of reaction rates as follows.

$$f = \frac{\text{rate of absorption of thermal neutrons by the fuel}}{\text{rate of absorption of thermal neutrons by all reactor materials}}$$

$$f = \frac{\Sigma_a^U \phi^U V^U}{\Sigma_a^U \phi^U V^U + \Sigma_a^m \phi^m V^m + \Sigma_a^p \phi^p V^p}$$

The superscripts U, m, and p refer to uranium, moderator, and poison, respectively. In a heterogeneous reactor, the flux will be different in the fuel region than in the moderator region due to the high absorption rate by the fuel. Also, the volumes of fuel, moderator, and poisons will be different. Although not shown in the above equation, other non-fuel materials, such as core construction materials, may absorb neutrons in a heterogeneous reactor. These other materials are often lumped together with the superscript designation OS, for "other stuff." To be completely accurate, the above equation for the thermal utilization factor should include all neutron-absorbing reactor materials when dealing with heterogeneous reactors. However, for the purposes of this text, the above equation is satisfactory.

In a homogeneous reactor the neutron flux seen by the fuel, moderator, and poisons will be the same. Also, since they are spread throughout the reactor, they all occupy the same volume. This allows the previous equation to be rewritten as shown below.

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^m + \Sigma_a^p} \qquad (3\text{-}1)$$

Equation (3-1) gives an approximation for a heterogeneous reactor if the fuel and moderator are composed of small elements distributed uniformly throughout the reactor.

Since absorption cross sections vary with temperature, it would appear that the thermal utilization factor would vary with a temperature change. But, substitution of the temperature correction formulas (see Module 2) in the above equation will reveal that all terms change by the same amount, and the ratio remains the same. In heterogeneous water-moderated reactors, there is another important factor. When the temperature rises, the water moderator expands, and a significant amount of it will be forced out of the reactor core. This means that N^m, the number of moderator atoms per cm^3, will be reduced, making it less likely for a neutron to be absorbed by a moderator atom. This reduction in N^m results in an increase in thermal utilization as moderator temperature increases because a neutron now has a better chance of hitting a fuel atom. Because of this effect, the temperature coefficient for the thermal utilization factor is positive. The amount of enrichment of uranium-235 and the poison concentration will affect the thermal utilization factor in a similar manner as can be seen from the equation above.

Example:

Calculate the thermal utilization factor for a homogeneous reactor. The macroscopic absorption cross section of the fuel is 0.3020 cm^{-1}, the macroscopic absorption cross section of the moderator is 0.0104 cm^{-1}, and the macroscopic absorption cross section of the poison is 0.0118 cm^{-1}.

Solution:

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^m + \Sigma_a^p}$$

$$= \frac{0.3020\ cm^{-1}}{0.3020\ cm^{-1} + 0.0104 cm^{-1} + 0.0118 cm^{-1}}$$

$$= 0.932$$

Reproduction Factor, (η)

Most of the neutrons absorbed in the fuel cause fission, but some do not. The *reproduction factor* (η) is defined as the ratio of the number of fast neurtons produces by thermal fission to the number of themal neutrons absorbed in the fuel. The reproduction factor is shown below.

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

The reproduction factor can also be stated as a ratio of rates as shown below.

$$\eta = \frac{\text{rate of production of fast neutrons by thermal fission}}{\text{rate of absorption of thermal neutrons by the fuel}}$$

The rate of production of fast neutrons by thermal fission can be determined by the product of the fission reaction rate ($\Sigma_f{}^u\phi^u$) and the average number of neutrons produced per fission (ν). The average number of neutrons released in thermal fission of uranium-235 is 2.42. The rate of absorption of thermal neutrons by the fuel is $\Sigma_a{}^u\phi^u$. Substituting these terms into the equation above results in the following equation.

$$\eta = \frac{\Sigma_f^U\ \phi^U\ \nu}{\Sigma_a^U\ \phi^U}$$

Table 1 lists values of ν and η for fission of several different materials by thermal neutrons and fast neutrons.

TABLE 1
Average Number of Neutrons Liberated in Fission

Fissile Nucleus	Thermal Neutrons		Fast Neutrons	
	ν	η	ν	η
Uranium-233	2.49	2.29	2.58	2.40
Uranium-235	2.42	2.07	2.51	2.35
Plutonium-239	2.93	2.15	3.04	2.90

In the case where the fuel contains several fissionable materials, it is necessary to account for each material. In the case of a reactor core containing both uranium-235 and uranium-238, the reproduction factor would be calculated as shown below.

$$\eta = \frac{N^{U-235}\,\sigma_f^{U-235}\,\nu^{U-235}}{N^{U-235}\,\sigma_a^{U-235} + N^{U-238}\,\sigma_a^{U-238}} \qquad (3\text{-}2)$$

Example:

Calculate the reproduction factor for a reactor that uses 10% enriched uranium fuel. The microscopic absorption cross section for uranium-235 is 694 barns. The cross section for uranium-238 is 2.71 barns. The microscopic fission cross section for uranium-235 is 582 barns. The atom density of uranium-235 is 4.83×10^{21} atoms/cm^3. The atom density of uranium-238 is 4.35×10^{22} atoms/cm^3. ν is 2.42.

Solution:

Use Equation (3-2) to calculate the reproduction factor.

$$\eta = \frac{N^{U-235}\,\sigma_f^{U-235}\,\nu^{U-235}}{N^{U-235}\,\sigma_a^{U-235} + N^{U-238}\,\sigma_a^{U-238}}$$

$$= \frac{\left(4.83 \times 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(582 \times 10^{-24}\ \text{cm}^2\right)\left(2.42\right)}{\left(4.83 \times 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(694 \times 10^{-24}\ \text{cm}^2\right) + \left(4.35 \times 10^{22}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(2.71 \times 10^{-24}\ \text{cm}^2\right)}$$

$$= 1.96$$

As temperature varies, each absorption and fission microscopic cross section varies according to the 1/v relationship (see Module 2). Since both the numerator and the denominator change equally, the net change in η is zero. Therefore, η changes only as uranium-235 enrichment changes. η increases with enrichment because there is less uranium-238 in the reactor making it more likely that a neutron absorbed in the fuel will be absorbed by uranium-235 and cause fission.

To determine the reproduction factor for a single nuclide rather than for a mixture, the calculation may be further simplified to the one shown below.

$$\eta = \frac{\sigma_f \nu}{\sigma_a}$$

Effective Multiplication Factor

The infinite multiplication factor can fully represent only a reactor that is infinitely large, because it assumes that no neutrons leak out of the reactor. To completely describe the neutron life cycle in a real, finite reactor, it is necessary to account for neutrons that leak out. The multiplication factor that takes leakage into account is the *effective multiplication factor* (k_{eff}), which is defined as the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption and leakage in the preceding generation.

The effective multiplication factor may be expressed mathematically as shown below.

$$k_{eff} = \frac{\text{neutron production from fission in one generation}}{\text{neutron absorption in the preceding generation} + \text{neutron leakage in the preceding generation}}$$

So, the value of k_{eff} for a self-sustaining chain reaction of fissions, where the neutron population is neither increasing nor decreasing, is one. The condition where the neutron chain reaction is self-sustaining and the neutron population is neither increasing nor decreasing is referred to as the *critical* condition and can be expressed by the simple equation $k_{eff} = 1$.

If the neutron production is greater than the absorption and leakage, the reactor is called supercritical. In a *supercritical* reactor, k_{eff} is greater than one, and the neutron flux increases each generation. If, on the other hand, the neutron production is less than the absorption and leakage, the reactor is called subcritical. In a *subcritical* reactor, k_{eff} is less than one, and the flux decreases each generation.

When the multiplication factor of a reactor is not equal to exactly one, the neutron flux will change and cause a change in the power level. Therefore, it is essential to know more about how this factor depends upon the contents and construction of the reactor. The balance between production of neutrons and their absorption in the core and leakage out of the core determines the value of the multiplication factor. If the leakage is small enough to be neglected, the multiplication factor depends upon only the balance between production and absorption, and is called the infinite multiplication factor (k_∞) since an infinitely large core can have no leakage. When the leakage is included, the factor is called the effective multiplication factor (k_{eff}).

The effective multiplication factor (k_{eff}) for a finite reactor may be expressed mathematically in terms of the infinite multiplication factor and two additional factors which account for neutron leakage as shown below.

$$k_{eff} = k_\infty \mathcal{L}_f \mathcal{L}_t$$

Fast Non-Leakage Probability ($\mathcal{L}_f$)

In a realistic reactor of finite size, some of the fast neutrons leak out of the boundaries of the reactor core before they begin the slowing down process. The *fast non-leakage probability* ($\mathcal{L}_f$) is defined as the ratio of the number of fast neutrons that do not leak from the reactor core to the number of fast neutrons produced by all fissions. This ratio is stated as follows.

$$\mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

Thermal Non-Leakage Probability ($\mathcal{L}_t$)

Neutrons can also leak out of a finite reactor core after they reach thermal energies. The *thermal non-leakage probability* ($\mathcal{L}_t$) is defined as the ratio of the number of thermal neutrons that do not leak from the reactor core to the number of neutrons that reach thermal energies. The thermal non-leakage probability is represented by the following.

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

The fast non-leakage probability ($\mathcal{L}_f$) and the thermal non-leakage probability ($\mathcal{L}_t$) may be combined into one term that gives the fraction of all neutrons that do not leak out of the reactor core. This term is called the *total non-leakage probability* and is given the symbol $\mathcal{L}_T$, where $\mathcal{L}_T = \mathcal{L}_f\mathcal{L}_t$. $\mathcal{L}_f$ and $\mathcal{L}_t$ are both effected by a change in coolant temperature in a heterogeneous water-cooled, water-moderated reactor. As coolant temperature rises, the coolant expands. The density of the moderator is lower; therefore, neutrons must travel farther while slowing down. This effect increases the probability of leakage and thus decreases the non-leakage probability. Consequently, the temperature coefficient (defined later) for the non-leakage probabilities is negative, because as temperature rises, $\mathcal{L}_f$ and $\mathcal{L}_t$ decrease.

Six Factor Formula

With the inclusion of these last two factors it is possible to determine the fraction of neutrons that remain after every possible process in a nuclear reactor. The effective multiplication factor (k_{eff}) can then be determined by the product of six terms.

$$k_{eff} = \varepsilon \, \mathcal{L}_f \, p \, \mathcal{L}_t \, f \, \eta \qquad (3\text{-}3)$$

Equation (3-3) is called the *six factor formula*. Using this six factor formula, it is possible to trace the entire neutron life cycle from production by fission to the initiation of subsequent fissions. Figure 1 illustrates a neutron life cycle with nominal values provided for each of the six factors. Refer to Figure 1 for the remainder of the discussion on the neutron life cycle and sample calculations. The generation begins with 1000 neutrons. This initial number is represented by N_o. The first process is fast fission and the population has been increased by the neutrons from this fast fission process. From the definition of the fast fission factor it is possible to calculate its value based on the number of neutrons before and after fast fission occur.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

$$= \frac{1040}{1000}$$

$$= 1.04$$

The total number of fast neutrons produced by thermal and fast fission is represented by the quantity $N_o\varepsilon$.

Next, it can be seen that 140 neutrons leak from the core before reaching the thermal energy range. The fast non-leakage probability is calculated from its definition, as shown below.

$$\mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

$$= \frac{1040 - 140}{1040}$$

$$= 0.865$$

The number of neutrons that remain in the core during the slowing down process is represented by the quantity $N_o \, \varepsilon \, \mathcal{L}_f$.

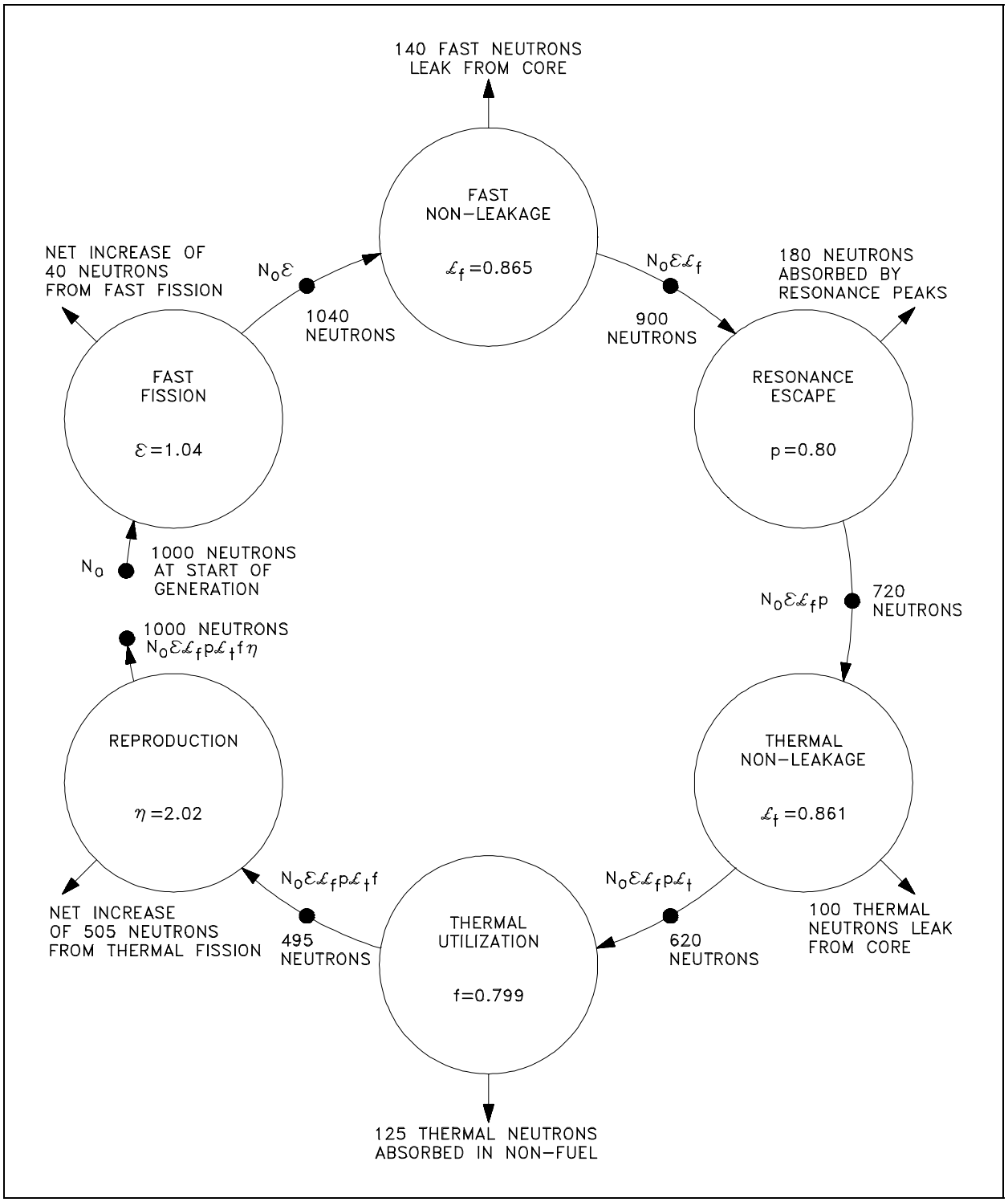


Figure 1 Neutron Life Cycle with k_{eff} = 1

The next step in the analysis is to consider the number of neutrons that are absorbed in the intermediate energy level. The probability of escaping this resonance absorption (p) is stated as follows.

$$p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

$$= \frac{720}{900}$$

$$= 0.80$$

The number of neutrons entering the thermal energy range is now represented by the quantity $N_o \varepsilon \mathcal{L}_f p$.

After reaching thermal energies, 100 neutrons leak from the core. The value for $\mathcal{L}_t$ can be calculated by substitution of the known values in the definition as shown below.

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

$$= \frac{620}{720}$$

$$= 0.861$$

The number of thermal neutrons available for absorption anywhere in the core is represented by the quantity $N_o \varepsilon \mathcal{L}_f p \mathcal{L}_t$.

Figure 1 indicates that 125 neutrons were absorbed in non-fuel materials. Since a total of 620 thermal neutrons were absorbed, the number absorbed by the fuel equals 620 - 125 = 495. Therefore, the thermal utilization factor can be calculated as follows.

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in any reactor material}}$$

$$= \frac{495}{620}$$

$$= 0.799$$

The final factor numerically describes the production of fission neutrons resulting from thermal neutrons being absorbed in the fuel. This factor is called the reproduction factor (η). The value for the reproduction factor can be determined as shown below.

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

$$= \frac{1000}{495}$$

$$= 2.02$$

The number of fission neutrons that exist at the end of the life cycle which are available to start a new generation and cycle is represented by the quantity $N_o\ \varepsilon\ \mathcal{L}_f\ p\ \mathcal{L}_t\ f\ \eta$.

In the example illustrated in Figure 1, k_{eff} is equal to one. Therefore, 1000 neutrons are available to start the next generation.

Example:

10,000 neutrons exist at the beginning of a generation. The values for each factor of the six factor formula are listed below. Calculate the number of neutrons that exist at the points in the neutron life cycle listed below.

1) Number of neutrons that exist after fast fission.
2) Number of neutrons that start to slow down in the reactor.
3) Number of neutrons that reach thermal energies.
4) Number of thermal neutrons that are absorbed in the reactor.
5) Number of thermal neutrons absorbed in the fuel.
6) Number of neutrons produced from thermal fission.

$\varepsilon = 1.031$ $\quad \mathcal{L}_f = 0.889$ $\quad f = 0.751$
$p = 0.803$ $\quad \mathcal{L}_t = 0.905$ $\quad \eta = 2.012$

Solution:

1) $N = N_o\ \varepsilon = 10{,}310$
2) $N = N_o\ \varepsilon\ \mathcal{L}_f = 9{,}166$
3) $N = N_o\ \varepsilon\ \mathcal{L}_f\ p = 7{,}360$
4) $N = N_o\ \varepsilon\ \mathcal{L}_f\ p\ \mathcal{L}_t = 6{,}661$
5) $N = N_o\ \varepsilon\ \mathcal{L}_f\ p\ \mathcal{L}_t\ f = 5{,}002$
6) $N = N_o\ \varepsilon\ \mathcal{L}_f\ p\ \mathcal{L}_t\ f\ \eta = 10{,}065$

Neutron Life Cycle of a Fast Reactor

The neutron life cycle in a fast reactor is markedly different than that for a thermal reactor. In a fast reactor, care is taken during the reactor design to minimize thermalization of neutrons. Virtually all fissions taking place in a fast reactor are caused by fast neutrons. Due to this, many factors that are taken into account by the thermal reactor neutron life cycle are irrelevant to the fast reactor neutron life cycle. The resonance escape probability is not significant because very few neutrons exist at energies where resonance absorption is significant. The thermal non-leakage probability does not exist because the reactor is designed to avoid the thermalization of neutrons. A separate term to deal with fast fission is not necessary because all fission is fast fission and is handled by the reproduction factor.

The thermal utilization factor is modified to describe the utilization of fast neutrons instead of thermal neutrons. The reproduction factor is similarly modified to account for fast fission instead of thermal fission.

Summary

The important information in this chapter is summarized on the following pages.

Neutron Life Cycle Summary

- The infinite multiplication factor, k_{∞}, is the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation.
- The effective multiplication factor, k_{eff}, is the ratio of the number of neutrons produced by fission in one generation to the number of neutrons lost through absorption and leakage in the preceding generation.
- Critical is the condition where the neutron chain reaction is self-sustaining and the neutron population is neither increasing nor decreasing.
- Subcritical is the condition in which the neutron population is decreasing each generation.
- Supercritical is the condition in which the neutron population is increasing each generation.
- The six factor formula is stated as $k_{eff} = \varepsilon \, \mathcal{L}_f \, p \, \mathcal{L}_t \, f \, \eta$. Each of the six factors is defined below.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

$$\mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

$$p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}}$$

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

Neutron Life Cycle Summary (Cont.)

- The thermal utilization factor can be calculated from the macroscopic cross section for absorption of reactor materials using Equation (3-1).

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^m + \Sigma_a^p}$$

- The reproduction factor can be calculated based on the characteristics of the reactor fuel using Equation (3-2).

$$\eta = \frac{N^{U-235}\ \sigma_f^{U-235}\ \nu^{U-235}}{N^{U-235}\ \sigma_a^{U-235} + N^{U-238}\ \sigma_a^{U-238}}$$

- The number of neutrons present at any point in the neutron life cycle can be calculated as the product of the number of neutrons present at the start of the generation and all the factors preceding that point in the life cycle.
- The thermal utilization factor is effected by the enrichment of uranium-235, the amount of neutron poisons, and the moderator-to-fuel ratio.
- The reproduction factor is effected by the enrichment of uranium-235.
- The resonance escape probability is effected by the enrichment of uranium-235, the temperature of the fuel, and the temperature of the moderator.
- An increase in moderator temperature will have the following effects.

  Increase the thermal utilization factor
  Decrease resonance escape probability
  Decrease fast non-leakage probability
  Decrease thermal non-leakage probability

REACTIVITY

Reactivity is a measure of the departure of a reactor from criticality. The reactivity is related to the value of k_{eff}. Reactivity is a useful concept to predict how the neutron population of a reactor will change over time.

EO 1.8 **Given the number of neutrons in a reactor core and the effective multiplication factor, CALCULATE the number of neutrons present after any number of generations.**

EO 1.9 **DEFINE the term reactivity.**

EO 1.10 **CONVERT between reactivity and the associated value of k_{eff}.**

EO 1.11 **CONVERT measures of reactivity between the following units:**

a. **$\Delta k/k$**
b. **$\% \Delta k/k$**
c. **$10^{-4}\ \Delta k/k$**
d. **Percent millirho (pcm)**

EO 1.12 **EXPLAIN the relationship between reactivity coefficients and reactivity defects.**

Application of the Effective Multiplication Factor

When k_{eff} remains constant from generation to generation, it is possible to determine the number of neutrons beginning any particular generation by knowing only the value of k_{eff} and the number of neutrons starting the first generation. If N_o neutrons start the first generation, then $N_o(k_{eff})$ neutrons start the second generation. Equation (3-4) can be used to calculate the number of neutrons after the completion of "n" generations.

$$N_n = N_o \left(k_{eff}\right)^n \qquad (3\text{-}4)$$

Example:

The number of neutrons in the core at time zero is 1000 and k_{eff} = 1.002. Calculate the number of neutrons after 50 generations.

Solution:

Use Equation (3-4) to calculate the number of neutrons.

$$N_n = N_o \left(k_{eff}\right)^n$$

$$N_{50} = 1000 \text{ neutrons } \left(1.002\right)^{50}$$

$$= 1105 \text{ neutrons}$$

Reactivity

If there are N_o neutrons in the preceding generation, then there are $N_o(k_{eff})$ neutrons in the present generation. The numerical change in neutron population is $(N_o k_{eff} - N_o)$. The gain or loss in neutron population $(N_o k_{eff} - N_o)$, expressed as a fraction of the present generation $(N_o k_{eff})$, is shown below.

$$\frac{N_o\ k_{eff} - N_o}{N_o\ k_{eff}}$$

This relationship represents the fractional change in neutron population per generation and is referred to as *reactivity* (ρ). Cancelling out the term N_o from the numerator and denominator, the reactivity is determined as shown in the equation below.

