$5\times0+1\times2+4\times1+5\times5+5\times3=46$

	$V_1=0$	$V_2=2$	$V_3=6$	
$u_1=0$	(0) 5	(2) $1-\theta$	(1) θ $\boxed{5}$	~~6~~ ~~1~~ 0
$u_2=-1$	(2) $\boxed{-3}$	(1) $4+\theta$	(5) $5-\theta$	~~9~~ ~~5~~ 0
$u_3=-3$	(2) $\boxed{-5}$	(4) $\boxed{-5}$	(3) 5	5
	~~5~~ 0	~~5~~ ~~4~~ 0	~~10~~ 5	

Since $u_1+V_3-C_{13}=5$, then continue

the entring var. is X_{13}

and the leaving var. is X_{12},

$\theta=1$.

	$V_1=0$	$V_2=-3$	$V_3=1$	
$u_1=0$	(0) $5-\theta$	(2) $\boxed{-5}$	(1) $1+\theta$	6
$u_2=4$	(2) θ $\boxed{2}$	(1) 5	(5) $4-\theta$	9
$u_3=2$	(2) $\boxed{0}$	(4) $\boxed{-5}$	(3) 5	5
	5	5	10	

$Z=5\times0+1\times1+5\times1+4\times5+5\times3=41$

Since $u_2+V_1-C_{21}=2$, then Continue

X_{21} is the entring var. and X_{23} is the leaving var, $\theta=4$.

	$V_1=0$	$V_2=-3$	$V_3=1$	
$u_1=0$	(0) 1	(2) $\boxed{-5}$	(1) 5	6
$u_2=2$	(2) 4	(1) 5	(5) $\boxed{-2}$	9
$u_3=2$	(2) $\boxed{0}$	(4) $\boxed{-5}$	(3) 5	5
	5	5	10	

$Z=1\times0+5\times1+4\times2+5\times1+5\times3=33$

Since all $u_i+v_j-c_{ij}\leq0$, $\forall$ non basic var. Stop.

Optimal Solution is $Z=33$, when

$X_{11}=1, X_{13}=5, X_{21}=4, X_{22}=5, X_{33}=5$

H.W ① Find the optimal Sol. for the following trans. models.

	1	2	3	
1	0	4	2	8
2	2	3	4	5
3	1	2	0	6
	7	6	6	

(a)

	1	2	3	
1	−	3	5	4
2	7	4	9	7
3	1	8	6	19
	5	6	19	

(b)

② In the following trans. problem the total demand exceeds the total supply. Suppose that the costs per unit of unsatisfied demand are 5, 3 and 2 for dest. 1, 2 and 3, resp. Determine the optimum solution.

	1	2	3	
1	5	1	7	10
2	6	4	6	80
3	3	2	5	15
	75	20	50	

③ The following trans. problem solved by northwest-corner with $z = 38$, find the unknowns p and q.

	1	2	3	
1	2 6	0 2	4	8
2	4	3 $p-2$	2 1	q
3	0	2	1 6	6
	6	p	7	

18/0[illegible]

[Ch]apter Three - Assignment Model

Introduction

"The best person for job" is an efficient description of what the assignment model seeks to accomplish. The situation can be illustrated by the assignment of workers to jobs. The objective of the model is to determine the optimum (Least-cost) assignment of workers to jobs. The general assignment model with n workers and n jobs is represented in the following table:

jobs

Workers	1	2	...	n	
1	c_{11}	c_{12}	...	c_{1n}	1
2	c_{21}	c_{22}	...	c_{2n}	1
⋮	⋮	⋮		⋮	⋮
n	c_{n1}	c_{n2}	...	c_{nn}	1
	1	1	...	1	

The element c_{ij} represents the cost of assigning worker i to job j $(i,j = 1,2,\dots,n)$. The assignment model is actually a special case of the transportation model in which the workers represents the sources, and the jobs represent the distinations.

Now we will discuss the assignment model.

3-2 Permutation Method (PM).

In this method we will seek all the possible states of the model

Ex 1: Solve the following assignment by using PM.

