

The weighted arithmetic mean.

Sometimes we associate with the number x_1, x_2, \dots, x_n certain weighting factors (weights) w_1, w_2, \dots, w_n , depending on the significance to the number. In this case:

$$\bar{x} = \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{w_1 + w_2 + \dots + w_n} = \frac{\sum (wx)}{\sum w}$$

Q1 Find the weighted mean for this numbers.

marks student	90	85	70
quiz	1	3	1

$$\bar{x} = \frac{\sum (wx)}{\sum w} = \frac{90 \times 1 + 85 \times 3 + 70 \times 1}{1 + 3 + 1} = 83.$$

Q1 Mark 62, 80, 75, 88, 84, 89, 90
hours 2 2 2 3 3 3 3

Find the weighted mean?

The Harmonic mean H .

The harmonic mean H of a set of N numbers $X_1, X_2, X_3, \dots, X_N$ is the reciprocal of the arithmetic mean of the reciprocals of the numbers

$$H = \frac{1}{\frac{1}{N} \sum_{i=1}^N \frac{1}{X_i}} = \frac{N}{\sum \frac{1}{X}}$$

Example: Find the harmonic mean of the numbers 2, 4 and 8.

$$H = \frac{N=3}{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}} = \frac{3}{\frac{7}{8}} = 3.43.$$

Example: Find the harmonic mean of the numbers
7.34 5, 7, 9 and 11.

For Grouped data.

The Harmonic mean H is

$$H = \frac{\sum f_i}{\sum \frac{f_i}{X_i}}$$

The Geometric mean G

The geometric mean G of a set of N numbers $x_1, x_2, x_3, \dots, x_N$ is the N th root of the product of numbers;

$$G = \sqrt[N]{x_1 x_2 x_3 \dots x_N}$$

Example: Find the geometric mean of the numbers 2, 4 and 8.

$$G = \sqrt[3]{(2)(4)(8)} = \sqrt[3]{64} = 4$$

Example: Find the geometric mean of the numbers 9, 6 and 4.

$$G = \sqrt[3]{(9)(6)(4)}$$

$$G = \sqrt[3]{216} = 6$$

Example: Find the geometric mean of the numbers: 20, 20, 30, 40, 50 and 60.

Example: Find the Harmonic mean H of the classes and frequencies:

class	F_i
50 -	8
60 -	20
70 -	10
80 -	24
90 -	10
100 -	5
110 - 120	2
120 -	

The relation between the arithmetic, geometric and Harmonic means:

- ① The geometric mean of a set of positive numbers x_1, x_2, \dots, x_n is less than or equal to their arithmetic mean but is greater than or equal to their harmonic mean. In symbols,

$$H \leq G \leq \bar{x}$$

The set 2, 4, 8 has arithmetic mean 4.67, geometric mean 4 and harmonic mean 3.43.

② $\bar{x} H = G^2$

- Q / Find the arithmetic, geometric and Harmonic means for the set numbers.

1, 2, 3, 4, 5

$$N = 5$$

$$\bar{x} = 3$$

$$G = 2.6$$

$$H = 2.18$$