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \tag{3-5}$$

From Equation (3-5) it may be seen that ρ may be positive, zero, or negative, depending upon the value of k_{eff}. The larger the absolute value of reactivity in the reactor core, the further the reactor is from criticality. It may be convenient to think of reactivity as a measure of a reactor's departure from criticality.

Example:

Calculate the reactivity in the reactor core when k_{eff} is equal to 1.002 and 0.998.

Solution:

The reactivity for each case is determined by substituting the value of k_{eff} into Equation (3-5).

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \qquad \rho = \frac{k_{eff} - 1}{k_{eff}}$$

$$= \frac{1.002 - 1}{1.002} \qquad = \frac{0.998 - 1}{0.998}$$

$$= 0.001996 \qquad = -0.0020$$

Units of Reactivity

Reactivity is a dimensionless number. It does not have dimensions of time, length, mass, or any combination of these dimensions. It is simply a ratio of two quantities that are dimensionless. As shown in the calculation in the previous example, the value of reactivity is often a small decimal value. In order to make this value easier to express, artificial units are defined.

By definition, the value for reactivity that results directly from the calculation of Equation (3-5) is in units of $\Delta k/k$. Alternative units for reactivity are $\%\Delta k/k$ and pcm (percent millirho). The conversions between these units of reactivity are shown below.

$$1\% \ \frac{\Delta k}{k} = 0.01 \ \frac{\Delta k}{k}$$

$$1 \text{ pcm} = 0.00001 \ \frac{\Delta k}{k}$$

Another unit of reactivity that is used at some reactors is equivalent to 10^{-4} $\Delta k/k$. This unit of reactivity does not have a unique name. Special units for reactivity that do have unique names are dollars and cents. These units and their applications will be described in a later chapter.

Example:

Convert the values of reactivity listed below to the indicated units.

a. 0.000421 $\Delta k/k$ = _____ pcm
b. 0.0085 $\Delta k/k$ = _____ % $\Delta k/k$
c. 16×10^{-4} $\Delta k/k$ = _____ $\Delta k/k$

Solution:

a. 42.1 pcm
b. 0.85% $\Delta k/k$
c. 0.0016 $\Delta k/k$

If the reactivity is known, the effective multiplication factor can be determined by solving Equation (3-5) for k_{eff} in terms of the reactivity. This results in the following relationship.

$$k_{eff} = \frac{1}{1 - \rho} \tag{3-6}$$

Reactivity must be in units of $\Delta k/k$ for use in Equation (3-6).

Example:

Given a reactivity of -20.0×10^{-4} $\Delta k/k$, calculate k_{eff}.

Solution:

$$k_{eff} = \frac{1}{1 - \rho}$$

$$= \frac{1}{1 - (-20.0 \times 10^{-4})}$$

$$= 0.998$$

Reactivity Coefficients and Reactivity Defects

The amount of reactivity (ρ) in a reactor core determines what the neutron population, and consequently the reactor power, are doing at any given time. The reactivity can be effected by many factors (for example, fuel depletion, temperature, pressure, or poisons). The next several chapters discuss the factors affecting reactivity and how they are used to control or predict reactor behavior.

To quantify the effect that a variation in parameter (that is, increase in temperature, control rod insertion, increase in neutron poison) will have on the reactivity of the core, *reactivity coefficients* are used. Reactivity coefficients are the amount that the reactivity will change for a given change in the parameter. For instance, an increase in moderator temperature will cause a decrease in the reactivity of the core. The amount of reactivity change per degree change in the moderator temperature is the moderator temperature coefficient. Typical units for the moderator temperature coefficient are pcm/^{o}F. Reactivity coefficients are generally symbolized by α_x, where x represents some variable reactor parameter that affects reactivity. The definition of a reactivity coefficient in equation format is shown below.

$$\alpha_x = \frac{\Delta \rho}{\Delta x}$$

If the parameter x increases and positive reactivity is added, then α_x is positive. If the parameter x increases and negative reactivity is added, then α_x is negative.

Reactivity defects ($\Delta\rho$) are the total reactivity change caused by a variation in a parameter. Reactivity defects can be determined by multiplying the change in the parameter by the average value of the reactivity coefficient for that parameter. The equation below shows the general method for relating reactivity coefficients to reactivity defects.

$$\Delta\rho = \alpha_x \, \Delta x$$

Example:

The moderator temperature coefficient for a reactor is -8.2 pcm/^{o}F. Calculate the reactivity defect that results from a temperature decrease of 5^{o}F.

Solution:

$$\Delta \rho = \alpha_T \, \Delta T$$

$$= \left(-8.2 \ \frac{\text{pcm}}{^{\circ}\text{F}} \right) \left(-5 \ ^{\circ}\text{F} \right)$$

$$= 41 \ \text{pcm}$$

The reactivity addition due to the temperature decrease was positive because of the negative temperature coefficient.

Summary

The important information in this chapter is summarized below.

Reactivity Summary

- The number of neutrons present in the core after a given number of generations is calculated using Equation (3-4).

$$N_n = N_o \left(k_{eff}\right)^n$$

- Reactivity is the fractional change in neutron population per generation.
- Reactivity and k_{eff} are represented in Equation (3-5) and Equation (3-6), respectively.

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \qquad k_{eff} = \frac{1}{1 - \rho}$$

- The relationship between units of reactivity are listed below.

$$1\% \ \frac{\Delta k}{k} = 0.01 \ \frac{\Delta k}{k}$$

$$1 \text{ pcm} = 0.00001 \ \frac{\Delta k}{k}$$

- A reactivity coefficient is the amount of change in reactivity per unit change in the parameter. A reactivity defect is the total reactivity change caused by a change in the parameter. The reactivity defect is the product of the reactivity coefficient and the magnitude of the parameter change.

REACTIVITY COEFFICIENTS

Changes in the physical properties of the materials in the reactor will result in changes in the reactivity. Reactivity coefficients are useful in quantifying the reactivity change that will occur due to the change in a physical property such as the temperature of the moderator or fuel.

EO 2.1 **EXPLAIN the conditions of over moderation and under moderation.**

EO 2.2 **EXPLAIN why many reactors are designed to be operated in an under moderated condition.**

EO 2.3 **STATE the effect that a change in moderator temperature will have on the moderator to fuel ratio.**

EO 2.4 **DEFINE the temperature coefficient of reactivity.**

EO 2.5 **EXPLAIN why a negative temperature coefficient of reactivity is desirable.**

EO 2.6 **EXPLAIN why the fuel temperature coefficient is more effective than the moderator temperature coefficient in terminating a rapid power rise.**

EO 2.7 **EXPLAIN the concept of Doppler broadening of resonance absorption peaks.**

EO 2.8 **LIST two nuclides that are present in some types of reactor fuel assemblies that have significant resonance absorption peaks.**

EO 2.9 **DEFINE the pressure coefficient of reactivity.**

EO 2.10 **EXPLAIN why the pressure coefficient of reactivity is usually negligible in a reactor cooled and moderated by a subcooled liquid.**

EO 2.11 **DEFINE the void coefficient of reactivity.**

EO 2.12 **IDENTIFY the moderator conditions under which the void coefficient of reactivity becomes significant.**

Moderator Effects

As discussed in the previous module, a moderator possesses specific desirable characteristics.

(a) large neutron scattering cross section
(b) low neutron absorption cross section
(c) large neutron energy loss per collision

With the exception of the Liquid Metal Fast Breeder Reactor (LMFBR), the remaining major reactor types that are currently employed use moderating materials to reduce fission neutron energies to the thermal range. Light moderators (composed of light nuclei) are found to be more effective than heavy moderators because the light moderator removes more energy per collision than a heavy moderator. Therefore, the neutrons reach thermal energy more rapidly and they are less likely to be lost through resonance absorption.

As discussed in a previous module, the ability of a given material to slow down neutrons is referred to as the macroscopic slowing down power (MSDP) and is defined as the product of the logarithmic energy decrement per collision (ξ) times the macroscopic scattering cross section for neutrons as follows.

$$MSDP = \xi \Sigma_s$$

Macroscopic slowing down power indicates how rapidly slowing down occurs in the material in question, but it does not completely define the effectiveness of the material as a moderator. An element such as boron has a high logarithmic energy decrement and a good slowing down power, but is a poor moderator. It is a poor moderator because of its high probability of absorbing neutrons, and may be accounted for by dividing the macroscopic slowing down power by the macroscopic absorption cross section. This relationship is called the moderating ratio (MR).

$$MR = \frac{\xi \Sigma_s}{\Sigma_a}$$

The moderating ratio is merely the ratio of slowing down power to the macroscopic absorption cross section. The higher the moderating ratio, the more effectively the material performs as a moderator.

Another ratio, the *moderator-to-fuel ratio* (N^m/N^u), is very important in the discussion of moderators. As the reactor designer increases the amount of moderator in the core (that is, N^m/N^u increases), neutron leakage decreases. Neutron absorption in the moderator (Σ_a^m) increases and causes a decrease in the thermal utilization factor. Having insufficient moderator in the core (that is, N^m/N^u decreases) causes an increase in slowing down time and results in a greater loss of neutrons by resonance absorption. This also causes an increase in neutron leakage. The effects of varying the moderator-to-fuel ratio on the thermal utilization factor and the resonance probability are shown in Figure 2.

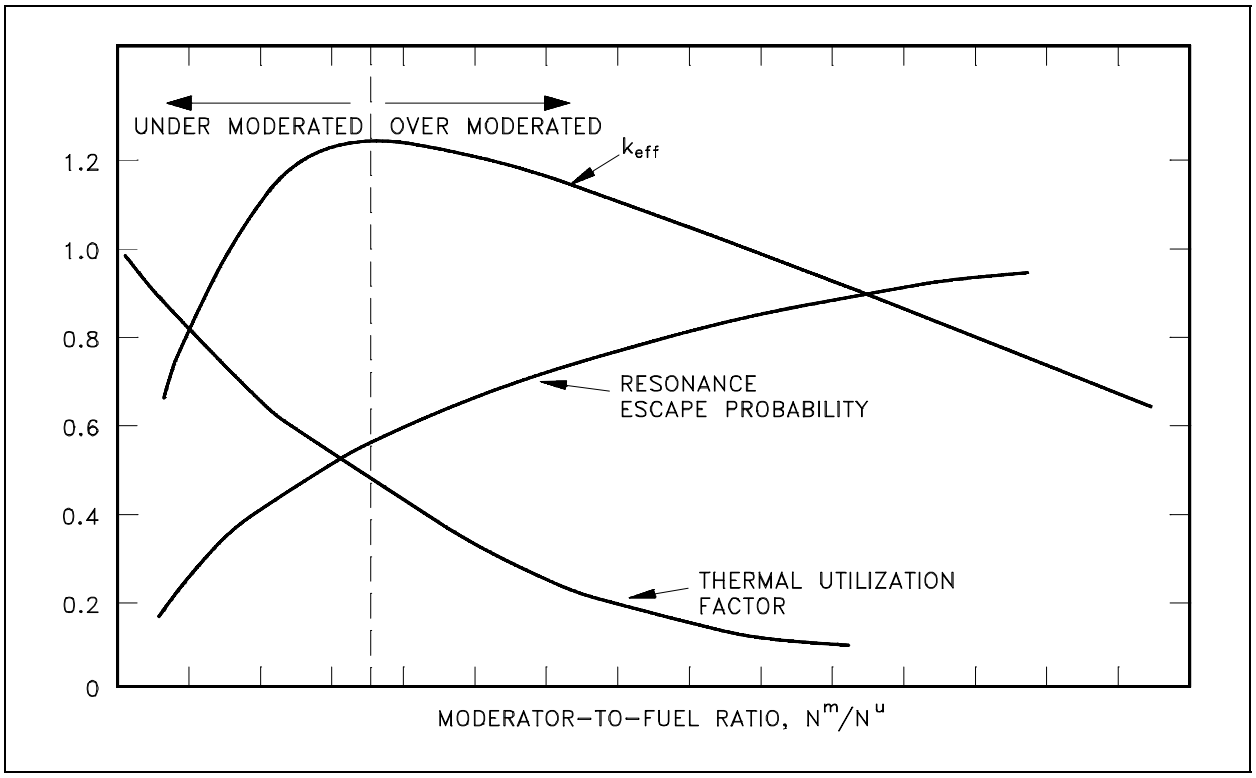


Figure 2 Effects of Over and Under Moderation on k_{eff}

Because the moderator-to-fuel ratio affects the thermal utilization factor and the resonance escape probability, it also affects k_{eff}. The remaining factors in the six factor formula are also affected by the moderator-to-fuel ratio, but to a lesser extent than f and p. As illustrated in Figure 2, which is applicable to a large core fueled with low-enriched fuel, there is an optimum point above which increasing the moderator-to-fuel ratio decreases k_{eff} due to the dominance of the decreasing thermal utilization factor. Below this point, a decrease in the moderator-to-fuel ratio decreases k_{eff} due to the dominance of the increased resonance absorption in the fuel. If the ratio is above this point, the core is said to be over moderated, and if the ratio is below this point, the core is said to be under moderated.

In practice, water-moderated reactors are designed with a moderator-to-fuel ratio so that the reactor is operated in an under moderated condition. The reason that some reactors are designed to be under moderated is if the reactor were over moderated, an increase in temperature would decrease the N^m/N^u due to the expansion of the water as its density became lower. This decrease in N^m/N^u would be a positive reactivity addition, increasing k_{eff} and further raising power and temperature in a dangerous cycle. If the reactor is under moderated, the same increase in temperature results in the addition of negative reactivity, and the reactor becomes more self-regulating.

Moderator Temperature Coefficient

The change in reactivity per degree change in temperature is called the *temperature coefficient of reactivity*. Because different materials in the reactor have different reactivity changes with temperature and the various materials are at different temperatures during reactor operation, several different temperature coefficients are used. Usually, the two dominant temperature coefficients are the moderator temperature coefficient and the fuel temperature coefficient.

The change in reactivity per degree change in moderator temperature is called the *moderator temperature coefficient* of reactivity. The magnitude and sign (+ or -) of the moderator temperature coefficient is primarily a function of the moderator-to-fuel ratio. If a reactor is under moderated, it will have a negative moderator temperature coefficient. If a reactor is over moderated, it will have a positive moderator temperature coefficient. A negative moderator temperature coefficient is desirable because of its self-regulating effect. For example, an increase in reactivity causes the reactor to produce more power. This raises the temperature of the core and adds negative reactivity, which slows down, or turns, the power rise.

Fuel Temperature Coefficient

Another temperature coefficient of reactivity, the fuel temperature coefficient, has a greater effect than the moderator temperature coefficient for some reactors. The *fuel temperature coefficient* is the change in reactivity per degree change in fuel temperature. This coefficient is also called the "prompt" temperature coefficient because an increase in reactor power causes an immediate change in fuel temperature. A negative fuel temperature coefficient is generally considered to be even more important than a negative moderator temperature coefficient because fuel temperature immediately increases following an increase in reactor power. The time for heat to be transferred to the moderator is measured in seconds. In the event of a large positive reactivity insertion, the moderator temperature cannot turn the power rise for several seconds, whereas the fuel temperature coefficient starts adding negative reactivity immediately.

Another name applied to the fuel temperature coefficient of reactivity is the fuel doppler reactivity coefficient. This name is applied because in typical low enrichment, light water-moderated, thermal reactors the fuel temperature coefficient of reactivity is negative and is the result of the doppler effect, also called doppler broadening. The phenomenon of the doppler effect is caused by an apparent broadening of the resonances due to thermal motion of nuclei as illustrated in Figure 3. Stationary nuclei absorb only neutrons of energy E_o. If the nucleus is moving away from the neutron, the velocity (and energy) of the neutron must be greater than E_o to undergo resonance absorption. Likewise, if the nucleus is moving toward the neutron, the neutron needs less energy than E_o to be absorbed. Raising the temperature causes the nuclei to vibrate more rapidly within their lattice structures, effectively broadening the energy range of neutrons that may be resonantly absorbed in the fuel. Two nuclides present in large amounts in the fuel of some reactors with large resonant peaks that dominate the doppler effect are uranium-238 and plutonium-240.

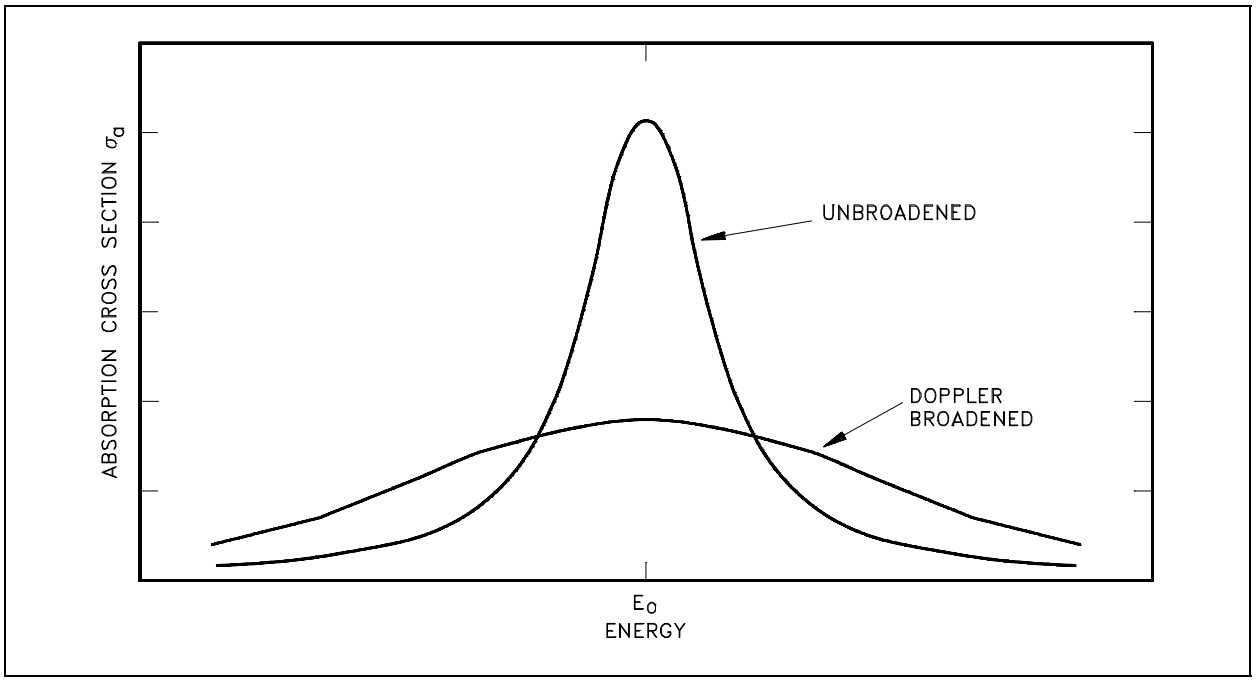


Figure 3 Effect of Fuel Temperature on Resonance Absorption Peaks

Pressure Coefficient

The reactivity in a reactor core can be affected by the system pressure. The *pressure coefficient* of reactivity is defined as the change in reactivity per unit change in pressure. The pressure coefficient of reactivity for the reactor is the result of the effect of pressure on the density of the moderator. For this reason, it is sometimes referred to as the moderator density reactivity coefficient. As pressure increases, density correspondingly increases, which increases the moderator-to-fuel ratio in the core. In the typical under moderated core the increase in the moderator-to-fuel ratio will result in a positive reactivity addition. In reactors that use water as a moderator, the absolute value of the pressure reactivity coefficient is seldom a major factor because it is very small compared to the moderator temperature coefficient of reactivity.

Void Coefficient

In systems with boiling conditions, such as boiling water reactors (BWR), the pressure coefficient becomes an important factor due to the larger density changes that occur when the vapor phase of water undergoes a pressure change. Of prime importance during operation of a BWR, and a factor in some other water-moderated reactors, is the void coefficient. The void coefficient is caused by the formation of steam voids in the moderator. The *void coefficient* of reactivity is defined as the change in reactivity per percent change in void volume. As the reactor power is raised to the point where the steam voids start to form, voids displace moderator from the coolant channels within the core. This displacement reduces the moderator-to-fuel ratio, and in an under moderated core, results in a negative reactivity addition, thereby limiting reactor power rise. The void coefficient is significant in water-moderated reactors that operate at or near saturated conditions.

Summary

The important information in this chapter is summarized below.

Reactivity Coefficients Summary

- The temperature coefficient of reactivity is the change in reactivity per degree change in temperature.
- A reactor is under moderated when a decrease in the moderator-to-fuel ratio decreases k_{eff} due to the increased resonance absorption. A reactor is over moderated when an increase in the moderator-to-fuel ratio decreases k_{eff} due to the decrease in the thermal utilization factor.
- Reactors are usually designed to operate in an under moderated condition so that the moderator temperature coefficient of reactivity is negative.
- Increasing the moderator temperature will decrease the moderator-to-fuel ratio. Decreasing the moderator temperature will increase the moderator-to-fuel ratio.
- A negative temperature coefficient of reactivity is desirable because it makes the reactor more self-regulating. An increase in power, resulting in an increase in temperature, results in negative reactivity addition due to the temperature coefficient. The negative reactivity addition due to the temperature increase will slow or stop the power increase.
- The fuel temperature coefficient is more effective than the moderator temperature coefficient in terminating a rapid power rise because the fuel temperature immediately increases following a power increase, while the moderator temperature does not increase for several seconds.
- The Doppler broadening of resonance peaks occurs because the nuclei may be moving either toward or away from the neutron at the time of interaction. Therefore, the neutron may actually have either slightly more or slightly less than the resonant energy, but still appear to be at resonant energy relative to the nucleus.
- Uranium-238 and plutonium-240 are two nuclides present in some reactor fuels that have large resonance absorption peaks.

Reactivity Coefficients Summary (Cont.)

- The pressure coefficient of reactivity is the change in reactivity per unit change in pressure.
- The pressure coefficient of reactivity is usually negligible in reactors moderated by subcooled liquids because the density of the liquid does not change significantly within the operating pressure range.
- The void coefficient of reactivity is the change in reactivity per unit change in void volume.
- The void coefficient of reactivity becomes significant in a reactor in which the moderator is at or near saturated conditions.

NEUTRON POISONS

In some reactors, neutron-absorbing materials called poisons are intentionally designed into the reactor for specific purposes. Some of these poisons deplete as they absorb neutrons during reactor operation, and others remain relatively constant.

EO 3.1 DEFINE the following terms:

a. Burnable poison
b. Non-burnable poison
c. Chemical shim

EO 3.2 EXPLAIN the use of burnable neutron poisons in a reactor core.

EO 3.3 LIST the advantages and disadvantages of chemical shim over fixed burnable poisons.