	1	2	3
1	15	10	9
2	9	15	10
3	10	12	8

	1	2	3	
1.	1	2	3	$= 15 + 15 + 8 = 38$
2.	1	3	2	$= 15 + 10 + 12 = 37$
3.	2	1	3	$10 + 9 + 8 = 27$
4.	2	3	1	$10 + 10 + 10 = 30$
5.	3	1	2	$9 + 9 + 12 = 30$
6.	3	2	1	$9 + 15 + 10 = 34$

1-2, 2-1, 3-3, $Z = 27$

3-3 Least-Cost Method (LCM)

This method depends on finding the most minimum cost in the assignment table, after assigning the worker to the job, cross the row and the column of this cost value, then repeating the process until completing all the assignment model.

EX2, call EX1.

first we identify 3-3 since its the most minimum cost = 8, then cross row3 and col3. In the same process we choose 2-1 with cost 9, Lastly 1-2 with cost 10

	1	2	3
1	15	10	9
2	9	15	10
3	10	12	8

The final result is 1-2, 2-1, 3-3

$Z = 10 + 9 + 8 = 27.$

In this manner we can introduce another two methods:

① - Top-to-Bottom Least-Cost method (TBLCM)

This method applies starting from top to bottom using LCM.

② Bottom-to-Top Least-Cost Method (BTLCM)

while this method applies starting from bottom to top using LCM.

	1	2	3
1	15	10	9
2	9	15	10
3	10	12	8

TBLCM

1-3, 2-1, 3-2

$Z = 9 + 9 + 12 = \underline{30}$

	1	2	3
1	15	10	9
2	9	15	10
3	10	12	8

BTLCM

1-2, 2-1, 3-3

$Z = 10 + 9 + 8 = \underline{27}$

Notes

① LCM is more efficient than TBLCM and BTLCM.

② LCM and its related methods are not necessary to give optimum sol.

③ PM considered as brute force (exhustive search) attack.

solve the following assignment model using PM, LCM, TBLCM and BTLCM.

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

① PM:

1. 1-1, 2-2, 3-3, 4-4 ⇒ $Z = 24$
2. 1-1, 2-2, 3-4, 4-3 ⇒ $Z = 23$
3. 1-1, 2-3, 3-2, 4-4 ⇒ $Z = 21$
4. 1-1, 2-3, 3-4, 4-2 ⇒ $Z = 25$

⋮

23. 1-4, 2-3, 3-1, 4-2 ⇒ $Z = 24$
24. 1-4, 2-3, 3-2, 4-1 ⇒ $Z = 26$

The best (minimum cost is $Z = 21$) for 1-1, 2-3, 3-2, 4-4

② LCM:

The best sol. is:

1-1, 2-3, 3-2, 4-4

$Z = 1 + 10 + 5 + 5 = 21$

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

③ TBLCM:

The best sol. is:

1-1, 2-2, 3-4, 4-3

$Z = 1 + 7 + 7 + 8 = 23$

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

④ BTLCM:

The best sol. is:

1-3, 2-2, 3-1, 4-4

$Z = 6 + 7 + 4 + 5 = 22$

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

3-4 The Hungarian Method (HM)

Step1: Identify each row's minimum, and Subtract it from all the entries of the row.

Step2: For the new matrix, identify each Col's min and Subtract it from all the entries of the Col.

Step3: If no feasible assignment can be secured from Steps 1 and 2:

(i) Draw the min. number of horizontal and vertical lines in the last reduced matrix that will cover all the Zero entries.

(ii) Select the smallest uncoverd element, and Subtract it from every encoverd element; then add it to every element at the intersection of two lines.

(iii) If no feasible assignment can be found among the resulting Zero entries, repeat step 2.

Step4: Identify the optimal assignment as the one associated with Zero element of the matrix obtained in Step3.

Ex4: Solve the following assignment using the Hangarian method.

$n = 4$

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

Step(1)

	1	2	3	4
1	0	3	5	2
2	2	0	3	2
3	0	1	7	3
4	3	2	3	0

العدد (3) [illegible]

Step(2)

	1	2	3	4
1	0	3	2	2
2	2	0	0	2
3	0	1	4	3
4	3	2	0	0

	1	2	3	4
1	0	2	1	1
2	3	0	0	2
3	0	0	3	2
4	4	2	0	0

The optimum Solution is

1–1, 2–3, 3–2, 4–4

$Z = 1 + 10 + 5 + 5 = 21$