Why is electron spin denoted by the quantum number 1/2?

Solution:

1- The ± 12 term comes by solving the Dirac equation for electrons, which explicitly comes out to be fractional for **fermions** (like electrons) and integral for **bosons** (like photons, Higgs etc.).

2- It can have spin 1 (=1.$\hbar$). Then if we change the particle's spin by $\hbar$ we can get spin 0 and spin -1.$\hbar$. So if you measure the spin of a spin1 particle (a **boson**) you can get three results: spin +1, 0 or -1. Like the earth, it depends on the "angle" you are looking at.

So is spin 1/4 possible? No, because if you look at it from the opposite direction you get spin -1/4, and the difference is only 12.$\hbar$.
So what is the smallest possible spin apart from spin zero? It's spin 1/2. Change it by 1.$\hbar$ and you go from spin 1/2 to spin -1/2.
That is why particles can have spin 1/2 basically because it is the smallest number that can change by 1 and still be "symmetrical".

3- Spin quantum number can have values, that are ..
1) Non negative integers
2) Non negative half-integers

The negative sign denotes spin direction

For example... 1, 1/2, 2, 3/2....

Assume you want to know why spin has values of +1/2 and -1/2, instead of +1 and -1...
Its to differentiate between **Bosons** and **Fermions.**
Bosons have integer spins (1, 2, 3...)
Fermions have half-integer spins (1/2, 3/2...)
(Electrons are fermions)