EO 3.4 STATE two reasons why fixed non-burnable neutron poisons are used in reactor cores.

EO 3.5 STATE an example of a material used as a fixed non-burnable neutron poison.

Fixed Burnable Poisons

During operation of a reactor the amount of fuel contained in the core constantly decreases. If the reactor is to operate for a long period of time, fuel in excess of that needed for exact criticality must be added when the reactor is built. The positive reactivity due to the excess fuel must be balanced with negative reactivity from neutron-absorbing material. Moveable control rods containing neutron-absorbing material are one method used to offset the excess fuel. Control rods will be discussed in detail in a later chapter. Using control rods alone to balance the excess reactivity may be undesirable or impractical for several reasons. One reason for a particular core design may be that there is physically insufficient room for the control rods and their large mechanisms.

To control large amounts of excess fuel without adding additional control rods, burnable poisons are loaded into the core. *Burnable poisons* are materials that have a high neutron absorption cross section that are converted into materials of relatively low absorption cross section as the result of neutron absorption. Due to the burnup of the poison material, the negative reactivity of the burnable poison decreases over core life. Ideally, these poisons should decrease their negative reactivity at the same rate the fuel's excess positive reactivity is depleted. Fixed burnable poisons are generally used in the form of compounds of boron or gadolinium that are shaped into separate lattice pins or plates, or introduced as additives to the fuel. Since they can usually be distributed more uniformly than control rods, these poisons are less disruptive to the core power distribution.

Soluble Poisons

Soluble poisons, also called *chemical shim*, produce a spatially uniform neutron absorption when dissolved in the water coolant. The most common soluble poison in commercial pressurized water reactors (PWR) is boric acid, which is often referred to as "soluble boron," or simply "solbor." The boric acid in the coolant decreases the thermal utilization factor, causing a decrease in reactivity. By varying the concentration of boric acid in the coolant (a process referred to as boration and dilution), the reactivity of the core can be easily varied. If the boron concentration is increased, the coolant/moderator absorbs more neutrons, adding negative reactivity. If the boron concentration is reduced (dilution), positive reactivity is added. The changing of boron concentration in a PWR is a slow process and is used primarily to compensate for fuel burnout or poison buildup. The variation in boron concentration allows control rod use to be minimized, which results in a flatter flux profile over the core than can be produced by rod insertion. The flatter flux profile is due to the fact that there are no regions of depressed flux like those that would be produced in the vicinity of inserted control rods.

DOE reactors typically do not use soluble neutron poisons during normal operation. Some DOE reactors do, however, include emergency shutdown systems that inject solutions containing neutron poisons into the system that circulates reactor coolant. Various solutions, including sodium polyborate and gadolinium nitrate, are used.

Fixed burnable poisons possess some advantages over chemical shim. Fixed burnable poisons may be discretely loaded in specific locations in order to shape or control flux profiles in the core. Also, fixed burnable poisons do not make the moderator temperature reactivity coefficient less negative as chemical shim does. With chemical shim, as temperature rises and the moderator expands, some moderator is pushed out of the active core area. Boron is also moved out, and this has a positive effect on reactivity. This property of chemical shim limits the allowable boron concentration because any greater concentration makes the moderator temperature coefficient of reactivity positive.

Non-Burnable Poisons

A *non-burnable poison* is one that maintains a constant negative reactivity worth over the life of the core. While no neutron poison is strictly non-burnable, certain materials can be treated as non-burnable poisons under certain conditions. One example is hafnium. The removal (by absorption of neutrons) of one isotope of hafnium leads to the production of another neutron absorber, and continues through a chain of five absorbers. This absorption chain results in a long-lived burnable poison which approximates non-burnable characteristics. Absorbers with low neutron absorption cross sections can also be treated as non-burnable under most conditions.

It is possible to make the reactivity of a poison material that is usually a burnable poison more uniform over core life through the use of self-shielding. In self-shielding, the poison material is thick enough that only the outer layer of the poison is exposed to the neutron flux. The absorptions that take place in the outer layers reduce the number of neutrons that penetrate to the inner material. As the outer layers of poison absorb neutrons and are converted to non-poison materials, the inner layers begin absorbing more neutrons, and the negative reactivity of the poison is fairly uniform.

The normal use of fixed non-burnable poisons is in power shaping, or to prevent excessive flux and power peaking near moderator regions of the reactor.

Summary

The important information in this chapter is summarized below.

Neutron Poisons Summary

- A burnable neutron poison is a material that has a high neutron absorption cross section that is converted into a material of relatively low absorption cross section as the result of neutron absorption.
- A non-burnable neutron poison is a material that has relatively constant neutron absorption characteristics over core life. The absorption of a neutron by one isotope in the material produces another isotope that also has a high absorption cross section.
- Chemical shim is a soluble neutron poison that is circulated in the coolant during normal operation.
- Burnable neutron poisons are used in reactor cores to compensate for the excess positive reactivity of the fuel when the reactor is initially started up.
- Chemical shim has several advantages over fixed burnable poisons.
  Has a spatially uniform effect
  Possible to increase or decrease amount of poison in the core during reactor operation
- Fixed burnable poisons have several advantages over chemical shim.
  Can be used to shape flux profiles
  Do not have an adverse effect on moderator temperature coefficient
- Two reasons for using non-burnable neutron poisons in reactor cores are to shape power and to prevent excessive flux and power peaking near moderator regions.
- An example of a material that is used as a fixed non-burnable neutron poison is hafnium.

XENON

Xenon-135 has a tremendous impact on the operation of a nuclear reactor. It is important to understand the mechanisms that produce and remove xenon from the reactor to predict how the reactor will respond following changes in power level.

EO 4.1 LIST two methods of production and two methods of removal for xenon-135 during reactor operation.

EO 4.2 STATE the equation for equilibrium xenon-135 concentration.

EO 4.3 DESCRIBE how equilibrium xenon-135 concentration varies with reactor power level.

EO 4.4 DESCRIBE the causes and effects of a xenon oscillation.

EO 4.5 DESCRIBE how xenon-135 concentration changes following a reactor shutdown from steady-state conditions.

EO 4.6 EXPLAIN the effect that pre-shutdown power levels have on the xenon-135 concentration after shutdown.

EO 4.7 STATE the approximate time following a reactor shutdown at which the reactor can be considered "xenon free."

EO 4.8 EXPLAIN what is meant by the following terms:

a. Xenon precluded startup
b. Xenon dead time

EO 4.9 DESCRIBE how xenon-135 concentration changes following an increase or a decrease in the power level of a reactor.

Fission Product Poisons

Fission fragments generated at the time of fission decay to produce a variety of fission products. Fission products are of concern in reactors primarily because they become parasitic absorbers of neutrons and result in long term sources of heat. Although several fission products have significant neutron absorption cross sections, xenon-135 and samarium-149 have the most substantial impact on reactor design and operation. Because these two fission product poisons remove neutrons from the reactor, they will have an impact on the thermal utilization factor and thus k_{eff} and reactivity.

Production and Removal of Xenon-135

Xenon-135 has a 2.6 x 10^6 barns neutron absorption cross section. It is produced directly by some fissions, but is more commonly a product of the tellurium-135 decay chain shown below. The fission yield (γ) for xenon-135 is about 0.3%, while γ for tellurium-135 is about 6%.

$$^{135}_{52}\text{Te} \xrightarrow[19.0\ \text{sec}]{\beta^-} {}^{135}_{53}\text{I} \xrightarrow[6.57\ \text{hr}]{\beta^-} {}^{135}_{54}\text{Xe} \xrightarrow[9.10\ \text{hr}]{\beta^-} {}^{135}_{55}\text{Cs} \xrightarrow[2.3\text{x}10^6\ \text{yr}]{\beta^-} {}^{135}_{56}\text{Ba} \quad \text{(stable)}$$

The half-life for tellurium-135 is so short compared to the other half-lives that it can be assumed that iodine-135 is produced directly from fission. Iodine-135 is not a strong neutron absorber, but decays to form the neutron poison xenon-135. Ninety-five percent of all the xenon-135 produced comes from the decay of iodine-135. Therefore, the half-life of iodine-135 plays an important role in the amount of xenon-135 present.

The rate of change of iodine concentration is equal to the rate of production minus the rate of removal. This can be expressed in the equation below.

rate of change of iodine concentration = yield from fission - decay rate - burnup rate

or

$$\frac{dN_I}{dt} = \gamma_I \Sigma_f^{fuel} \phi - \lambda_I N_I - \sigma_a^I N_I \phi$$

where:

N_I = ^{135}I concentration

γ_I = fission yield of ^{135}I

Σ_f^{fuel} = macroscopic fission cross section fuel

ϕ = thermal neutron flux

λ_I = decay constant for ^{135}I

σ_a^I = microscopic absorption cross section ^{135}I

Since the σ_a^I is very small, the burn up rate term may be ignored, and the expression for the rate of change of iodine concentration is modified as shown below.

$$\frac{dN_I}{dt} = \gamma_I \Sigma_f^{fuel} \phi - \lambda_I N_I$$

When the rate of production of iodine equals the rate of removal of iodine, equilibrium exists. The iodine concentration remains constant and is designated $N_I(eq)$. The following equation for the equilibrium concentration of iodine can be determined from the preceding equation by setting the two terms equal to each other and solving for $N_I(eq)$.

$$N_I(eq) = \frac{\gamma_I \Sigma_f^{fuel} \phi}{\lambda_I}$$

Since the equilibrium iodine concentration is proportional to the fission reaction rate, it is also proportional to reactor power level.

The rate of change of the xenon concentration is equal to the rate of production minus the rate of removal. Recall that 5% of xenon comes directly from fission and 95% comes from the decay of iodine. The rate of change of xenon concentration is expressed by the following equations.

$$\begin{pmatrix}\text{rate of change of}\\ \text{xenon-135 concentration}\end{pmatrix} = \begin{pmatrix}\text{xenon-135 yield}\\ \text{from fission}\end{pmatrix} + \begin{pmatrix}\text{iodine-135}\\ \text{decay}\end{pmatrix} - \begin{pmatrix}\text{xenon-135}\\ \text{decay}\end{pmatrix} - \begin{pmatrix}\text{xenon-135}\\ \text{burnup}\end{pmatrix}$$

$$\frac{dN_{Xe}}{dt} = \gamma_{Xe} \Sigma_f^{fuel} \phi + \lambda_I N_I - \lambda_{Xe} N_{Xe} - \sigma_a^{Xe} N_{Xe} \phi$$

where:

N_{Xe} = ^{135}Xe concentration

γ_{Xe} = fission yield of ^{135}Xe

Σ_f^{fuel} = macroscopic fission cross section of the fuel

ϕ = thermal neutron flux

λ_I = decay constant for ^{135}I

N_I = ^{135}I concentration

λ_{Xe} = decay constant for ^{135}Xe

σ_a^{Xe} = microscopic absorption cross section ^{135}Xe

The xenon burnup term above refers to neutron absorption by xenon-135 by the following reaction.

$$^{135}_{54}Xe + ^{1}_{0}n \rightarrow ^{136}_{54}Xe + \gamma$$

Xenon-136 is not a significant neutron absorber; therefore, the neutron absorption by xenon-135 constitutes removal of poison from the reactor. The burnup rate of xenon-135 is dependent upon the neutron flux and the xenon-135 concentration.

The equilibrium concentration of xenon-135 is designated $N_{Xe}(eq)$, and is represented as shown below.

$$N_{Xe}(eq) = \frac{\gamma_{Xe} \Sigma_f^{fuel} \phi + \lambda_I N_I}{\lambda_{Xe} + \sigma_a^{Xe} \phi}$$

For xenon-135 to be in equilibrium, iodine-135 must also be in equilibrium. Substituting the expression for equilibrium iodine-135 concentration into the equation for equilibrium xenon results in the following.

$$N_{Xe}\ (eq) = \frac{\left(\gamma_{Xe} + \gamma_I\right) \Sigma_f^{fuel}\ \phi}{\lambda_{Xe} + \sigma_a^{Xe}\ \phi}$$

From this equation it can be seen that the equilibrium value for xenon-135 increases as power increases, because the numerator is proportional to the fission reaction rate. Thermal flux is also in the denominator; therefore, as the thermal flux exceeds 10^{12} neutrons/cm^2-sec, the $\sigma_a^{Xe}\phi$ term begins to dominate, and at approximately 10^{15} neutrons/cm^2-sec, the xenon-135 concentration approaches a limiting value. The equilibrium iodine-135 and xenon-135 concentrations as a function of neutron flux are illustrated in Figure 4.

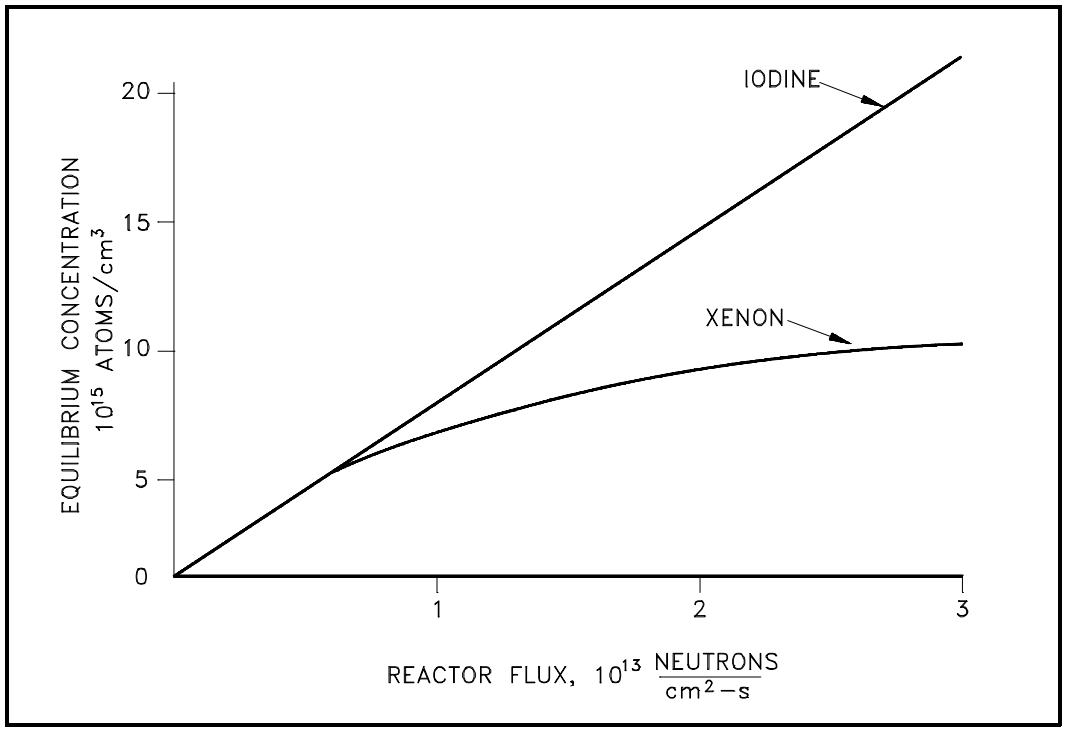


Figure 4 Equilibrium Iodine-135 and Xenon-135 Concentrations Versus Neutron Flux

The higher the power level, or flux, the higher the equilibrium xenon-135 concentration, but equilibrium xenon-135 is not directly proportional to power level. For example, equilibrium xenon-135 at 25% power is more than half the value for equilibrium xenon-135 at 100% power for many reactors. Because the xenon-135 concentration directly affects the reactivity level in the reactor core, the negative reactivity due to the xenon concentrations for different power levels or conditions are frequently plotted instead of the xenon concentration.

Xenon-135 Response to Reactor Shutdown

When a reactor is shutdown, the neutron flux is reduced essentially to zero. Therefore, after shutdown, xenon-135 is no longer produced by fission and is no longer removed by burnup. The only remaining production mechanism is the decay of the iodine-135 which was in the core at the time of shutdown. The only removal mechanism for xenon-135 is decay.

$$\frac{dN_{Xe}}{dt} = \lambda_I N_I - \lambda_{Xe} N_{Xe}$$

Because the decay rate of iodine-135 is faster than the decay rate of xenon-135, the xenon concentration builds to a peak. The peak is reached when the product of the terms $\lambda_I N_I$ is equal to $\lambda_{Xe} N_{Xe}$ (in about 10 to 11 hours). Subsequently, the production from iodine decay is less than the removal of xenon by decay, and the concentration of xenon-135 decreases. The greater the flux level prior to shutdown, the greater the concentration of iodine-135 at shutdown; therefore, the greater the peak in xenon-135 concentration after shutdown. This phenomenon can be seen in Figure 5, which illustrates the negative reactivity value of xenon-135 following shutdown from various neutron flux levels.

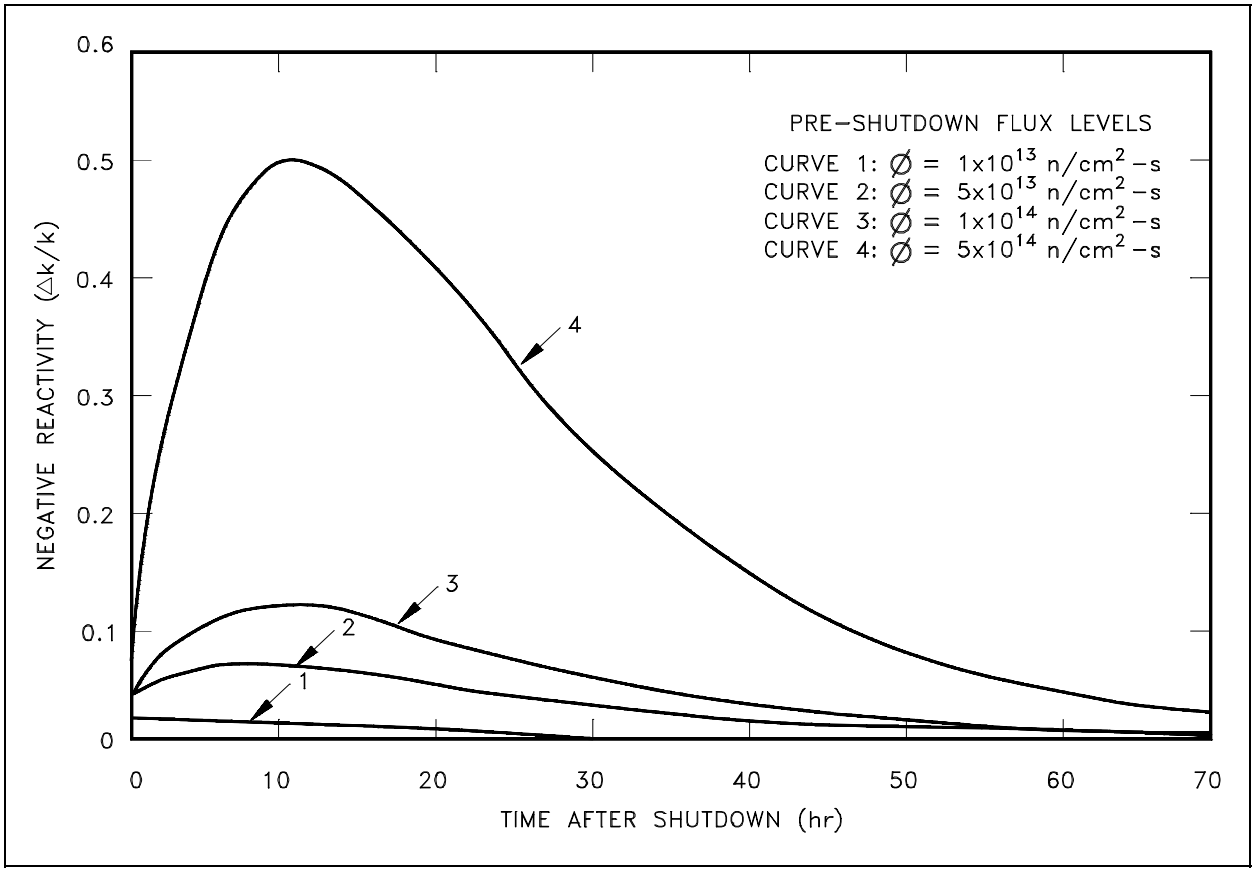


Figure 5 Xenon-135 Reactivity After Reactor Shutdown

Negative xenon reactivity, also called xenon poisoning, may provide sufficient negative reactivity to make the reactor inoperable because there is insufficient positive reactivity available from control rod removal or chemical shim dilution (if used) to counteract it. The inability of the reactor to be started due to the effects of xenon is sometimes referred to as a *xenon precluded startup*. The period of time where the reactor is unable to "override" the effects of xenon is called *xenon dead time*. Because the amount of excess core reactivity available to override the negative reactivity of the xenon is usually less than 10% $\Delta k/k$, thermal power reactors are normally limited to flux levels of about 5×10^{13} neutrons/cm^2-sec so that timely restart can be ensured after shutdown. For reactors with very low thermal flux levels ($\sim 5 \times 10^{12}$ neutrons/cm^2-sec or less), most xenon is removed by decay as opposed to neutron absorption. For these cases, reactor shutdown does not cause any xenon-135 peaking effect.

Following the peak in xenon-135 concentration about 10 hours after shutdown, the xenon-135 concentration will decrease at a rate controlled by the decay of iodine-135 into xenon-135 and the decay rate of xenon-135. For some reactors, the xenon-135 concentration about 20 hours after shutdown from full power will be the same as the equilibrium xenon-135 concentration at full power. About 3 days after shutdown, the xenon-135 concentration will have decreased to a small percentage of its pre-shutdown level, and the reactor can be assumed to be xenon free without a significant error introduced into reactivity calculations.

Xenon-135 Oscillations

Large thermal reactors with little flux coupling between regions may experience spatial power oscillations because of the non-uniform presence of xenon-135. The mechanism is described in the following four steps.

(1) An initial lack of symmetry in the core power distribution (for example, individual control rod movement or misalignment) causes an imbalance in fission rates within the reactor core, and therefore, in the iodine-135 buildup and the xenon-135 absorption.

(2) In the high-flux region, xenon-135 burnout allows the flux to increase further, while in the low-flux region, the increase in xenon-135 causes a further reduction in flux. The iodine concentration increases where the flux is high and decreases where the flux is low.

(3) As soon as the iodine-135 levels build up sufficiently, decay to xenon reverses the initial situation. Flux decreases in this area, and the former low-flux region increases in power.

(4) Repetition of these patterns can lead to xenon oscillations moving about the core with periods on the order of about 15 hours.

With little change in overall power level, these oscillations can change the local power levels by a factor of three or more. In a reactor system with strongly negative temperature coefficients, the xenon-135 oscillations are damped quite readily. This is one reason for designing reactors to have negative moderator-temperature coefficients.

