

$$5x_0 + 1x_2 + 4x_1 + 5x_5 + 5x_3 = 46$$

Since $U_1 + V_3 - C_{13} = 5$, then continue
the entering var. is x_{13}

and the leaving var. is x_{12} ,
 $\theta = 1$.

$$Z = 5x_0 + 1x_1 + 5x_1 + 4x_5 + 5x_3 = 41$$

Since $U_2 + V_1 - C_{21} = 2$, then continue

x_{21} is the entering var. and x_{23} is
the leaving var, $\theta = 4$.

$$Z = 1x_0 + 5x_1 + 4x_2 + 5x_1 + 5x_3 = 33$$

Since all $a_{ij} + v_j - c_{ij} \leq 0$, \forall non basic
var. Stop.

Optimal Solution is $Z = 33$, when

$$x_{11} = 1, x_{13} = 5, x_{21} = 4, x_{22} = 5, x_{33} = 5$$

H.W ① Find the optimal sol. for the following trans. models.

	1	2	3	
1	0	4	2	8
2	2	3	4	5
3	1	2	0	6
7	6	6		

(a)

	1	2	3	
1	-	3	5	4
2	7	4	9	7
3	1	8	6	19
5	6	19		

(b)

	$V_1=0$	$V_2=2$	$V_3=6$	
$U_1=0$	0	2	1	
$U_2=-1$	2	1	5	
$U_3=-3$	-3	4	3	
	8	8	10	
	0	4	5	

	$V_1=0$	$V_2=-3$	$V_3=1$	
$U_1=0$	0	2	1	
$U_2=4$	2	1	5	
$U_3=2$	0	4	3	
	5	5	10	

	$V_1=0$	$V_2=-3$	$V_3=1$	
$U_1=0$	0	2	1	
$U_2=2$	2	1	5	
$U_3=2$	0	4	3	
	5	5	10	

② In the following trans. problem the total demand exceeds the total supply. Suppose that the costs per unit of unsatisfied demand are 5, 3 and 2 for dest. 1, 2 and 3, resp. Determine the optimum solution.

	1	2	3	
1	5	1	7	10
2	6	4	6	80
3	3	2	5	15
	78	20	50	

③ The following trans. problem solved by northwest-corner with $Z = 38$, find the unknowns P and q .

	1	2	3	
1	6	2	0	4
2	4	3	2	9
3	0	2	1	6
	6	P	7	

Chapter Three - Assignment Model

Introduction

"The best person for job" is an efficient description of what the assignment model seeks to accomplish. The situation can be illustrated by the assignment of workers to jobs. The objective of the model is to determine the optimum (Least-Cost) assignment of workers to jobs. The general assignment model with n workers and n jobs is represented in the following table:

	1	2	...	n	jobs
1	c_{11}	c_{12}	...	c_{1n}	1
2	c_{21}	c_{22}	...	c_{2n}	1
:	:	:		:	:
n	c_{n1}	c_{n2}	...	c_{nn}	1
	1	1	...	1	

The element c_{ij} represents the cost of assigning worker i to job j ($i, j = 1, 2, \dots, n$). The assignment model is actually a special case of the transportation model in which the workers represent the sources, and the jobs represent the destinations.

Now we will discuss the assignment model.

3-2 Permutation Method (PM).

In this method we will seek all the possible states of the model

Ex 1: Solve the following assignment by using PM.

	1	2	3
1	15	10	(9)
2	9	15	10
3	10	12	8

	1	2	3
1.	1	2	3
2.	1	3	2
3.	2	1	3
4.	2	3	1
5.	3	1	2
6.	3	2	1

$1. 1 + 2 + 3 = 15 + 15 + 8 = 38$
 $2. 1 + 3 + 2 = 15 + 10 + 12 = 37$
 $3. 2 + 1 + 3 = 10 + 9 + 8 = 27$
 $4. 2 + 3 + 1 = 10 + 10 + 10 = 30$
 $5. 3 + 1 + 2 = 9 + 9 + 12 = 30$
 $6. 3 + 2 + 1 = 9 + 18 + 10 = 34$

$$1-2, 2-1, 3-3, 2-2 = 27$$

3-3 Least-Cost Method (LCM)

This method depends on finding the most minimum cost assignment table, after assigning the worker to the job, then row and the column of this cost value, then repeating the process until completing all the assignment model.

