

# The primitive equation

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv + A_x \frac{\partial^2 u}{\partial x^2} + A_y \frac{\partial^2 u}{\partial y^2} + A_z \frac{\partial^2 u}{\partial z^2} \quad (1)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + A_x \frac{\partial^2 v}{\partial x^2} + A_y \frac{\partial^2 v}{\partial y^2} + A_z \frac{\partial^2 v}{\partial z^2} \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

$$\frac{\partial p}{\partial z} = -\rho g \quad (4)$$

Since the turbulent momentum transports are

$$\tau_{xx} = \rho A_x \frac{\partial u}{\partial x}, \quad \tau_{xy} = \rho A_y \frac{\partial u}{\partial y}, \quad \tau_{xz} = \rho A_z \frac{\partial u}{\partial z} \quad \text{etc}$$

We can also write the momentum equations in more general forms

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv + \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + \frac{1}{\rho} \left( \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \right)$$

At the sea surface ( $z=0$ ),  
turbulent transport is wind stress.

$$\tau_{xz}^0 = \rho A_z \frac{\partial u}{\partial z}, \quad \tau_{yz}^0 = \rho A_z \frac{\partial v}{\partial z}$$