Xenon-135 Response to Reactor Power Changes

During periods of steady state operation, at a constant neutron flux level, the xenon-135 concentration builds up to its equilibrium value for that reactor power in about 40 to 50 hours. Figure 6 illustrates a typical xenon transient that occurs as a result of a change in reactor power level. At time zero, reactor power is raised from 50% power to 100% power. When the reactor power is increased, xenon concentration initially decreases because the burnup is increased at the new higher power level. Because 95% of the xenon production is from iodine-135 decay, which has a 6 to 7 hour half-life, the production of xenon remains constant for several hours. After a few hours (roughly 4 to 6 hours depending on power levels) the rate of production of xenon from iodine and fission equals the rate of removal of xenon by burnup and decay. At this point, the xenon concentration reaches a minimum. The xenon concentration then increases to the new equilibrium level for the new power level in roughly 40 to 50 hours. It should be noted that the magnitude and the rate of change of xenon concentration during the initial 4 to 6 hours following the power change is dependent upon the initial power level and on the amount of change in power level. The xenon concentration change is greater for a larger change in power level.

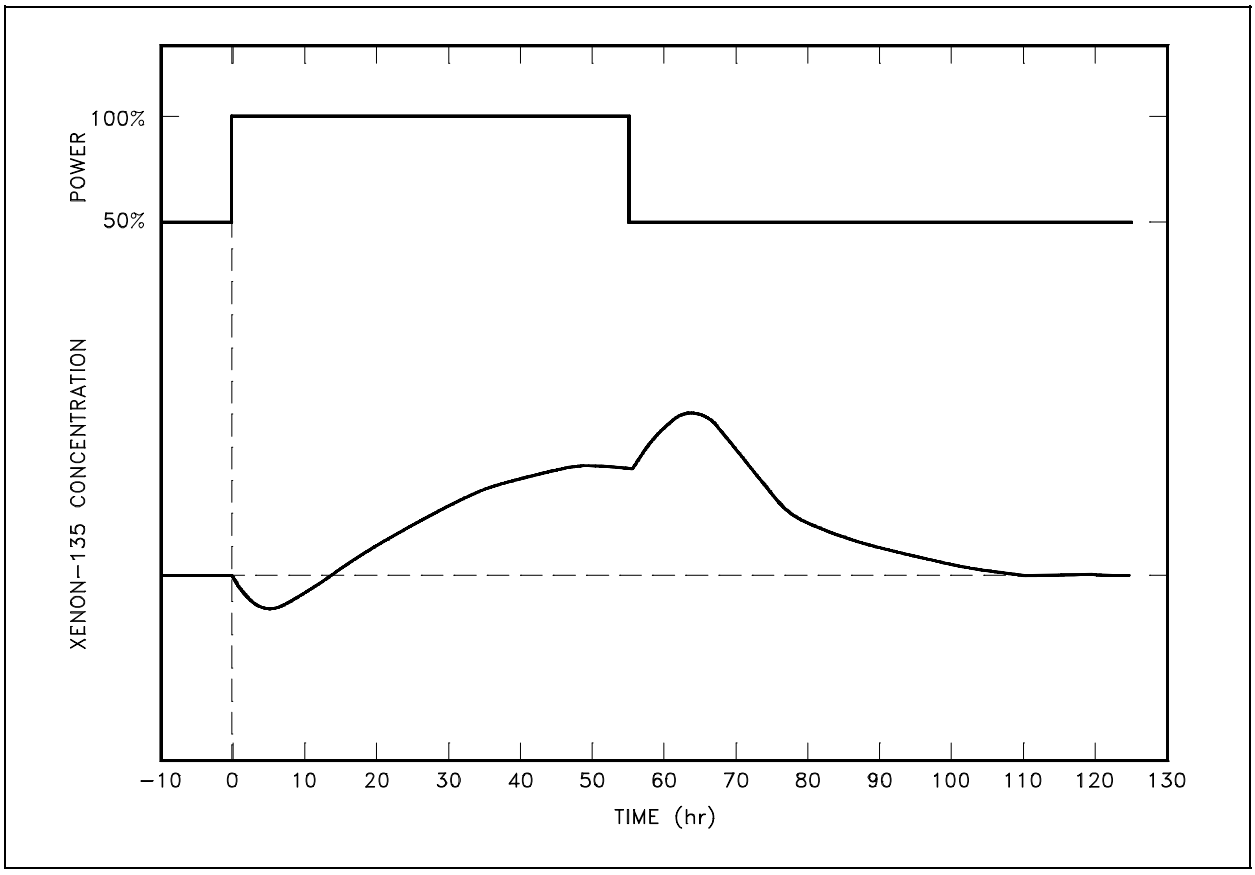


Figure 6 Xenon-135 Variations During Power Changes

When reactor power is decreased from 100% to 50% power (t = 55 hours), the process is reversed. There is an immediate decrease in xenon burnup, which results in an increase in xenon-135 concentration. The iodine-135 concentration is still at the higher equilibrium level for 100% power and is therefore still producing xenon-135 at the higher rate. The xenon-135 concentration continues to rise until the rate of production of xenon-135 becomes equal to the rate of removal (roughly 7 to 8 hours after the initial reduction in power level). The xenon-135 concentration then gradually decreases to the new equilibrium level in about 50 to 60 hours. The magnitude of the xenon peak is greatest if the initial power level is very high.

Maximum peak xenon occurs when a reactor that is operating at 100% equilibrium xenon concentration is suddenly shut down. The most rapid possible burnout of xenon occurs when a reactor is started up and operated at full power while this maximum peak xenon condition exists.

Summary

The important information in this chapter is summarized below.

Xenon Summary

- Xenon-135 is produced directly as a fission product and by the decay of iodine-135 during reactor operation. Xenon-135 is removed from the core by radioactive decay and by neutron absorption during reactor operation.
- The equilibrium concentration for xenon-135 is determined by the following equation.

$$N_{Xe}\ (eq) = \frac{\gamma_{Xe}\ \Sigma_f^{fuel}\ \phi + \lambda_I\ N_I}{\lambda_{Xe} + \sigma_a^{Xe}\ \phi} \quad \text{or} \quad N_{Xe}\ (eq) = \frac{(\gamma_{Xe} + \gamma_I)\ \Sigma_f^{fuel}\ \phi}{\lambda_{Xe} + \sigma_a^{Xe}\ \phi}$$

- The xenon-135 concentration increases with increasing power level in a non-linear manner. Equilibrium xenon-135 concentration reaches a maximum at a flux of about 10^{15} neutrons/cm^2-sec.
- After a power increase, xenon-135 concentration will initially decrease due to the increased removal by burnout. Xenon-135 will reach a minimum about 5 hours after the power increase and then increase to a new, higher equilibrium value as the production from iodine decay increases.

Xenon Summary (Cont.)

- A xenon-135 oscillation may be caused by a rapid perturbation in the core power distribution. The xenon-135 oscillation can change local power levels in the core by a factor of three or more.
- Following a reactor shutdown, xenon-135 concentration will increase due to the decay of the iodine inventory of the core. Xenon-135 will peak approximately 10 hours after the shutdown (from 100%) and then decrease as xenon-135 decay becomes greater than the iodine-135 decay.
- The greater the pre-shutdown power level, the greater the peak value of xenon.
- The core can be considered xenon-free about 3 days after shutdown.
- A xenon precluded startup occurs when there is insufficient reactivity in the control rods to overcome the negative reactivity of xenon-135.
- Xenon dead time is the period of time where the reactor is unable to override the effects of xenon.
- After a power decrease, xenon-135 concentration will initially increase due to production by iodine decay being greater than the burnout. Xenon-135 will reach a maximum about 8 hours after the power decrease and then decrease to a new, lower equilibrium value.

SAMARIUM AND OTHER FISSION PRODUCT POISONS

The fission product poison that has the most significant effect on reactor operations other than xenon-135 is samarium-149. Samarium-149 behaves significantly different from xenon-135 due to its different nuclear properties.

EO 4.10 DESCRIBE how samarium-149 is produced and removed from the reactor core during reactor operation.

EO 4.11 STATE the equation for equilibrium samarium-149 concentration.

EO 4.12 DESCRIBE how equilibrium samarium-149 concentration varies with reactor power level.

EO 4.13 DESCRIBE how samarium-149 concentration changes following a reactor shutdown from steady-state conditions.

EO 4.14 DESCRIBE how samarium-149 concentration changes following a reactor startup.

EO 4.15 STATE the conditions under which helium-3 will have a significant effect on the reactivity of a reactor.

Production and Removal of Samarium-149

Samarium-149 is the second most important fission-product poison because of its high thermal neutron absorption cross section of 4.1×10^4 barns. Samarium-149 is produced from the decay of the neodymium-149 fission fragment as shown in the decay chain below.

$$^{149}_{60}\text{Nd} \xrightarrow[1.72\ \text{hr}]{\beta^-} {}^{149}_{61}\text{Pm} \xrightarrow[53.1\ \text{hr}]{\beta^-} {}^{149}_{62}\text{Sm} \quad \text{(stable)}$$

For the purpose of examining the behavior of samarium-149, the 1.73 hour half-life of neodymium-149 is sufficiently shorter than the 53.1 hour value for promethium-149 that the promethium-149 may be considered as if it were formed directly from fission. This assumption, and neglecting the small amount of promethium burnup, allows the situation to be described as follows.

Rate of change of ^{149}Pm = yield from fission - decay ^{149}Pm concentration

therefore:

$$\frac{dN_{Pm}}{dt} = \gamma_{Pm} \Sigma_f^{fuel} \phi - \lambda_{Pm} N_{Pm}$$

where:

N_{Pm} = ^{149}Pm concentration

γ_{Pm} = ^{149}Pm fission yield

λ_{Pm} = decay constant for ^{149}Pm

Solving for the equilibrium value of promethium-149 gives the following.

$$N_{Pm}(eq) = \frac{\gamma_{Pm} \Sigma_f^{fuel} \phi}{\lambda_{Pm}}$$

The rate of samarium-149 formation is described as follows.

Rate of change of ^{149}Sm = yield from fission + ^{149}Pm decay - ^{149}Sm burnup

therefore:

$$\frac{dN_{Sm}}{dt} = \gamma_{Sm} \Sigma_f^{fuel} \phi + \lambda_{Pm} N_{Pm} - N_{Sm} \sigma_a^{Sm} \phi$$

where:

N_{Sm} = ^{149}Sm concentration

γ_{Sm} = ^{149}Sm fission yield

σ_a^{Sm} = microscopic absorption cross section of ^{149}Sm

The fission yield of samarium-149, however, is nearly zero; therefore, the equation becomes the following.

$$\frac{dN_{Sm}}{dt} = \lambda_{Pm} N_{Pm} - N_{Sm} \sigma_a^{Sm} \phi$$

Solving this equation for the equilibrium concentration of samarium-149 and substituting $\gamma_{Pm}\Sigma_f^{fuel} \phi / \lambda_{Pm}$ for $N_{Pm}(eq)$ yields the following.

$$N_{Sm}(eq) = \frac{\gamma_{Pm} \Sigma_f^{fuel}}{\sigma_a^{Sm}}$$

This expression for equilibrium samarium-149 concentration during reactor operation illustrates that equilibrium samarium-149 concentration is independent of neutron flux and power level. The samarium concentration will undergo a transient following a power level change, but it will return to its original value.

Samarium-149 Response to Reactor Shutdown

Since the neutron flux drops to essentially zero after reactor shutdown, the rate of samarium-149 production becomes the following.

$$\frac{dN_{Sm}}{dt} = \lambda_{Pm} N_{Pm}$$

Because samarium-149 is not radioactive and is not removed by decay, it presents problems somewhat different from those encountered with xenon-135, as illustrated in Figure 7. The equilibrium concentration and the poisoning effect build to an equilibrium value during reactor operation. This equilibrium is reached in approximately 20 days (500 hours), and since samarium-149 is stable, the concentration remains essentially constant during reactor operation. When the reactor is shutdown, the samarium-149 concentration builds up as a result of the decay of the accumulated promethium-149. The buildup of samarium-149 after shutdown depends upon the power level before shutdown. Samarium-149 does not peak as xenon-135 does, but increases slowly to a maximum value as shown in Figure 7. After shutdown, if the reactor is then operated at power, samarium-149 is burned up and its concentration returns to the equilibrium value. Samarium poisoning is minor when compared to xenon poisoning. Although samarium-149 has a constant poisoning effect during long-term sustained operation, its behavior during initial startup and during post-shutdown and restart periods requires special considerations in reactor design.

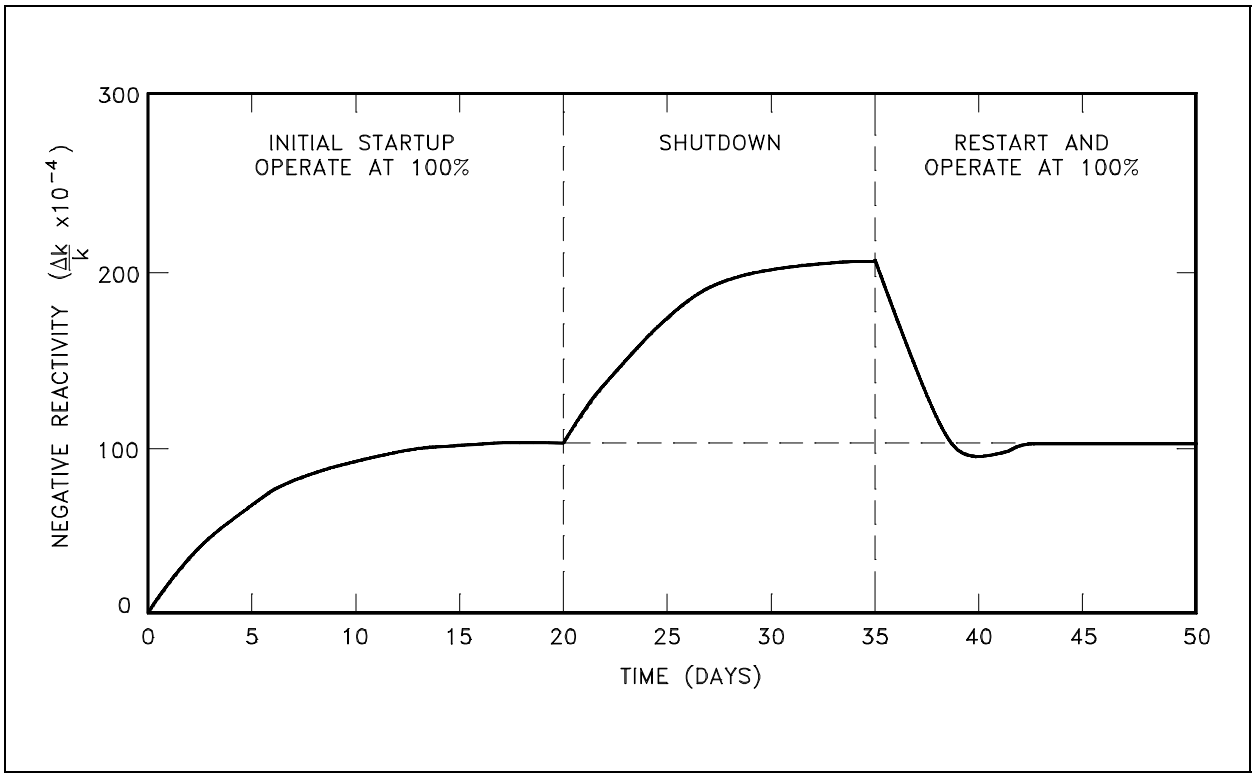


Figure 7 Behavior of Samarium-149 in a Typical Light Water Reactor

The xenon-135 and samarium-149 mechanisms are dependent on their very large thermal neutron cross sections and only affect thermal reactor systems. In fast reactors, neither these nor any other fission products have a major poisoning influence.

Other Neutron Poisons

There are numerous other fission products that, as a result of their concentration and thermal neutron absorption cross section, have a poisoning effect on reactor operation. Individually, they are of little consequence, but "lumped" together they have a significant impact. These are often characterized as "lumped fission product poisons" and accumulate at an average rate of 50 barns per fission event in the reactor.

In addition to fission product poisons, other materials in the reactor decay to materials that act as neutron poisons. An example of this is the decay of tritium to helium-3. Since tritium has a half-life of 12.3 years, normally this decay does not significantly affect reactor operations because the rate of decay of tritium is so slow. However, if tritium is produced in a reactor and then allowed to remain in the reactor during a prolonged shutdown of several months, a

sufficient amount of tritium may decay to helium-3 to add a significant amount of negative reactivity. Any helium-3 produced in the reactor during a shutdown period will be removed during subsequent operation by a neutron-proton reaction.

Summary

The important information in this chapter is summarized below.

Samarium and Other Fission Product Poisons Summary

- Samarium-149 is produced directly from fission and from the decay of promethium-149 during reactor operation. Samarium-149 is removed from the core by neutron absorption.
- The equation for equilibrium samarium-149 concentration is stated below.

$$N_{Sm}(eq) = \frac{\gamma_{Pm}\,\Sigma_f^{fuel}}{\sigma_a^{Sm}}$$

- The equilibrium samarium-149 concentration is independent of power level.
- Following a reactor shutdown, the samarium-149 concentration increases due to the decay of the promethium-149 inventory of the core and the loss of the burnup factor.
- If the reactor is restarted following a shutdown, the samarium-149 concentration decreases as samarium is burned up and returns to its equilibrium operating value.
- Helium-3 will become a significant neutron poison if significant amounts of tritium are left in a reactor during a shutdown period that lasts longer than several months.

CONTROL RODS

Most reactors contain control rods made of neutron absorbing materials that are used to adjust the reactivity of the core. Control rods can be designed and used for coarse control, fine control, or fast shutdowns.

EO 5.1 DESCRIBE the difference between a "grey" neutron absorbing material and a "black" neutron absorbing material.

EO 5.2 EXPLAIN why a "grey" neutron absorbing material may be preferable to a "black" neutron absorbing material for use in control rods.

EO 5.3 EXPLAIN why resonance absorbers are sometimes preferred over thermal absorbers as a control rod material.

EO 5.4 DEFINE the following terms:

a. Integral control rod worth
b. Differential control rod worth

EO 5.5 DESCRIBE the shape of a typical differential control rod worth curve and explain the reason for the shape.

EO 5.6 DESCRIBE the shape of a typical integral control rod worth curve and explain the reason for the shape.

EO 5.7 Given an integral or differential control rod worth curve, CALCULATE the reactivity change due to a control rod movement between two positions.

EO 5.8 Given differential control rod worth data, PLOT differential and integral control rod worth curves.

Selection of Control Rod Materials

Rods of neutron-absorbing material are installed in most reactors to provide precise, adjustable control of reactivity. These rods are able to be moved into or out of the reactor core and typically contain elements such as silver, indium, cadmium, boron, or hafnium.

The material used for the control rods varies depending on reactor design. Generally, the material selected should have a good absorption cross section for neutrons and have a long lifetime as an absorber (not burn out rapidly). The ability of a control rod to absorb neutrons can be adjusted during manufacture. A control rod that is referred to as a "black" absorber absorbs essentially all incident neutrons. A "grey" absorber absorbs only a part of them. While it takes more grey rods than black rods for a given reactivity effect, the grey rods are often preferred because they cause smaller depressions in the neutron flux and power in the vicinity of the rod. This leads to a flatter neutron flux profile and more even power distribution in the core.

If grey rods are desired, the amount of material with a high absorption cross section that is loaded in the rod is limited. Material with a very high absorption cross section may not be desired for use in a control rod, because it will burn out rapidly due to its high absorption cross section. The same amount of reactivity worth can be achieved by manufacturing the control rod from material with a slightly lower cross section and by loading more of the material. This also results in a rod that does not burn out as rapidly.

Another factor in control rod material selection is that materials that resonantly absorb neutrons are often preferred to those that merely have high thermal neutron absorption cross sections. Resonance neutron absorbers absorb neutrons in the epithermal energy range. The path length traveled by the epithermal neutrons in a reactor is greater than the path length traveled by thermal neutrons. Therefore, a resonance absorber absorbs neutrons that have their last collision farther (on the average) from the control rod than a thermal absorber. This has the effect of making the area of influence around a resonance absorber larger than around a thermal absorber and is useful in maintaining a flatter flux profile.

Types of Control Rods

There are several ways to classify the types of control rods. One classification method is by the purpose of the control rods. Three purposes of control rods are listed below.

Shim rods - used for coarse control and/or to remove reactivity in relatively large amounts.

Regulating rods - used for fine adjustments and to maintain desired power or temperature.

Safety rods - provide a means for very fast shutdown in the event of an unsafe condition. Addition of a large amount of negative reactivity by rapidly inserting the safety rods is referred to as a "scram" or "trip."

Not all reactors have different control rods to serve the purposes mentioned above. Depending upon the type of reactor and the controls necessary, it is possible to use dual-purpose or even triple-purpose rods. For example, consider a set of control rods that can insert enough reactivity to be used as shim rods. If the same rods can be operated at slow speeds, they will function as regulating rods. Additionally, these same rods can be designed for rapid insertion, or scram. These rods serve a triple function yet meet other specifications such as precise control, range of control, and efficiency.

Control Rod Effectiveness

The effectiveness of a control rod depends largely upon the value of the ratio of the neutron flux at the location of the rod to the average neutron flux in the reactor. The control rod has maximum effect (inserts the most negative reactivity) if it is placed in the reactor where the flux is maximum. If a reactor has only one control rod, the rod should be placed in the center of the reactor core. The effect of such a rod on the flux is illustrated in Figure 8.

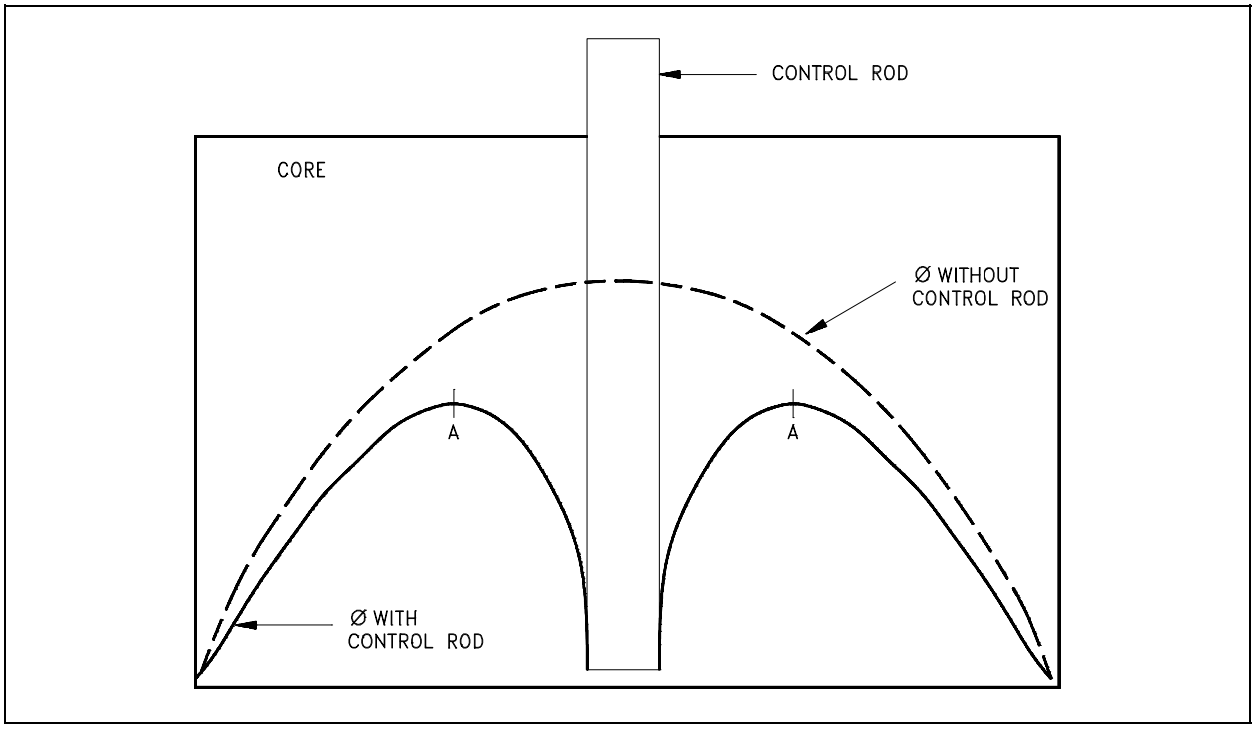


Figure 8 Effect of Control Rod on Radial Flux Distribution

If additional rods are added to this simple reactor, the most effective location is where the flux is maximum, that is, at point A. Numerous control rods are required for a reactor that has a large amount of excess reactivity (that amount of reactivity in excess of that needed to be critical). The exact amount of reactivity that each control rod inserts depends upon the reactor design. The change in reactivity caused by control rod motion is referred to as control rod worth.