Ex 2, call Ex 1.

first we identify 3-3 since it's the most minimum cost = 8, then cross row 3 and col 3. In the same process we choose 2-1 with cost 9, Lastly 1-2 with cost 10

The final result is 3-2, 2-1, 1-3

$$Z = 10 + 9 + 8 = 27$$

In this manner we can introduce another two methods:

① Top-to-Bottom Least-Cost Method (TBLCM)

This method applies starting from top to bottom using LCM.

② Bottom-to-Top Least-Cost Method (BTLCM)

while this method applies starting from bottom to top using LCM.

	1	2	3
1	15	10	9
2	9	15	10
3	10	12	8

TBLCM

1-3, 2-1, 3-2

$$Z = 9 + 9 + 12 = \underline{30}$$

	1	2	3
1	15	10	9
2	9	15	10
3	10	12	8

BTLCM

1-2, 2-1, 3-3

$$Z = 10 + 9 + 8 = \underline{27}$$

Notes

- ① LCM is more efficient than TBLCM and BTLCM.
- ② LCM and its related methods are not necessary to give optimum sol.
- ③ PM considered as brute force (exhaustive search) attack.

Solve the following assignment model using PM, LCM, TBLCM and BTLCM.

① PM:

$$\begin{aligned}
 & 1-1-1, 2-2, 3-3, 4-4 \Rightarrow Z = 24 \\
 & 2-1-1, 2-2, 3-4, 4-3 \Rightarrow Z = 23 \\
 & 3-1-1, 2-3, 3-2, 4-4 \Rightarrow Z = 21 \\
 & 4-1-1, 2-3, 3-4, 4-2 \Rightarrow Z = 25 \\
 & \vdots \quad \vdots \quad \vdots \quad \vdots \\
 & 23. 1-4, 2-3, 3-1, 4-2 \Rightarrow Z = 24 \\
 & 24. 1-4, 2-3, 3-2, 4-1 \Rightarrow Z = 26
 \end{aligned}$$

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

The best (minimum cost is $Z = 21$)
for $1-1, 2-3, 3-2, 4-4$

② LCM:

The best sol. is:

$$\begin{aligned}
 & 1-1, 2-3, 3-2, 4-4 \\
 & Z = 1+10+5+5 = 21
 \end{aligned}$$

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

③ TBLCM:

The best sol. is:

$$\begin{aligned}
 & 1-1, 2-2, 3-4, 4-3 \\
 & Z = 1+7+7+8 = 23
 \end{aligned}$$

④ BTLCM:

The best sol. is:

$$\begin{aligned}
 & 1-3, 2-2, 3-1, 4-4 \\
 & Z = 6+7+4+5 = 22
 \end{aligned}$$

3-4 The Hungarian Method (HM)

Step1: Identify each row's minimum, and Subtract it from all the entries of the row.

Step2: For the new matrix, identify each col's min and subtract it from all the entries of the col.

Step3: If no feasible assignment can be secured from steps 1 and 2:

(i) Draw the min. number of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries.

(ii) Select the smallest uncovered element, and subtract it from every covered element; then add it to every element at the intersection of two lines.

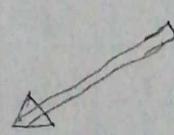
~ (iii). If no feasible assignment can be found among the resulting zero entries, repeat step 2.

Step4: Identify the optimal assignment as the one associated with zero elements of the matrix obtained in step 3.

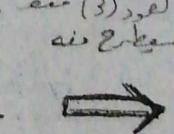
Ex4: Solve the following assignment using the Hungarian method.

$$n = 4$$

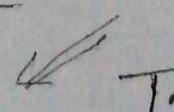
	①	②	③	④
1	1	2	3	4
2	0	3	5	2
3	2	0	3	2
4	3	1	7	3
③	4	3	2	3
④	1	2	3	0



Step(1)
min(3), el 1
row 2 b/s



	①	②	③	④
1	1	2	3	4
2	2	0	0	2
3	0	1	4	3
4	3	2	0	0
③	4	3	2	0
④	1	2	3	0



	①	②	③	④
1	2	1	1	1
2	3	0	2	2
3	0	2	3	2
4	4	2	0	0
③	4	2	0	0
④	1	2	3	0

The optimum solution is

$$1-1, 2-3, 3-2, 4-4$$

$$Z = 1 + 10 + 5 + 5 = 21$$