Integral and Differential Control Rod Worth

The exact effect of control rods on reactivity can be determined experimentally. For example, a control rod can be withdrawn in small increments, such as 0.5 inch, and the change in reactivity can be determined following each increment of withdrawal. By plotting the resulting reactivity versus the rod position, a graph similar to Figure 9 is obtained. The graph depicts integral control rod worth over the full range of withdrawal. The *integral control rod worth* is the total reactivity worth of the rod at that particular degree of withdrawal and is usually defined to be the greatest when the rod is fully withdrawn.

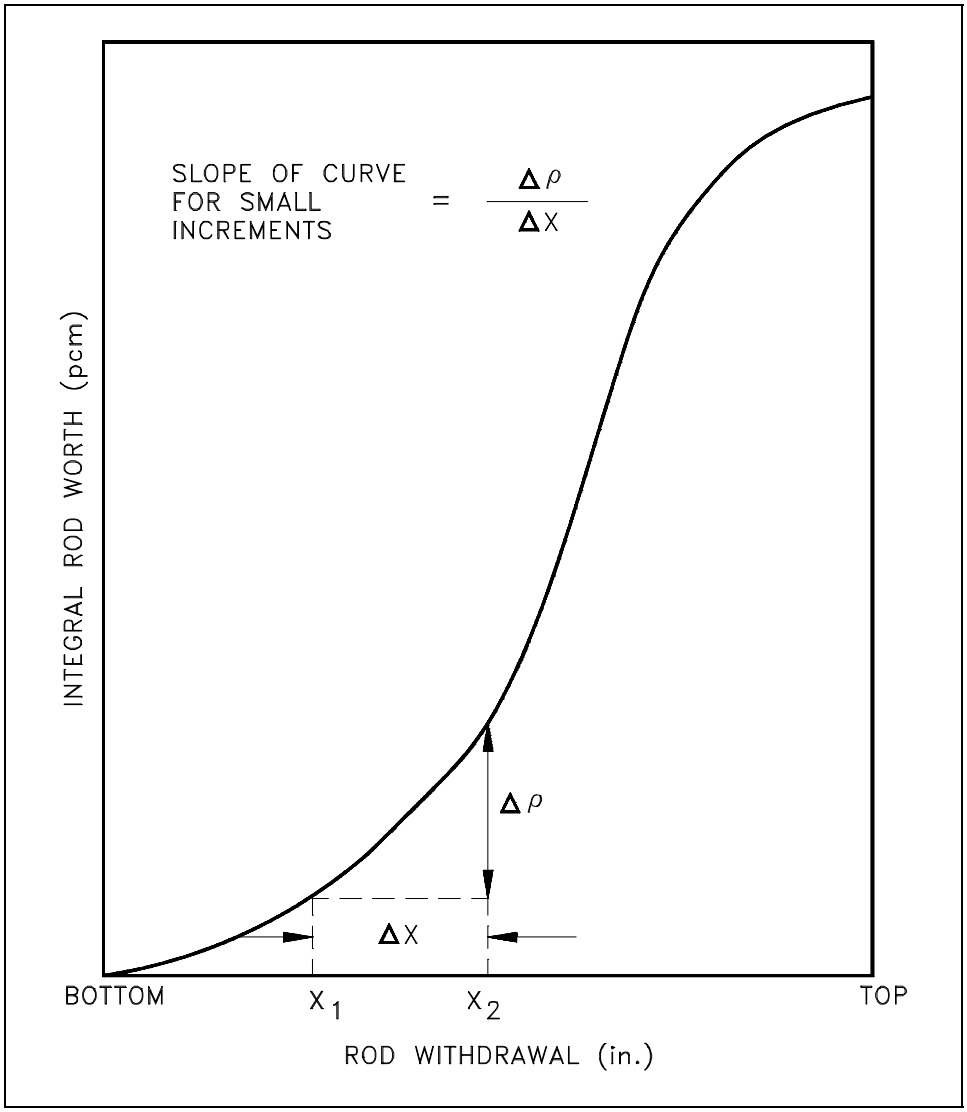


Figure 9 Integral Control Rod Worth

The slope of the curve ($\Delta\rho/\Delta x$), and therefore the amount of reactivity inserted per unit of withdrawal, is greatest when the control rod is midway out of the core. This occurs because the area of greatest neutron flux is near the center of the core; therefore, the amount of change in neutron absorption is greatest in this area. If the slope of the curve for integral rod worth in Figure 9 is taken, the result is a value for rate of change of control rod worth as a function of control rod position. A plot of the slope of the integral rod worth curve, also called the differential control rod worth, is shown in Figure 10. At the bottom of the core, where there are few neutrons, rod movement has little effect so the change in rod worth per inch varies little. As the rod approaches the center of the core its effect becomes greater, and the change in rod worth per inch is greater. At the center of the core the differential rod worth is greatest and varies little with rod motion. From the center of the core to the top, the rod worth per inch is basically the inverse of the rod worth per inch from the center to the bottom.

Differential control rod worth is the reactivity change per unit movement of a rod and is normally expressed as ρ/inch, $\Delta k/k$ per inch, or pcm/inch. The integral rod worth at a given withdrawal is merely the summation of all the differential rod worths up to that point of withdrawal. It is also the area under the differential rod worth curve at any given withdrawal position.

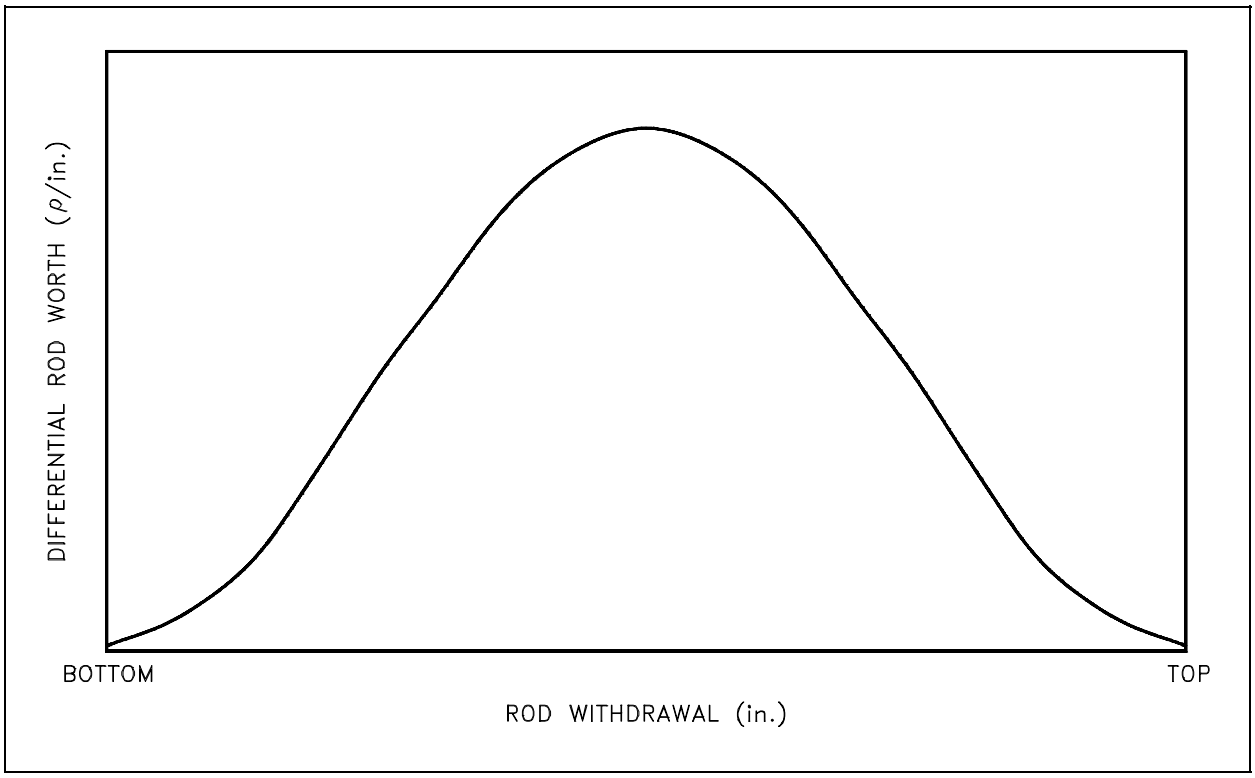


Figure 10 Differential Control Rod Worth

The following exercises are intended to reinforce an understanding of the concepts of integral and differential rod worth.

Example 1:

Using the integral rod worth curve provided in Figure 11, find the reactivity inserted by moving the rod from 12 inches withdrawn out to 18 inches withdrawn.

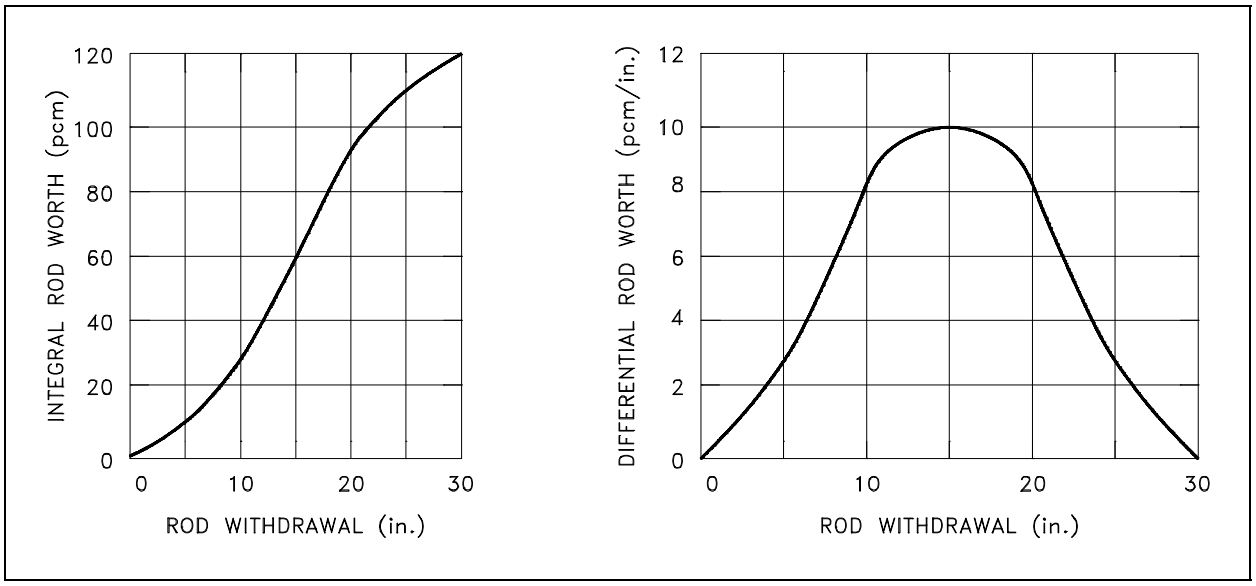


Figure 11 Rod Worth Curves for Example Problems

Solution:

The integral rod worth at 12 inches is 40 pcm and the integral rod worth at 18 inches is 80 pcm.

$$\Delta\rho = \rho_{final} - \rho_{initial}$$

$$= \rho_{18} - \rho_{12}$$

$$= 80 \text{ pcm} - 40 \text{ pcm}$$

$$= 40 \text{ pcm}$$

Example 2:

Using the differential rod worth curve provided in Figure 11, calculate the reactivity inserted by moving the rod from 10 inches withdrawn to 6 inches withdrawn.

Solution:

The solution is basically given by the area under the curve for the interval. The answers obtained in the following approximation may vary slightly depending upon the degree of approximation.

Method 1. Treating the range from 10 inches to 6 inches as a trapezoid, that is, taking the end values of pcm/inch and multiplying their average by the 4 inches moved yields the following.

$$\left(\frac{8\ \frac{\text{pcm}}{\text{inch}} + 3\ \frac{\text{pcm}}{\text{inch}}}{2}\right)(4\ \text{inches}) = -22\ \text{pcm}$$

This is negative because the rod was inserted.

Method 2. Using the central value of rod position at 8 inches yields an average rod worth of 5.5 pcm/inch. Multiplying by the 4 inches of rod travel yields the answer.

(5.5 pcm/in.)(4 in.) = -22 pcm

Method 3. Breaking the rod travel total into two parts (10 inches to 8 inches and 8 inches to 6 inches) yields:

$$\left(\frac{8\ \frac{\text{pcm}}{\text{inch}} + 5.5\ \frac{\text{pcm}}{\text{inch}}}{2}\right)(-2\ \text{inches}) = -13.5\ \text{pcm}$$

$$\left(\frac{5.5\ \frac{\text{pcm}}{\text{inch}} + 3\ \frac{\text{pcm}}{\text{inch}}}{2}\right)(-2\ \text{inches}) = -8.5\ \text{pcm}$$

(- 13.5 pcm) + (- 8.5 pcm) = -22 pcm

In this example the various approximations used did not cause any difference because the problem deals with a section of the curve with an approximately constant slope. To obtain the value over the interval between 8 inches and 20 inches, however, would require the use of several subintervals (as in the last approximation) to obtain an accurate answer.

Example 3:

For the differential rod worth data given below, construct differential and integral rod worth curves.

Interval (inches)	Reactivity Inserted (pcm)
0 to 2	10
2 to 4	20
4 to 6	40
6 to 8	60
8 to 10	60
10 to 12	40
12 to 14	20
14 to 16	10

Solution:

Differential rod worth:

For each interval, the number of pcm/inch must be determined. For example, in the first interval (0 inches to 2 inches), 10 pcm is added. Therefore, the differential rod worth equals an average 5 pcm/inch. This value of differential rod worth is plotted at the center of each interval. The center of the interval 0 inches to 2 inches is 1 inch. The values of pcm/inch for each interval are then listed as shown below and plotted on Figure 12.

Interval Center	pcm/inch
1	5
3	10
5	20
7	30
9	30
11	20
13	10
15	5

Integral rod worth:

To plot the integral rod worth, merely develop a cumulative total of the reactivity added after each interval and plot the summed reactivity insertion vs. rod position as shown in Figure 12.

Interval Endpoint	Summed Reactivity
2	10
4	30
6	70
8	130
10	190
12	230
14	250
16	260

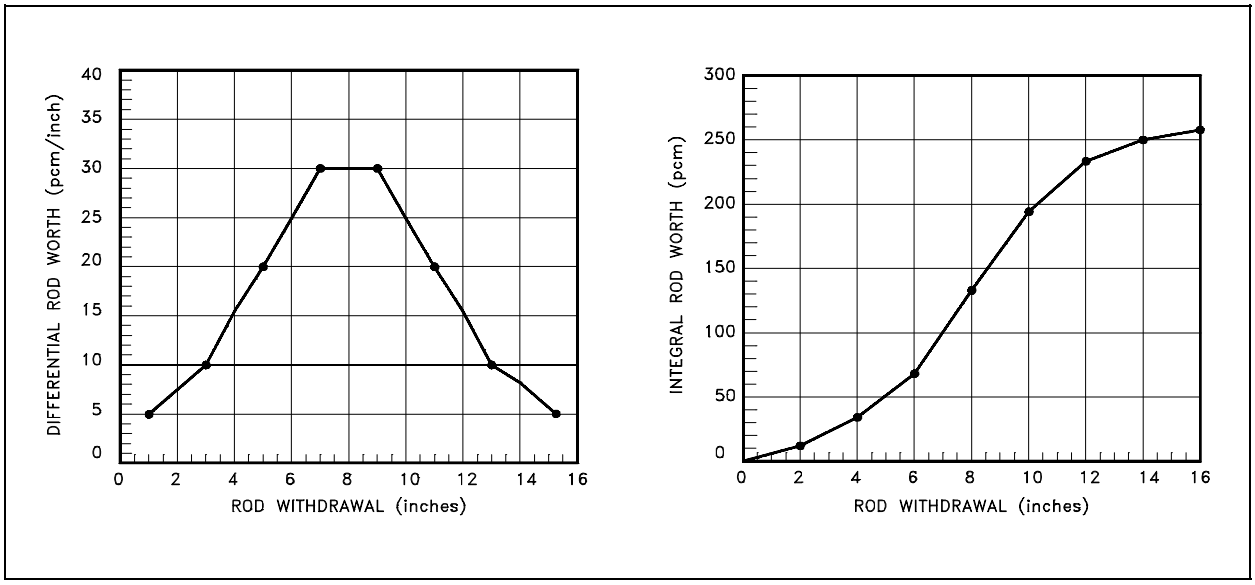


Figure 12 Rod Worth Curves From Example 3

If an integral rod worth curve is supplied, a differential rod worth curve can be generated from the integral rod worth data. Merely select a convenient interval of rod withdrawal, such as 1 inch or 2 inches. Then, determine from the curve the amount of reactivity added for each constant interval of rod withdrawal. A plot of this reactivity addition versus rod withdrawal represents differential rod worth.

Rod Control Mechanisms

The control rod insertion rates on a scram are designed to be sufficient to protect the reactor against damage in all transients that are expected to occur during the life of the reactor.

During normal rod motion, the control rods must be able to move rapidly enough to compensate for the most rapid rate at which positive reactivity is expected to build within the reactor in order to provide positive control. The transient that is normally considered when setting this minimum rod speed is the burnout of maximum peak xenon while at full power. Xenon burnout is usually the most rapid, non-accident transient expected. The maximum rod speed is normally limited in order to reduce the severity of an accident involving the continuous withdrawal of control rods.

Summary

The important information in this chapter is summarized on the following page.

Control Rods Summary

- A black neutron-absorbing material absorbs essentially all incident neutrons. A grey neutron-absorbing material absorbs only part of the incident neutrons.
- A grey neutron-absorbing material may be preferable to a black neutron-absorbing material in the construction of control rods because the grey absorber causes smaller depressions in neutron flux and power in the vicinity of the rod.
- Resonance absorbers are sometimes preferred to thermal absorbers as control rod materials because they have a larger area of influence and result in a flatter flux profile.
- Integral control rod worth is the total reactivity worth of the control rod at a particular degree of withdrawal from the core.
- Differential control rod worth is the reactivity change per unit movement of a control rod.
- The typical differential control rod worth curve has a bell shape. It has very low values at the top and bottom of the core and a maximum value at the center of the core. The curve has this shape because rod worth is related to neutron flux, and flux is highest in the center of the core.
- The typical integral control rod worth curve has an "S" shape. It has a relatively flat slope at the top and bottom of the core and a maximum slope at the center of the core.
- Integral or differential control rod worth curves can be used to determine the reactivity change due to a control rod movement between two positions.
- Integral or differential control rod worth curves can be plotted based on measured control rod worth data.

Department of Energy
Fundamentals Handbook

NUCLEAR PHYSICS AND REACTOR THEORY

Module 4

Reactor Theory (Reactor Operations)

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

REFERENCES

- Foster, Arthur R. and Wright, Robert L. Jr., Basic Nuclear Engineering, 3rd Edition, Allyn and Bacon, Inc., 1977.
- Jacobs, A.M., Kline, D.E., and Remick, F.J., Basic Principles of Nuclear Science and Reactors, Van Nostrand Company, Inc., 1960.
- Kaplan, Irving, Nuclear Physics, 2nd Edition, Addison-Wesley Company, 1962.
- Knief, Ronald Allen, Nuclear Energy Technology: Theory and Practice of Commercial Nuclear Power, McGraw-Hill, 1981.
- Lamarsh, John R., Introduction to Nuclear Engineering, Addison-Wesley Company, 1977.
- Lamarsh, John R., Introduction to Nuclear Reactor Theory, Addison-Wesley Company, 1972.
- General Electric Company, Nuclides and Isotopes: Chart of the Nuclides, 14th Edition, General Electric Company, 1989.
- Academic Program for Nuclear Power Plant Personnel, Volume III, Columbia, MD, General Physics Corporation, Library of Congress Card #A 326517, 1982.
- Glasstone, Samuel, Sourcebook on Atomic Energy, Robert F. Krieger Publishing Company, Inc., 1979.
- Glasstone, Samuel and Sesonske, Alexander, Nuclear Reactor Engineering, 3rd Edition, Van Nostrand Reinhold Company, 1981.

TERMINAL OBJECTIVE

1.0 Given the necessary information and equations, **EXPLAIN** how subcritical multiplication occurs.

ENABLING OBJECTIVES

1.1 **DEFINE** the following terms:

a. Subcritical multiplication
b. Subcritical multiplication factor

1.2 Given a neutron source strength and a subcritical system of known k_{eff}, **CALCULATE** the steady-state neutron level.

1.3 Given an initial count rate and k_{eff}, **CALCULATE** the final count rate that will result from the addition of a known amount of reactivity.

1.4 Given count rates vs. the parameter being adjusted, **ESTIMATE** the value of the parameter at which the reactor will become critical through the use of a 1/M plot.

TERMINAL OBJECTIVE

2.0 Given the necessary information and equations, **DESCRIBE** how power changes in a reactor that is near criticality.

ENABLING OBJECTIVES

2.1 **DEFINE** the following terms:

a. Reactor period
b. Doubling time
c. Reactor startup rate

2.2 **DESCRIBE** the relationship between the delayed neutron fraction, average delayed neutron fraction, and effective delayed neutron fraction.

2.3 **WRITE** the period equation and **IDENTIFY** each symbol.

2.4 Given the reactivity of the core and values for the effective average delayed neutron fraction and decay constant, **CALCULATE** the reactor period and the startup rate.

2.5 Given the initial power level and either the doubling or halving time, **CALCULATE** the power at any later time.

2.6 Given the initial power level and the reactor period, **CALCULATE** the power at any later time.

2.7 **EXPLAIN** what is meant by the terms prompt drop and prompt jump.

2.8 **DEFINE** the term prompt critical.

2.9 **DESCRIBE** reactor behavior during the prompt critical condition.

2.10 **EXPLAIN** the use of measuring reactivity in units of dollars.

TERMINAL OBJECTIVE

3.0 Without references, **EXPLAIN** the concepts concerning reactor startup, operation, and shutdown.

ENABLING OBJECTIVES

3.1 **EXPLAIN** why a startup neutron source may be required for a reactor.

3.2 **LIST** four variables typically involved in a reactivity balance.

3.3 **EXPLAIN** how a reactivity balance may be used to predict the conditions under which the reactor will become critical.

3.4 **LIST** three methods used to shape or flatten the core power distribution.

3.5 **DESCRIBE** the concept of power tilt.

3.6 **DEFINE** the term shutdown margin.

3.7 **EXPLAIN** the rationale behind the one stuck rod criterion.

3.8 **IDENTIFY** five changes that will occur during and after a reactor shutdown that will affect the reactivity of the core.

3.9 **EXPLAIN** why decay heat is present following reactor operation.

3.10 **LIST** three variables that will affect the amount of decay heat present following reactor shutdown.

3.11 **ESTIMATE** the approximate amount of decay heat that will exist one hour after a shutdown from steady state conditions.

Intentionally Left Blank

SUBCRITICAL MULTIPLICATION

Subcritical multiplication is the phenomenon that accounts for the changes in neutron flux that takes place in a subcritical reactor due to reactivity changes. It is important to understand subcritical multiplication in order to understand reactor response to changes in conditions.

EO 1.1 DEFINE the following terms:

a. Subcritical multiplication
b. Subcritical multiplication factor

EO 1.2 Given a neutron source strength and a subcritical system of known k_{eff}, CALCULATE the steady-state neutron level.

EO 1.3 Given an initial count rate and k_{eff}, CALCULATE the final count rate that will result from the addition of a known amount of reactivity.

EO 1.4 Given count rates vs. the parameter being adjusted, ESTIMATE the value of the parameter at which the reactor will become critical through the use of a 1/M plot.

Subcritical Multiplication Factor

When a reactor is in a shutdown condition, neutrons are still present to interact with the fuel. These source neutrons are produced by a variety of methods that were discussed in Module 2. If neutrons and fissionable material are present in the reactor, fission will take place. Therefore, a reactor will always be producing a small number of fissions even when it is shutdown.

Consider a reactor in which k_{eff} is 0.6. If 100 neutrons are suddenly introduced into the reactor, these 100 neutrons that start the current generation will produce 60 neutrons (100 x 0.6) from fission to start the next generation. The 60 neutrons that start the second generation will produce 36 neutrons (60 x 0.6) to start the third generation. The number of neutrons produced by fission in subsequent generations due to the introduction of 100 source neutrons into the reactor is shown below.

Generation	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
Neutrons	100	60	36	22	13	8	5	3	2	1	0	0

Because the reactor is subcritical, neutrons introduced in the reactor will have a decreasing effect on each subsequent generation. The addition of source neutrons to the reactor containing fissionable material has the effect of maintaining a much higher stable neutron level due to the fissions occurring than the neutron level that would result from the source neutrons alone. The effects of adding source neutrons at a rate of 100 neutrons per generation to a reactor with a k_{eff} of 0.6 are shown below.

Generation	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
	100	60	36	22	13	8	5	3	2	1	0	0
		100	60	36	22	13	8	5	3	2	1	0
			100	60	36	22	13	8	5	3	2	1
				100	60	36	22	13	8	5	3	2
					100	60	36	22	13	8	5	3
						100	60	36	22	13	8	5
							100	60	36	22	13	8
								100	60	36	22	13
									100	60	36	22
										100	60	36
											100	60
												100
Total n	100	160	196	218	231	239	244	247	249	250	250	...

A neutron source strength of 100 neutrons per generation will result in 250 neutrons per generation being produced from a combination of sources and fission in a shutdown reactor with a k_{eff} of 0.6. If the value of k_{eff} were higher, the source neutrons would produce a greater number of fission neutrons and their effects would be felt for a larger number of subsequent generations after their addition to the reactor.

The effect of fissions in the fuel increasing the effective source strength of a reactor with a k_{eff} of less than one is *subcritical multiplication*. For a given value of k_{eff} there exists a *subcritical multiplication factor* (M) that relates the source level to the steady-state neutron level of the core. If the value of k_{eff} is known, the amount that the neutron source strength will be multiplied (M) can easily be determined by Equation (4-1).

$$M = \frac{1}{1 - k_{eff}} \qquad (4\text{-}1)$$

Example:

Calculate the subcritical multiplication factors for the following values of k_{eff}.

1) $k_{eff} = 0.6$
2) $k_{eff} = 0.986$

Solution:

1)

$$M = \frac{1}{1 - k_{eff}}$$

$$= \frac{1}{1 - 0.6}$$

$$= 2.5$$

2)

$$M = \frac{1}{1 - k_{eff}}$$

$$= \frac{1}{1 - 0.986}$$

$$= 71.4$$

The example above illustrates that the subcritical multiplication factor will increase as positive reactivity is added to a shutdown reactor, increasing the value of k_{eff}. If the source strength of this reactor were 1000 neutrons/sec, the neutron level would increase from 2500 neutrons/second at a k_{eff} of 0.6 to a neutron level of 71,400 neutrons/sec at a k_{eff} of 0.986.

Effect of Reactivity Changes on Subcritical Multiplication

In a subcritical reactor, the neutron level is related to the source strength by Equation (4-2).

$$N = (S)\ (M) \qquad (4\text{-}2)$$

where:

N = neutron level
S = neutron source strength
M = subcritical multiplication factor

If the term M in Equation (4-2) is replaced by the expression $1/1-k_{eff}$ from Equation (4-1), the following expression results.

$$N = S\left(\frac{1}{1 - k_{eff}}\right) \qquad (4\text{-}3)$$

Example:

A reactor contains a neutron source that produces 110,000 neutrons per second. The reactor has a k_{eff} of 0.986. Calculate the stable total neutron production rate in the reactor.

Solution:

The neutron production rate is calculated using Equation (4-3).

$$N = S\left(\frac{1}{1 - k_{eff}}\right)$$

$$= 110{,}000\ \frac{\text{neutrons}}{\text{second}}\left(\frac{1}{1 - 0.986}\right)$$

$$= 7.86 \times 10^{6}\ \frac{\text{neutrons}}{\text{second}}$$

To this point it has been necessary to know the neutron source strength of the reactor in order to use the concept of subcritical multiplication. In most reactors the actual strength of the neutron sources is difficult, if not impossible, to determine. Even though the actual source strength may not be known, it is still possible to relate the change in reactivity to a change in neutron level.

Consider a reactor at two different times when k_{eff} is two different values, k_1 and k_2. The neutron level at each time can be determined based on the neutron source strength and the subcritical multiplication factor using Equation (4-3).

$$N_1 = S\left(\frac{1}{1 - k_1}\right) \qquad N_2 = S\left(\frac{1}{1 - k_2}\right)$$

The equation for N_1 can be divided by the equation for N_2.

$$\frac{N_1}{N_2} = \frac{S\left(\frac{1}{1 - k_1}\right)}{S\left(\frac{1}{1 - k_2}\right)}$$

$$\frac{N_1}{N_2} = \frac{1 - k_2}{1 - k_1}$$

Because the source strength appears in both the numerator and denominator, it cancels out of the equation. Therefore, the neutron level at any time can be determined based on the neutron level present at any other time provided the values of k_{eff} or reactivity for both times are known.

The neutron level in a shutdown reactor is typically monitored using instruments that measure the neutron leakage out of the reactor. The neutron leakage is proportional to the neutron level in the reactor. Typical units for displaying the instrument reading are counts per second (cps). Because the instrument count rate is proportional to the neutron level, the above equation can be restated as shown in Equation (4-4).

$$\frac{CR_1}{CR_2} = \frac{1 - k_2}{1 - k_1} \qquad (4\text{-}4)$$

where:

CR_1 = count rate at time 1
CR_2 = count rate at time 2
k_1 = k_{eff} at time 1
k_2 = k_{eff} at time 2

Equation (4-4) is very useful during the shutdown operation of a reactor. Before adding positive reactivity to a reactor, it is possible to predict the effect the reactivity addition will have on the neutron level.

Example:

A reactor that has a reactivity of -1000 pcm has a count rate of 42 counts per second (cps) on the neutron monitoring instrumentation. Calculate what the neutron level should be after a positive reactivity insertion of 500 pcm from the withdrawal of control rods.

Solution:

Step 1: Determine the initial value of k_{eff} for the core.

$$k_1 = \frac{1}{1 - \rho_1}$$

$$= \frac{1}{1 - (-0.01000)}$$

$$= 0.9901$$

Step 2: Determine the final value of k_{eff} for the core. The final value of reactivity will be -500 pcm (-1000 + 500).

$$k_2 = \frac{1}{1 - \rho_2}$$

$$= \frac{1}{1 - (-0.00500)}$$

$$= 0.9950$$

Step 3: Use Equation (4-4) to determine the final count rate.

$$\frac{CR_1}{CR_2} = \frac{1 - k_2}{1 - k_1}$$

$$CR_2 = CR_1 \left(\frac{1 - k_1}{1 - k_2} \right)$$

$$= 42 \text{ cps} \left(\frac{1 - 0.9901}{1 - 0.9950} \right)$$

$$= 83 \text{ cps}$$

Notice from this example that the count rate doubled as the reactivity was halved (e.g., reactivity was changed from -1000 pcm to -500 pcm).

Use of 1/M Plots

Because the subcritical multiplication factor is related to the value of k_{eff}, it is possible to monitor the approach to criticality through the use of the subcritical multiplication factor. As positive reactivity is added to a subcritical reactor, k_{eff} will get nearer to one. As k_{eff} gets nearer to one, the subcritical multiplication factor (M) gets larger. The closer the reactor is to criticality, the faster M will increase for equal step insertions of positive reactivity. When the reactor becomes critical, M will be infinitely large. For this reason, monitoring and plotting M during an approach to criticality is impractical because there is no value of M at which the reactor clearly becomes critical.

Instead of plotting M directly, its inverse (1/M) is plotted on a graph of 1/M versus rod height.

$$M = \frac{1}{1 - k_{eff}}$$

$$\frac{1}{M} = 1 - k_{eff}$$

As control rods are withdrawn and k_{eff} approaches one and M approaches infinity, 1/M approaches zero. For a critical reactor, 1/M is equal to zero. A true 1/M plot requires knowledge of the neutron source strength. Because the actual source strength is usually unknown, a reference count rate is substituted, and the calculation of the factor 1/M is through the use of Equation (4-5).

$$\frac{1}{M} = \frac{CR_o}{CR} \qquad (4\text{-}5)$$

where:

1/M = inverse multiplication factor
CR_o = reference count rate
CR = current count rate

In practice, the reference count rate used is the count rate prior to the beginning of the reactivity change. The startup procedures for many reactors include instructions to insert positive reactivity in incremental steps with delays between the reactivity insertions to allow time for subcritical multiplication to increase the steady-state neutron population to a new, higher level and allow more accurate plotting of 1/M. The neutron population will typically reach its new steady-state value within 1-2 minutes, but the closer the reactor is to criticality, the longer the time will be to stabilize the neutron population.

Example:

Given the following rod withdrawal data, construct a 1/M plot and estimate the rod position when criticality would occur. The initial count rate on the nuclear instrumentation prior to rod withdrawal is 50 cps.

Rod Withdrawal (inches)	Count Rate (cps)
2	55
4	67
6	86
8	120
10	192
12	500

Solution:

Step 1: Calculate 1/M for each of the rod positions using equation (4-5). The reference count rate is 50 cps at a rod position of zero.

Rod Withdrawal (inches)	Count Rate (cps)	CR_o/CR
0	50	1
2	55	0.909
4	67	0.746
6	86	0.581
8	120	0.417
10	192	0.260
12	500	0.100

Step 2: Plotting these values, as shown in Figure 1, and extrapolating to a 1/M value of 0 reveals that the reactor will go critical at approximately 13 inches of rod withdrawal.

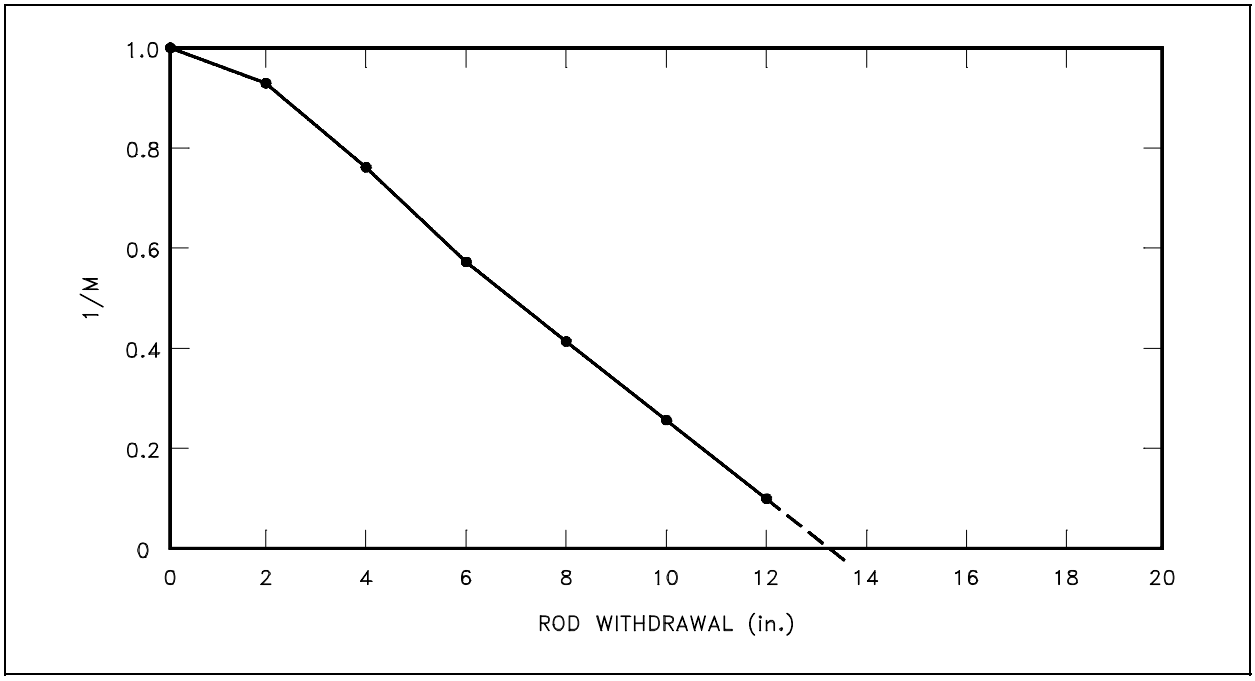


Figure 1 1/M Plot vs. Rod Withdrawal

Summary

The important information in this chapter is summarized below.

Subcritical Multiplication Summary

- Subcritical multiplication is the effect of fissions in the fuel increasing the effective source strength of a reactor with a k_{eff} less than one.
- Subcritical multiplication factor is the factor that relates the source level to the steady-state neutron level of the core.
- The steady-state neutron level of a subcritical reactor can be calculated based on the source strength and k_{eff} using Equation (4-3).

$$N = S\left(\frac{1}{1 - k_{eff}}\right)$$

- The count rate expected in a subcritical reactor following a change in reactivity can be calculated based on the initial count rate, initial k_{eff}, and amount of reactivity addition using Equation (4-4).

$$\frac{CR_1}{CR_2} = \frac{1 - k_2}{1 - k_1}$$

- 1/M plots can be used to predict the point of criticality.

REACTOR KINETICS

The response of neutron flux and reactor power to changes in reactivity is much different in a critical reactor than in a subcritical reactor. The reliance of the chain reaction on delayed neutrons makes the rate of change of reactor power controllable.

EO 2.1 DEFINE the following terms:

a. Reactor period
b. Doubling time
c. Reactor startup rate

EO 2.2 DESCRIBE the relationship between the delayed neutron fraction, average delayed neutron fraction, and effective delayed neutron fraction.

EO 2.3 WRITE the period equation and IDENTIFY each symbol.

EO 2.4 Given the reactivity of the core and values for the effective average delayed neutron fraction and decay constant, CALCULATE the reactor period and the startup rate.

EO 2.5 Given the initial power level and either the doubling or halving time, CALCULATE the power at any later time.

EO 2.6 Given the initial power level and the reactor period, CALCULATE the power at any later time.

EO 2.7 EXPLAIN what is meant by the terms prompt drop and prompt jump.

EO 2.8 DEFINE the term prompt critical.

EO 2.9 DESCRIBE reactor behavior during the prompt critical condition.

EO 2.10 EXPLAIN the use of measuring reactivity in units of dollars.

Reactor Period (τ)

The *reactor period* is defined as the time required for reactor power to change by a factor of "e," where "e" is the base of the natural logarithm and is equal to about 2.718. The reactor period is usually expressed in units of seconds. From the definition of reactor period, it is possible to develop the relationship between reactor power and reactor period that is expressed by Equation (4-6).

$$P = P_o \, e^{t/\tau} \tag{4-6}$$

where:

P = transient reactor power
P_o = initial reactor power
τ = reactor period (seconds)
t = time during the reactor transient (seconds)

The smaller the value of τ, the more rapid the change in reactor power. If the reactor period is positive, reactor power is increasing. If the reactor period is negative, reactor power is decreasing.

There are numerous equations used to express reactor period, but Equation (4-7) shown below, or portions of it, will be useful in most situations. The first term in Equation (4-7) is the prompt term and the second term is the delayed term.

$$\tau = \frac{\ell^*}{\rho} + \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff} \, \rho + \dot{\rho}} \tag{4-7}$$

where:

ℓ^* = prompt generation lifetime
$\bar{\beta}_{eff}$ = effective delayed neutron fraction
ρ = reactivity
λ_{eff} = effective delayed neutron precursor decay constant
$\dot{\rho}$ = rate of change of reactivity

Effective Delayed Neutron Fraction

Recall that β, the *delayed neutron fraction*, is the fraction of all fission neutrons that are born as delayed neutrons. The value of β depends upon the actual nuclear fuel used. As discussed in Module 1, the delayed neutron precursors for a given type of fuel are grouped on the basis of half-life. The following table lists the fractional neutron yields for each delayed neutron group of three common types of fuel.

TABLE 1
Delayed Neutron Fractions for Various Fuels

Group	Half-Life (sec)	Uranium-235	Uranium-238	Plutonium-239
1	55.6	0.00021	0.0002	0.00021
2	22.7	0.00141	0.0022	0.00182
3	6.22	0.00127	0.0025	0.00129
4	2.30	0.00255	0.0061	0.00199
5	0.61	0.00074	0.0035	0.00052
6	0.23	0.00027	0.0012	0.00027
TOTAL	-	0.00650	0.0157	0.00200

The term $\bar{\beta}$ (pronounced beta-bar) is the *average delayed neutron fraction*. The value of $\bar{\beta}$ is the weighted average of the total delayed neutron fractions of the individual types of fuel. Each total delayed neutron fraction value for each type of fuel is weighted by the percent of total neutrons that the fuel contributes through fission. If the percentage of fissions occurring in the different types of fuel in a reactor changes over the life of the core, the average delayed neutron fraction will also change. For a light water reactor using low enriched fuel, the average delayed neutron fraction can change from 0.0070 to 0.0055 as uranium-235 is burned out and plutonium-239 is produced from uranium-238.

Delayed neutrons do not have the same properties as prompt neutrons released directly from fission. The average energy of prompt neutrons is about 2 MeV. This is much greater than the average energy of delayed neutrons (about 0.5 MeV). The fact that delayed neutrons are born at lower energies has two significant impacts on the way they proceed through the neutron life cycle. First, delayed neutrons have a much lower probability of causing fast fissions than prompt neutrons because their average energy is less than the minimum required for fast fission to occur. Second, delayed neutrons have a lower probability of leaking out of the core while they are at fast energies, because they are born at lower energies and subsequently travel a shorter distance as fast neutrons. These two considerations (lower fast fission factor and higher fast non-leakage probability for delayed neutrons) are taken into account by a term called the *importance factor* (I). The importance factor relates the average delayed neutron fraction to the effective delayed neutron fraction.

The *effective delayed neutron fraction* $\left(\bar{\beta}_{eff}\right)$ is defined as the fraction of neutrons at thermal energies which were born delayed. The effective delayed neutron fraction is the product of the average delayed neutron fraction and the importance factor.

$$\bar{\beta}_{eff} = \bar{\beta}\ I$$

where:

$\bar{\beta}_{eff}$ = effective delayed neutron fraction
$\bar{\beta}$ = average delayed neutron fraction
I = importance factor

In a small reactor with highly enriched fuel, the increase in fast non-leakage probability will dominate the decrease in the fast fission factor, and the importance factor will be greater than one. In a large reactor with low enriched fuel, the decrease in the fast fission factor will dominate the increase in the fast non-leakage probability and the importance factor will be less than one (about 0.97 for a commercial PWR).

Effective Delayed Neutron Precursor Decay Constant

Another new term has been introduced in the reactor period (τ) equation. That term is λ_{eff} (pronounced lambda effective), the *effective delayed neutron precursor decay constant*. The decay rate for a given delayed neutron precursor can be expressed as the product of precursor concentration and the decay constant (λ) of that precursor. The decay constant of a precursor is simply the fraction of an initial number of the precursor atoms that decays in a given unit time. A decay constant of 0.1 sec^{-1}, for example, implies that one-tenth, or ten percent, of a sample of precursor atoms decays within one second. The value for the effective delayed neutron precursor decay constant, λ_{eff}, varies depending upon the balance existing between the concentrations of the precursor groups and the nuclide(s) being used as the fuel.

If the reactor is operating at a constant power, all the precursor groups reach an equilibrium value. During an up-power transient, however, the shorter-lived precursors decaying at any given instant were born at a higher power level (or flux level) than the longer-lived precursors decaying at the same instant. There is, therefore, proportionately more of the shorter-lived and fewer of the longer-lived precursors decaying at that given instant than there are at constant power. The value of λ_{eff} is closer to that of the shorter-lived precursors.

During a down-power transient the longer-lived precursors become more significant. The longer-lived precursors decaying at a given instant were born at a higher power level (or flux level) than the shorter-lived precursors decaying at that instant. Therefore, proportionately more of the longer-lived precursors are decaying at that instant, and the value of λ_{eff} approaches the values of the longer-lived precursors.

Approximate values for λ_{eff} are 0.08 sec^{-1} for steady-state operation, 0.1 sec^{-1} for a power increase, and 0.05 sec^{-1} for a power decrease. The exact values will depend upon the materials used for fuel and the value of the reactivity of the reactor core.

Returning now to Equation (4-7) for reactor period.

$$\tau = \underbrace{\frac{\ell^*}{\rho}}_{\left(\begin{array}{c}\text{prompt}\\ \text{term}\end{array}\right)} + \underbrace{\frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho + \dot{\rho}}}_{\left(\begin{array}{c}\text{delayed}\\ \text{term}\end{array}\right)}$$

If the positive reactivity added is less than the value of $\bar{\beta}_{eff}$, the emission of prompt fission neutrons alone is not sufficient to overcome losses to non-fission absorption and leakage. If delayed neutrons were not being produced, the neutron population would decrease as long as the reactivity of the core has a value less than the effective delayed neutron fraction. The positive reactivity insertion is followed immediately by a small immediate power increase called the *prompt jump*. This power increase occurs because the rate of production of prompt neutrons changes abruptly as the reactivity is added. Recall from an earlier module that the generation time for prompt neutrons is on the order of 10^{-13} seconds. The effect can be seen in Figure 2. After the prompt jump, the rate of change of power cannot increase any more rapidly than the built-in time delay the precursor half-lives allow. Therefore, the power rise is controllable, and the reactor can be operated safely.

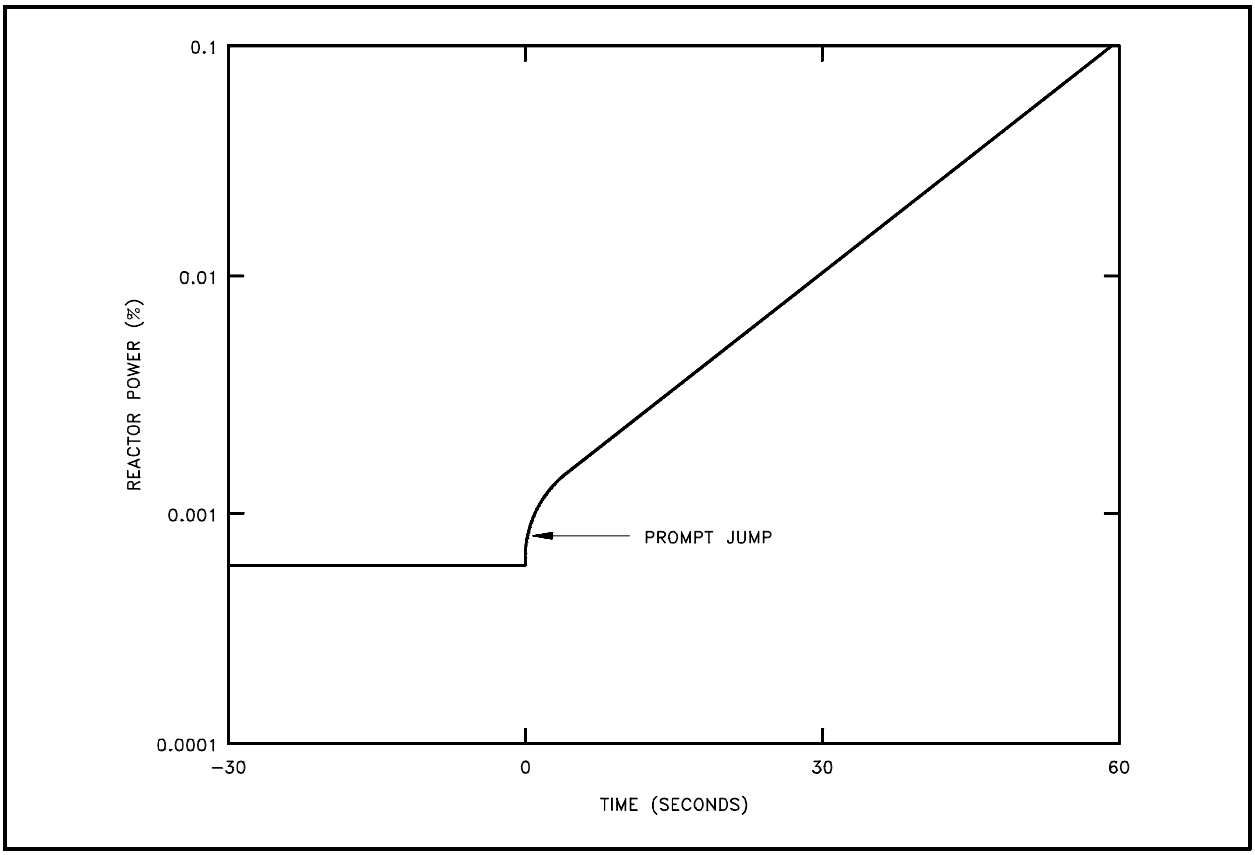


Figure 2 Reactor Power Response to Positive Reactivity Addition

Conversely, in the case where negative reactivity is added to the core there will be a prompt drop in reactor power. The *prompt drop* is the small immediate decrease in reactor power caused by the negative reactivity addition. The prompt drop is illustrated in Figure 3. After the prompt drop, the rate of change of power slows and approaches the rate determined by the delayed term of Equation (4-7).

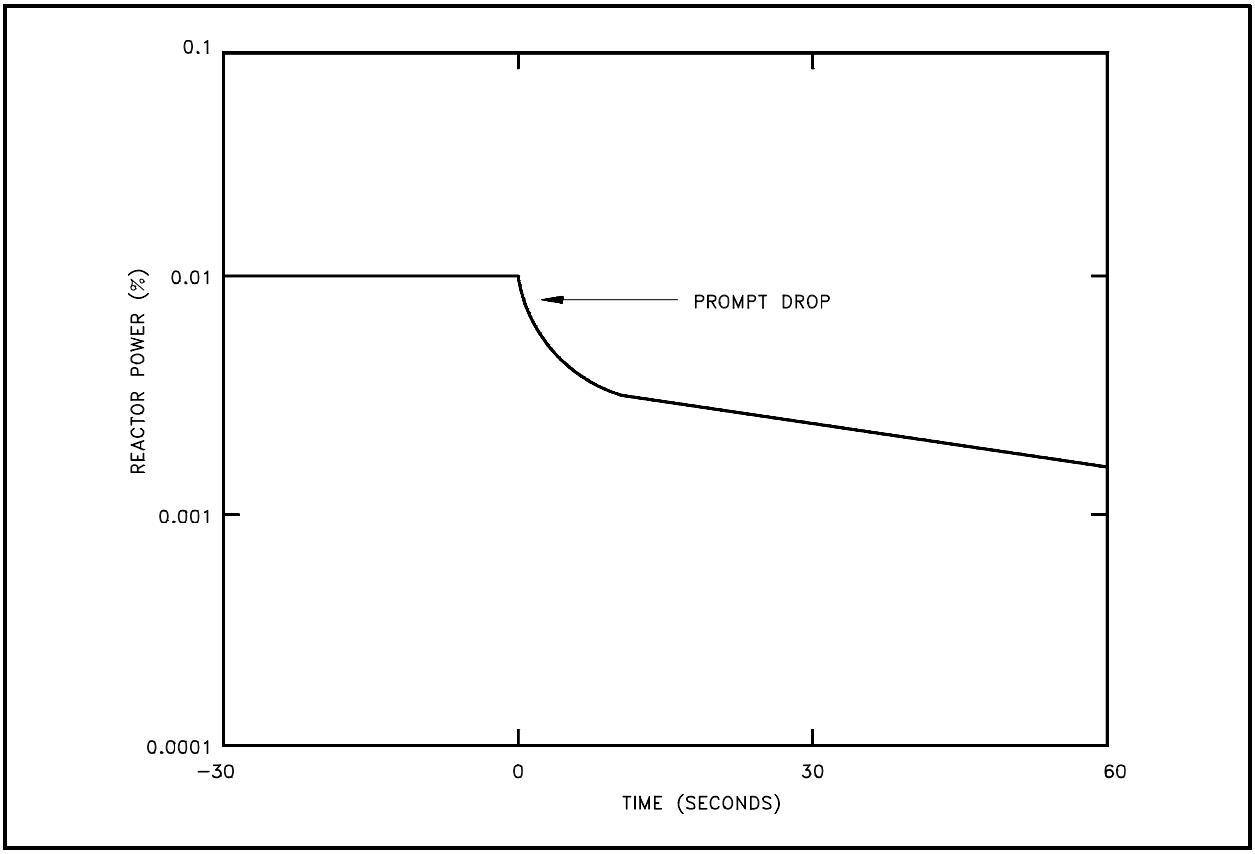


Figure 3 Reactor Power Response to Negative Reactivity Addition

Prompt Criticality

It can be readily seen from Equation (4-7) that if the amount of positive reactivity added equals the value of $\bar{\beta}_{eff}$, the reactor period equation becomes the following.

$$\tau = \frac{\ell^*}{\rho}$$

In this case, the production of prompt neutrons alone is enough to balance neutron losses and increase the neutron population. The condition where the reactor is critical on prompt neutrons, and the neutron population increases as rapidly as the prompt neutron generation lifetime allows is known as *prompt critical*. The prompt critical condition does not signal a dramatic change in neutron behavior. The reactor period changes in a regular manner between reactivities above and below this reference. Prompt critical is, however, a convenient condition for marking the transition from delayed neutron to prompt neutron time scales. A reactor whose reactivity even approaches prompt critical is likely to suffer damage due to the rapid rise in power to a very high level. For example, a reactor which has gone prompt critical could experience a several thousand percent power increase in less than one second.

Because the prompt critical condition is so important, a specific unit of reactivity has been defined that relates to it. The unit of reactivity is the dollar ($), where one dollar of reactivity is equivalent to the effective delayed neutron fraction $(\bar{\beta}_{eff})$. A reactivity unit related to the dollar is the cent, where one cent is one-hundredth of a dollar. If the reactivity of the core is one dollar, the reactor is prompt critical. Because the effective delayed neutron fraction is dependent upon the nuclides used as fuel, the value of the dollar is also dependent on the nuclides used as fuel.

Stable Period Equation

For normal reactor operating conditions, the value of positive reactivity in the reactor is never permitted to approach the effective delayed neutron fraction, and the reactor period equation is normally written as follows.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho + \dot{\rho}} \qquad (4\text{-}8)$$

Equation (4-8) is referred to as the *transient period equation* since it incorporates the $\dot{\rho}$ term to account for the changing amount of reactivity in the core. The ℓ^*/ρ term (prompt period) is normally negligible with respect to the remainder of the equation and is often not included.

For conditions when the amount of reactivity in the core is constant ($\dot{\rho} = 0$), and the reactor period is unchanging, Equation (4-8) can be simplified further to Equation (4-9) which is known as the stable period equation.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho} \qquad (4\text{-}9)$$

Reactor Startup Rate (SUR)

The *reactor startup rate* (SUR) is defined as the number of factors of ten that power changes in one minute. The units of SUR are powers of ten per minute, or decades per minute (DPM). Equation (4-10) shows the relationship between reactor power and startup rate.

$$P = P_o \, 10^{SUR\,(t)} \tag{4-10}$$

where:

SUR = reactor startup rate (DPM)
t = time during reactor transient (minutes)

The relationship between reactor period and startup rate can be developed by considering Equations (4-6) and (4-10).

$$P = P_o \, e^{t/\tau} \quad \text{and} \quad P = P_o \, 10^{SUR\,(t)}$$

$$\frac{P}{P_o} = e^{t/\tau} = 10^{SUR\,(t)}$$

Changing the base of the exponential term on the right side to "e" ($10 = e^{2.303}$) and solving the result yields the following.

$$e^{t\,(sec)/\tau} = e^{2.303\ SUR\ (t\ (min))}$$

$$\frac{t\ (sec)}{\tau} = 2.303\ SUR(t\ (min))$$

$$\frac{60}{\tau} = 2.303\ SUR$$

$$SUR = \frac{26.06}{\tau} \tag{4-11}$$

Doubling Time

Sometimes it is useful to discuss the rate of change of reactor power in terms similar to those used in radioactive decay calculations. *Doubling or halving time* are terms that relate to the amount of time it takes reactor power to double or be reduced to one-half the initial power level. If the stable reactor period is known, doubling time can be determined as follows.

Doubling time (DT) = τ (ln 2)

where:

τ = stable reactor period
ln 2 = natural logarithm of 2

When the doubling time is known, the power level change from P_o is given by the following equation.

$$P = P_o \, 2^{t/DT} \qquad (4\text{-}12)$$

where:

t = time interval of transient
DT = doubling time

The following example problems reinforce the concepts of period and startup rate.

Example 1:

A reactor has a λ_{eff} of 0.10 sec^{-1} and an effective delayed neutron fraction of 0.0070. If k_{eff} is equal to 1.0025, what is the stable reactor period and the SUR?

Solution:

Step 1: First solve for reactivity using Equation (3-5).

$$\rho = \frac{k_{eff} - 1}{k_{eff}}$$

$$= \frac{1.0025 - 1}{1.0025}$$

$$= 0.00249 \ \Delta k/k$$

Step 2: Use this value of reactivity in Equation (4-9) to calculate reactor period.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff} \, \rho}$$

$$= \frac{0.0070 - 0.00249}{(0.10 \ sec^{-1}) \, 0.00249}$$

$$= 18.1 \ sec$$

Step 3: The startup rate can be calculated from the reactor period using Equation (4-11).

$$SUR = \frac{26.06}{\tau}$$

$$= \frac{26.06}{18.1 \text{ sec}}$$

$$= 1.44 \text{ DPM}$$

Example 2:

130 pcm of negative reactivity is added to a reactor that is initially critical at a power of 100 watts. λ_{eff} for the reactor is 0.05 sec^{-1} and the effective delayed neutron fraction is 0.0068. Calculate the steady state period and startup rate. Also calculate the power level 2 minutes after the reactivity insertion.

Solution:

Step 1: Use Equation (4-9) to calculate the reactor period.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff} \rho}$$

$$= \frac{0.0068 - (-0.00130)}{(0.05 \text{ sec}^{-1})(-0.00130)}$$

$$= -124.6 \text{ sec}$$

Step 2: The startup rate can be calculated from the reactor period using Equation (4-11).

$$SUR = \frac{26.06}{\tau}$$

$$= \frac{26.06}{-124.6 \text{ sec}}$$

$$= -0.2091 \text{ DPM}$$

Step 3: Use either Equation (4-1) or Equation (4-10) to calculate the reactor power two minutes after the reactivity insertion.

$$P = P_o e^{t/\tau}$$

$$= (100 \text{ W}) e^{(120 \text{ s}/-124.6 \text{ s})}$$

$$= 38.2 \text{ W}$$

$$P = P_o 10^{SUR (t)}$$

$$= (100 \text{ W}) 10^{(-0.2091 \text{ DPM}) (2 \text{ min})}$$

$$= 38.2 \text{ W}$$

Example 3:

A reactor has a power level of 1000 watts and a doubling time of 2 minutes. What is the reactor power level 10 minutes later?

Solution:

Use Equation (4-12) to calculate the final power level.

$$P = P_o\ (2)^{t/DT}$$

$$= (1{,}000\ W)\ (2)^{10\ min/2\ min}$$

$$= 32{,}000\ W$$

Summary

The important information in this chapter is summarized below.

Reactor Kinetics Summary

- Reactor period is the time required for reactor power to change by a factor of e (2.718).
- Doubling time is the time required for reactor power to double.
- Reactor startup rate is the number of factors of ten that reactor power changes in one minute.
- The delayed neutron fraction (β) is the fraction of all fission neutrons that are born as delayed neutrons for a particular type of fuel (that is, uranium-235 and plutonium-239).
- The average delayed neutron fraction ($\bar{\beta}$) is the weighted average of the total delayed neutron fractions of the different types of fuel used in a particular reactor.
- The effective delayed neutron fraction ($\bar{\beta}_{eff}$) is the average delayed neutron fraction multiplied by an Importance Factor which accounts for the fact that delayed neutrons are born at lower average energies than fast neutrons.
- The reactor period equation is stated below.

$$\tau = \underbrace{\frac{\ell^*}{\rho}}_{\left(\begin{smallmatrix}\text{prompt}\\ \text{term}\end{smallmatrix}\right)} + \underbrace{\frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho + \dot{\rho}}}_{\left(\begin{smallmatrix}\text{delayed}\\ \text{term}\end{smallmatrix}\right)}$$

where:

τ = reactor period
ℓ^* = prompt generation lifetime
$\bar{\beta}_{eff}$ = effective delayed neutron fraction
ρ = reactivity
λ_{eff} = effective delayed neutron precursor decay constant
$\dot{\rho}$ = rate of change of reactivity

Reactor Kinetics Summary (Cont.)

- Equations (4-9) and (4-11) can be used to calculate the stable reactor period and startup rate.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\ \rho} \qquad SUR = \frac{26.06}{\tau}$$

- The concept of doubling time can be used in a similar manner to reactor period to calculate changes in reactor power using Equation (4-12).

$$P = P_o\ 2^{t/DT}$$

- The reactor period or the startup rate can be used to determine the reactor power using Equations (4-6) and (4-10).

$$P = P_o\ e^{t/\tau} \qquad P = P_o\ 10^{SUR\ (t)}$$

- Prompt jump is the small, immediate power increase that follows a positive reactivity insertion related to an increase in the prompt neutron population.
- Prompt drop is the small, immediate power decrease that follows a negative reactivity insertion related to a decrease in the prompt neutron population.
- Prompt critical is the condition when the reactor is critical on prompt neutrons alone.
- When a reactor is prompt critical, the neutron population, and hence power, can increase as quickly as the prompt neutron generation time allows.
- Measuring reactivity in units of dollars is useful when determining if a reactor is prompt critical. A reactor that contains one dollar of positive reactivity is prompt critical since one dollar of reactivity is equivalent to λ_{eff}.

REACTOR OPERATION

It is important to understand the principles that determine how a reactor responds during all modes of operation. Special measures must be taken during the startup of a reactor to ensure that expected responses are occurring. During power operation, control of the flux shape is necessary to ensure operation within limits and maximum core performance. Even when a reactor is shut down, the fact that the fission products created by the fission process continue to generate heat results in a need to monitor support systems to ensure adequate cooling of the core.

EO 3.1 **EXPLAIN why a startup neutron source may be required for a reactor.**

EO 3.2 **LIST four variables typically involved in a reactivity balance.**

EO 3.3 **EXPLAIN how a reactivity balance may be used to predict the conditions under which the reactor will become critical.**

EO 3.4 **LIST three methods used to shape or flatten the core power distribution.**

EO 3.5 **DESCRIBE the concept of power tilt.**

EO 3.6 **DEFINE the term shutdown margin.**

EO 3.7 **EXPLAIN the rationale behind the one stuck rod criterion.**

EO 3.8 **IDENTIFY five changes that will occur during and after a reactor shutdown that will affect the reactivity of the core.**

EO 3.9 **EXPLAIN why decay heat is present following reactor operation.**

EO 3.10 **LIST three variables that will affect the amount of decay heat present following reactor shutdown.**

EO 3.11 **ESTIMATE the approximate amount of decay heat that will exist one hour after a shutdown from steady state conditions.**

Startup

When a reactor is started up with unirradiated fuel, or on those occasions when the reactor is restarted following a long shutdown period, the source neutron population will be very low. In some reactors, the neutron population is frequently low enough that it cannot be detected by the nuclear instrumentation during the approach to criticality. Installed neutron sources, such as those discussed in Module 2, are frequently used to provide a safe, easily monitored reactor startup. The neutron source, together with the subcritical multiplication process, provides a sufficiently large neutron population to allow monitoring by the nuclear instruments throughout the startup procedure. Without the installed source, it may be possible to withdraw the control rods to the point of criticality, and then continue withdrawal without detecting criticality because the reactor goes critical below the indicating range. Continued withdrawal of control rods at this point could cause reactor power to rise at an uncontrollable rate before neutron level first becomes visible on the nuclear instruments.

An alternative to using a startup source is to limit the rate of rod withdrawal, or require waiting periods between rod withdrawal increments. By waiting between rod withdrawal increments, the neutron population is allowed to increase through subcritical multiplication. Subcritical multiplication is the process where source neutrons are used to sustain the chain reaction in a reactor with a multiplication factor (k_{eff}) of less than one. The chain reaction is not "self-sustaining," but if the neutron source is of sufficient magnitude, it compensates for the neutrons lost through absorption and leakage. This process can result in a constant, or increasing, neutron population even though k_{eff} is less than one.

Estimated Critical Position

In the first chapter of this module, 1/M plots were discussed. These plots were useful for monitoring the approach to criticality and predicting when criticality will occur based on indications received while the startup is actually in progress. Before the reactor startup is initiated, the operator calculates an estimate of the amount of rod withdrawal that will be necessary to achieve criticality. This process provides an added margin of safety because a large discrepancy between actual and estimated critical rod positions would indicate that the core was not performing as designed. Depending upon a reactor's design or age, the buildup of xenon within the first several hours following a reactor shutdown may introduce enough negative reactivity to cause the reactor to remain shutdown even with the control rods fully withdrawn. In this situation it is important to be able to predict whether criticality can be achieved, and if criticality cannot be achieved, the startup should not be attempted.

For a given set of conditions (such as time since shutdown, temperature, pressure, fuel burnup, samarium and xenon poisoning) there is only one position of the control rods (and boron concentrations for a reactor with chemical shim) that results in criticality, using the normal rod withdrawal sequence. Identification of these conditions allows accurate calculation of control rod position at criticality. The calculation of an *estimated critical position* (ECP) is simply a mathematical procedure that takes into account all of the changes in factors that significantly affect reactivity that have occurred between the time of reactor shutdown and the time that the reactor is brought critical again.

For most reactor designs, the only factors that change significantly after the reactor is shut down are the average reactor temperature and the concentration of fission product poisons. The reactivities normally considered when calculating an ECP include the following.

Basic Reactivity of the Core- The reactivity associated with the critical control rod position for a xenon-free core at normal operating temperature. This reactivity varies with the age of the core (amount of fuel burnup).

Direct Xenon Reactivity - The reactivity related to the xenon that was actually present in the core at the time it was shutdown. This reactivity is corrected to allow for xenon decay.

Indirect Xenon Reactivity - The reactivity related to the xenon produced by the decay of iodine that was present in the core at the time of shutdown.

Temperature Reactivity - The reactivity related to the difference between the actual reactor temperature during startup and the normal operating temperature.

To arrive at an ECP of the control rods, the basic reactivity, direct and indirect xenon reactivity, and temperature reactivity are combined algebraically to determine the amount of positive control rod reactivity that must be added by withdrawing control rods to attain criticality. A graph of control rod worth versus rod position is used to determine the estimated critical position.

Core Power Distribution

In order to ensure predictable temperatures and uniform depletion of the fuel installed in a reactor, numerous measures are taken to provide an even distribution of flux throughout the power producing section of the reactor. This shaping, or flattening, of the neutron flux is normally achieved through the use of *reflectors* that affect the flux profile across the core, or by the installation of poisons to suppress the neutron flux where desired. The last method, although effective at shaping the flux, is the least desirable since it reduces neutron economy by absorbing the neutrons.

A reactor core is frequently surrounded by a "reflecting" material to reduce the ratio of peak flux to the flux at the edge of the core fuel area. Reflector materials are normally not fissionable, have a high scattering cross section, and have a low absorption cross section. Essentially, for thermal reactors a good moderator is a good reflector. Water, heavy water, beryllium, zirconium, or graphite are commonly used as reflectors. In fast reactor systems, reflectors are not composed of moderating materials because it is desired to keep neutron energy high. The reflector functions by scattering some of the neutrons, which would have leaked from a bare (unreflected) core, back into the fuel to produce additional fissions.

Figure 4 shows the general effect of reflection in the thermal reactor system where core power is proportional to the thermal flux. Notice that a reflector can raise the power density of the core periphery and thus increase the core average power level without changing the peak power. As illustrated in Figure 4, the thermal flux in the reflector may actually be higher than that in the outermost fuel since there are very few absorptions in the reflector.

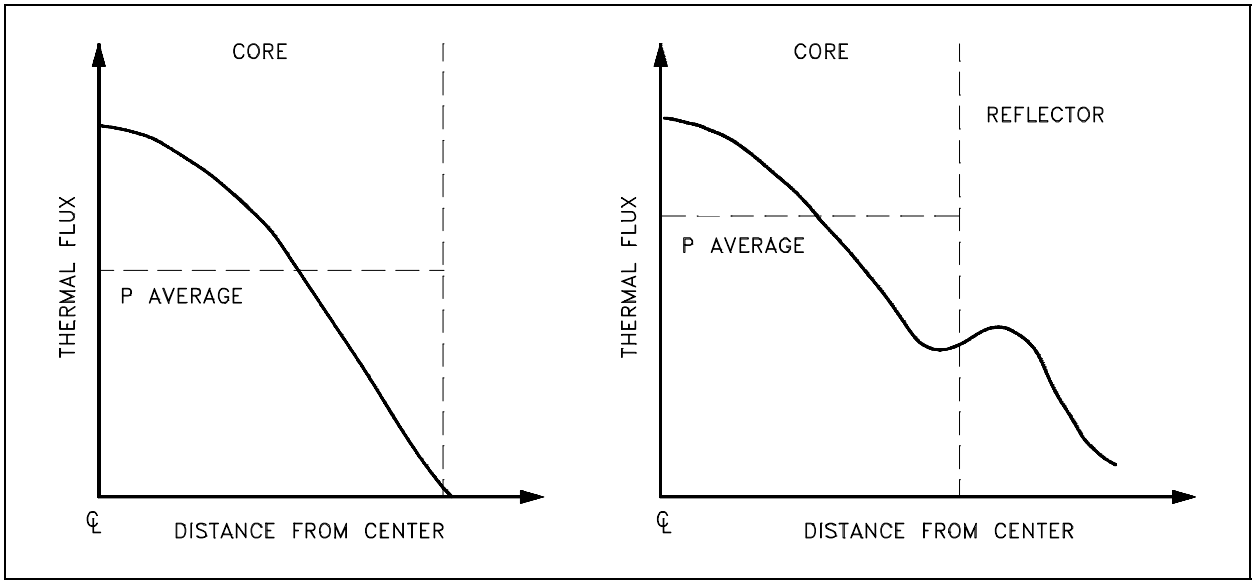


Figure 4 Neutron Radial Flux Shapes for Bare and Reflected Cores

Varying the fuel enrichment or fuel concentrations in the core radially, axially, or both, can readily be used to control power distribution. The simplified example illustrated in Figure 5 shows the effect of using a higher enrichment in the outer regions of the core. Varying fuel concentrations or poison loading for flux shaping is frequently referred to as zoning. In the example illustrated the large central peak is reduced, but the average power level remains the same.

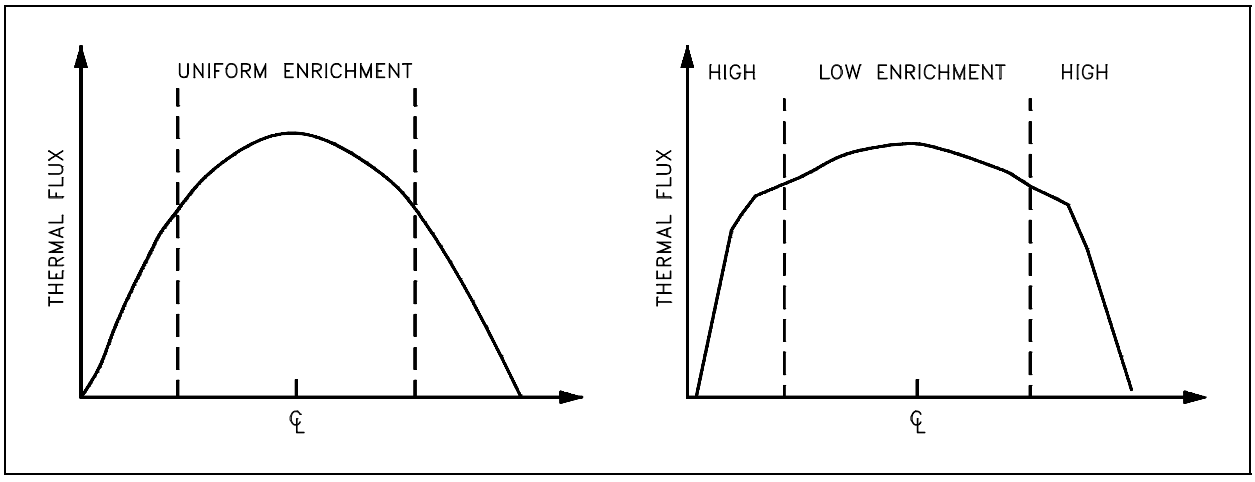


Figure 5 Effect of Non-Uniform Enrichment on Radial Flux Shape

The previous examples discuss changes in radial power distribution. Large variations also exist in axial power distribution. Figure 6(A) illustrates the power distribution that may exist for a reactor with a cylindrical geometry. The control rods in this reactor are inserted from the top, and the effect of inserting control rods further is shown in Figure 6(B). The thermal flux is largely suppressed in the vicinity of the control rods, and the majority of the power is generated low in the core. This flux profile can be flattened by the use of axial fuel and/or poison zoning.

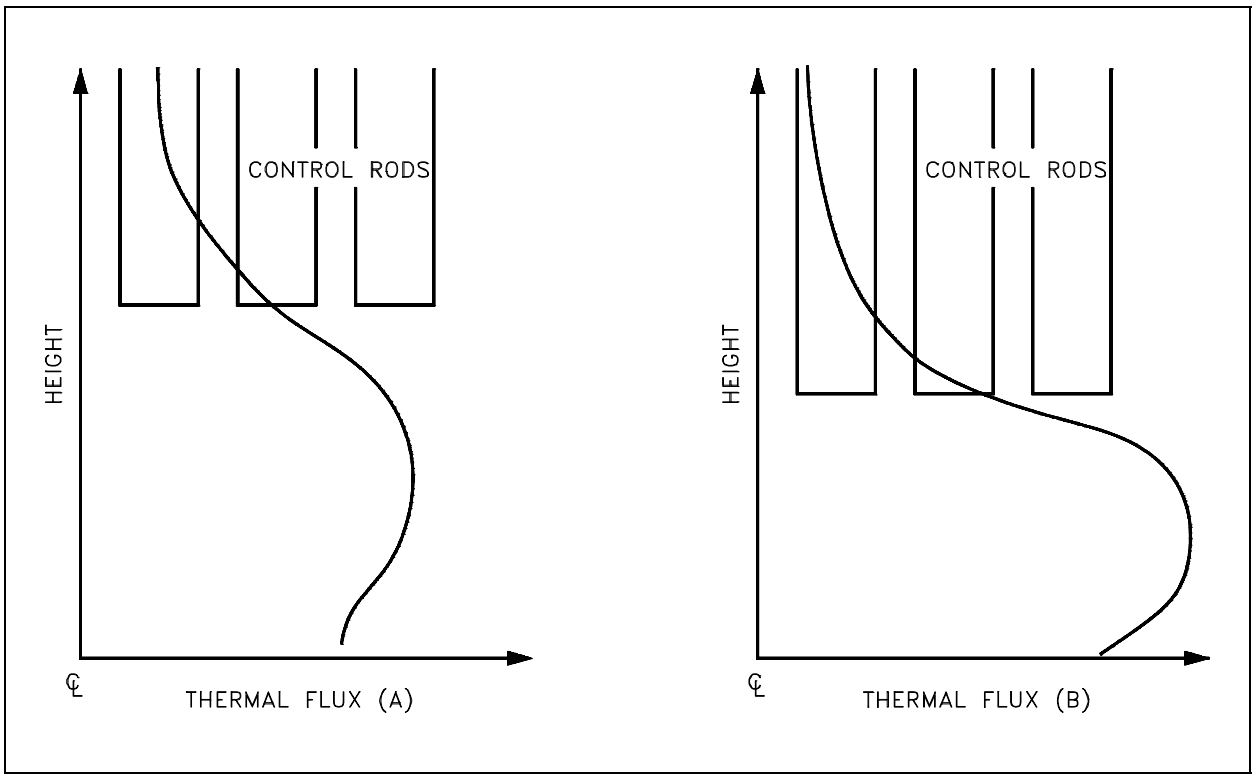


Figure 6 Effect of Control Rod Position on Axial Flux Distribution

Power Tilt

A *power tilt*, or flux tilt, is a specific type of core power distribution problem. It is a non-symmetrical variation of core power in one quadrant of the core relative to the others. The power in one portion might be suppressed by over-insertion of control rods in that portion of the core, which, for a constant overall power level, results in a relatively higher flux in the remainder of the core. This situation can lead to xenon oscillations, which were previously discussed.

Shutdown Margin

Shutdown margin is the instantaneous amount of reactivity by which a reactor is subcritical or would be subcritical from its present condition assuming all control rods are fully inserted except for the single rod with the highest integral worth, which is assumed to be fully withdrawn. Shutdown margin is required to exist at all times, even when the reactor is critical. It is important that there be enough negative reactivity capable of being inserted by the control rods to ensure complete shutdown at all times during the core lifetime. A shutdown margin in the range of one to five percent reactivity is typically required.

The stuck rod criterion refers to the fact that the shutdown margin does not take credit for the insertion of the highest worth control rod. The application of the stuck rod criterion ensures that the failure of a single control rod will not prevent the control rod system from shutting down the reactor.

Operation

During reactor operation, numerous parameters such as temperature, pressure, power level, and flow are continuously monitored and controlled to ensure safe and stable operation of the reactor. The specific effects of variations in these parameters vary greatly depending upon reactor design, but generally the effects for thermal reactors are as follows.

Temperature

The most significant effect of a variation in temperature upon reactor operation is the addition of positive or negative reactivity. As previously discussed, reactors are generally designed with negative temperature coefficients of reactivity (moderator and fuel temperature coefficients) as a self-limiting safety feature. A rise in reactor temperature results in the addition of negative reactivity. If the rise in temperature is caused by an increase in reactor power, the negative reactivity addition slows, and eventually turns the increase in reactor power. This is a highly desirable effect because it provides a negative feedback in the event of an undesired power excursion.

Negative temperature coefficients can also be utilized in water cooled and moderated power reactors to allow reactor power to automatically follow energy demands that are placed upon the system. For example, consider a reactor operating at a stable power level with the heat produced being transferred to a heat exchanger for use in an external closed cycle system. If the energy demand in the external system increases, more energy is removed from reactor system causing the temperature of the reactor coolant to decrease. As the reactor temperature decreases, positive reactivity is added and a corresponding increase in reactor power level results.

As reactor power increases to a level above the level of the new energy demand, the temperature of the moderator and fuel increases, adding negative reactivity and decreasing reactor power level to near the new level required to maintain system temperature. Some slight oscillations above and below the new power level occur before steady state conditions are achieved. The final result is that the average temperature of the reactor system is essentially the same as the initial temperature, and the reactor is operating at the new higher required power level. The same inherent stability can be observed as the energy demand on the system is decreased.

If the secondary system providing cooling to the reactor heat exchanger is operated as an open system with once-through cooling, the above discussion is not applicable. In these reactors, the temperature of the reactor is proportional to the power level, and it is impossible for the reactor to be at a higher power level and the same temperature.

Pressure

The pressure applied to the reactor system can also affect reactor operation by causing changes in reactivity. The reactivity changes result from changes in the density of the moderator in response to the pressure changes. For example, as the system pressure rises, the moderator density increases and results in greater moderation, less neutron leakage, and therefore the insertion of positive reactivity. A reduction in system pressure results in the addition of negative reactivity. Typically, in pressurized water reactors (PWR), the magnitude of this effect is considerably less than that of a change in temperature. In two-phase systems such as boiling water reactors (BWR), however, the effects of pressure changes are more noticeable because there is a greater change in moderator density for a given change in system pressure.

Power Level

A change in reactor power level can result in a change in reactivity if the power level change results in a change in system temperature.

The power level at which the reactor is producing enough energy to make up for the energy lost to ambient is commonly referred to as the *point of adding heat*. If a reactor is operating well below the point of adding heat, then variations in power level produce no measurable variations in temperature. At power levels above the point of adding heat, temperature varies with power level, and the reactivity changes will follow the convention previously described for temperature variations.

The inherent stability and power turning ability of a negative temperature coefficient are ineffective below the point of adding heat. If a power excursion is initiated from a very low power level, power will continue to rise unchecked until the point of adding heat is reached, and the subsequent temperature rise adds negative reactivity to slow, and turn, the rise of reactor power. In this region, reactor safety is provided by automatic reactor shutdown systems and operator action.

Flow

At low reactor power levels, changing the flow rate of the coolant through the reactor does not result in a measurable reactivity change because fuel and moderator temperatures and the fraction of steam voids occurring in the core are not changed appreciably.

When the flow rate is varied, however, the change in temperature that occurs across the core (outlet versus inlet temperature) will vary inversely with the flow rate. At higher power levels, on liquid cooled systems, increasing flow will lower fuel and coolant temperatures slightly, resulting in a small positive reactivity insertion. A positive reactivity addition also occurs when flow is increased in a two-phase (steam-water) cooled system. Increasing the flow rate decreases the fraction of steam voids in the coolant and results in a positive reactivity addition. This property of the moderator in a two-phase system is used extensively in commercial BWRs. Normal power variations required to follow load changes on BWRs are achieved by varying the coolant/moderator flow rate.

Core Burnup

As a reactor is operated, atoms of fuel are constantly consumed, resulting in the slow depletion of the fuel frequently referred to as core burnup. There are several major effects of this fuel depletion. The first, and most obvious, effect of the fuel burnup is that the control rods must be withdrawn or chemical shim concentration reduced to compensate for the negative reactivity effect of this burnup.

Some reactor designs incorporate the use of supplemental burnable poisons in addition to the control rods to compensate for the reactivity associated with excess fuel in a new core. These fixed burnable poisons burn out at a rate that approximates the burnout of the fuel and they reduce the amount of control rod movement necessary to compensate for fuel depletion early in core life.

As control rods are withdrawn to compensate for fuel depletion, the effective size of the reactor is increased. By increasing the effective size of the reactor, the probability that a neutron slows down and is absorbed while it is still in the reactor is also increased. Therefore, neutron leakage decreases as the effective reactor size is increased. The magnitude of the moderator negative temperature coefficient is determined in part by the change in neutron leakage that occurs as the result of a change in moderator temperature. Since the fraction of neutrons leaking out is less with the larger core, a given temperature change will have less of an effect on the leakage. Therefore, the magnitude of the moderator negative temperature coefficient decreases with fuel burnup.

There is also another effect that is a consideration only on reactors that use dissolved boron in the moderator (chemical shim). As the fuel is burned up, the dissolved boron in the moderator is slowly removed (concentration diluted) to compensate for the negative reactivity effects of fuel burnup. This action results in a larger (more negative) moderator temperature coefficient of reactivity in a reactor using chemical shim. This is due to the fact that when water density is decreased by rising moderator temperature in a reactor with a negative temperature coefficient, it results in a negative reactivity addition because some moderator is forced out of the core. With a coolant containing dissolved poison, this density decrease also results in some poison being forced out of the core, which is a positive reactivity addition, thereby reducing the magnitude of the negative reactivity added by the temperature increase. Because as fuel burnup increases the concentration of boron is slowly lowered, the positive reactivity added by the above poison removal process is lessened, and this results in a larger negative temperature coefficient of reactivity.

The following effect of fuel burnup is most predominant in a reactor with a large concentration of uranium-238. As the fission process occurs in a thermal reactor with low or medium enrichment, there is some conversion of uranium-238 into plutonium-239. Near the end of core life in certain reactors, the power contribution from the fission of plutonium-239 may be comparable to that from the fission of uranium-235. The value of the delayed neutron fraction (β) for uranium-235 is 0.0064 and for plutonium-239 is 0.0021. Consequently, as core burnup progresses, the effective delayed neutron fraction for the fuel decreases appreciably. It follows then that the amount of reactivity insertion needed to produce a given reactor period decreases with burnup of the fuel.

Shutdown

A reactor is considered to be shut down when it is subcritical and sufficient shutdown reactivity exists so there is no immediate probability of regaining criticality. Shutdown is normally accomplished by insertion of some (or all) of the control rods, or by introduction of soluble neutron poison into the reactor coolant.

The rate at which the reactor fission rate decays immediately following shutdown is similar for all reactors provided a large amount of negative reactivity is inserted. After a large negative reactivity addition the neutron level undergoes a rapid decrease of about two decades (prompt drop) until it is at the level of production of delayed neutrons. Then the neutron level slowly drops off as the delayed neutron precursors decay, and in a short while only the longest-lived precursor remains in any significant amount. This precursor determines the final rate of decrease in reactor power until the neutron flux reaches the steady state level corresponding to the subcritical multiplication of the neutron source.

The half-life of the longest lived delayed neutron precursor results in a reactor period of around -80 seconds or a startup rate of -1/3 DPM for most reactors after a reactor shutdown. One noticeable exception to this is a heavy water reactor. In a heavy water reactor, the photo-neutron source is extremely large after shutdown due to the amount of deuterium in the moderator and the large number of high energy gammas from short-lived fission product decay. The photo-neutron source is large enough to have a significant impact on neutron population immediately after shutdown. The photo-neutron source has the result of flux levels decreasing more slowly so that a heavy water reactor will have a significantly larger negative reactor period after a shutdown.

Throughout the process of reactor shutdown the nuclear instrumentation is closely monitored to observe that reactor neutron population is decreasing as expected, and that the instrumentation is functioning properly to provide continuous indication of neutron population. Instrumentation is observed for proper overlap between ranges, comparable indication between multiple instrument channels, and proper decay rate of neutron population.

A distinction should be made between indicated reactor power level after shutdown and the actual thermal power level. The indicated reactor power level is the power produced directly from fission in the reactor core, but the actual thermal power drops more slowly due to decay heat production as previously discussed. Decay heat, although approximately 5 to 6% of the steady state reactor power prior to shutdown, diminishes to less than 1% of the pre-shutdown power level after about one hour.

After a reactor is shutdown, provisions are provided for the removal of decay heat. If the reactor is to be shut down for only a short time, operating temperature is normally maintained. If the shutdown period will be lengthy or involves functions requiring cooldown of the reactor, the reactor temperature can be lowered by a number of methods. The methods for actually conducting cooldown of the reactor vary depending on plant design, but in all cases limitations are imposed on the maximum rate at which the reactor systems may be cooled. These limits are provided to reduce the stress applied to system materials, thereby reducing the possibility of stress induced failure.

Although a reactor is shut down, it must be continuously monitored to ensure the safety of the reactor. Automatic monitoring systems are employed to continuously collect and assess the data provided by remote sensors. It is ultimately the operator who must ensure the safety of the reactor.

Decay Heat

About 7 percent of the 200 MeV produced by an average fission is released at some time after the instant of fission. This energy comes from the decay of the fission products. When a reactor is shut down, fission essentially ceases, but decay energy is still being produced. The energy produced after shutdown is referred to as decay heat. The amount of decay heat production after shutdown is directly influenced by the power history of the reactor prior to shutdown. A reactor operated at full power for 3 to 4 days prior to shutdown has much higher decay heat generation than a reactor operated at low power for the same period. The decay heat produced by a reactor shutdown from full power is initially equivalent to about 5 to 6% of the thermal rating of the reactor. This decay heat generation rate diminishes to less than 1% approximately one hour after shutdown. However, even at these low levels, the amount of heat generated requires the continued removal of heat for an appreciable time after shutdown. Decay heat is a long-term consideration and impacts spent fuel handling, reprocessing, waste management, and reactor safety.

Summary

The important information in this chapter is summarized below.

Reactor Operation Summary

- An installed neutron source, together with the subcritical multiplication process, may be needed to increase the neutron population to a level where it can be monitored throughout the startup procedure.
- Reactivity balances, such as Estimated Critical Position calculations, typically consider the basic reactivity of the core and the reactivity effects of temperature, direct xenon, and indirect xenon.
- A reactivity balance called an Estimated Critical Position is used to predict the position of the control rods at which criticality will be achieved during a startup. To arrive at an ECP of the control rods, the basic reactivity, direct and indirect xenon reactivity, and temperature reactivity are added together to determine the amount of positive reactivity that must be added by withdrawing control rods to attain criticality. A graph of control rod worth versus rod position is used to determine the estimated critical position.

Reactor Operation Summary (Cont.)

- Three methods are used to shape or flatten the core power distribution.
  - Use of reflectors
  - Installation of neutron poisons
  - Axial or radial variation of fuel enrichment

- Power tilt is a non-symmetrical variation of core power in one quadrant of the core relative to the other quadrants.

- Shutdown margin is the instantaneous amount of reactivity by which a reactor is subcritical or would be subcritical from its present condition assuming all control rods are fully inserted except for the single rod with the highest integral worth, which is assumed to be fully withdrawn.

- The stuck rod criterion is applied to the shutdown margin to ensure that the failure of a single control rod will not prevent the control rod system from shutting down the reactor.

- Several factors may change during and after the shutdown of the reactor that affect the reactivity of the core.
  - Control rod position
  - Soluble neutron poison concentration
  - Temperature of the fuel and coolant
  - Xenon
  - Samarium

- Decay heat is always present following reactor operation due to energy resulting from the decay of fission products.

- The amount of decay heat present in the reactor is dependent on three factors.
  - The pre-shutdown power level
  - How long the reactor operated
  - The amount of time since reactor shutdown

- Decay heat immediately after shutdown is approximately 5-6% of the pre-shutdown power level. Decay heat will decrease to approximately 1% of the pre-shutdown power level within one hour of reactor shutdown.

end of text.

CONCLUDING MATERIAL

Review activities:

DOE - ANL-W, BNL, EG&G Idaho, EG&G Mound, EG&G Rocky Flats, LLNL, LANL, MMES, ORAU, REECo, WHC, WINCO, WEMCO, and WSRC.

Preparing activity:

DOE - NE-73
Project Number 6910-0025

Intentionally Left Blank