

5. Solve the following assignment model using Hungarian method

	1	2	3	4	5
1	3	8	2	10	3
2	8	7	2	9	7
3	6	4	2	7	5
4	8	4	2	3	5
5	9	10	6	9	10

⇒

	1	2	3	4	5
1	1	6	0	8	1
2	6	5	0	7	5
3	4	2	0	5	3
4	6	2	0	1	3
5	3	4	0	3	4

بزرگترین عدد از جدول

	1	2	3	4	5
1	0	5	0	7	0
2	5	4	0	6	4
3	3	1	0	4	2
4	5	1	0	0	2
5	2	3	0	2	3

	1	2	3	4	5
1	0	5	1	7	0
2	4	3	0	5	3
3	2	0	0	3	1
4	5	1	1	0	2
5	1	2	0	1	2

	1	2	3	4	5
1	0	5	2	7	0
2	3	2	0	4	2
3	2	0	1	3	1
4	5	1	2	0	2
5	0	1	0	0	1

The optimum solution is:  
 1-5, 2-3, 3-2, 4-4, 5-1

$Z = 3 + 2 + 4 + 3 + 9 = 21$

H.W: ① Solve the following assignment model using H.M.

	1	2	3	4	5
1	3	9	2	3	7
2	6	1	5	6	6
3	9	4	7	10	3
4	2	5	4	2	1
5	9	6	2	4	5

② A shop needs to assign four jobs it received to 4 workers. The varying skills of the workers give rise to varying costs for performing the jobs. The following table summarizes the cost data of the assignments. The data indicate that worker 1 cannot work on job 3, and worker 3 cannot work on job 4. Determine the optimal assignment.

	1	2	3	4
1	50	50	-	20
2	70	40	20	30
3	90	30	50	-
4	70	20	60	70

# Costs for Markers - Four - Game Theory

## Introduction

Game theory deals with decision situations in which two intelligent opponents have conflicting objectives. Typical examples include launching advertisement campaigns for competing products and planning war strategies for opposing armies.

In two conflict, two opponents, known as players, will each have a number of alternatives or strategies. Associated with each pair of strategies is a payoff that one player pays to the other. Such games are

known as two-person zero-sum games because the gain by one player equals the loss to the other. Designating

the two players as A and B with  $m$  and  $n$  strategies, resp, the game is usually represented by the payoff matrix to player A as:

	$B_1$	$B_2$	...	$B_n$
$A_1$	$a_{11}$	$a_{12}$	...	$a_{1n}$
$A_2$	$a_{21}$	$a_{22}$	...	$a_{2n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$
$A_m$	$a_{m1}$	$a_{m2}$	...	$a_{mn}$

### Ex 1:

In a game similar to matching pennies, player A picks heads or tails and player B attempts to guess the choice. A will pay B \$3 if both choose heads; A will pay B \$2 if both choose tails. If B guesses incorrectly, he will pay A \$5.

Sol. If we denote the gain of A by positive entries and B

his losses by negative entries ::

	H	T
A	H -3 5	T 5 -2

Ex 2: Tami and Lura, simultaneously each show one or two fingers. If the total number of fingers shown is even, Tami pays Lura that number of dimes, if its odd, Lura pays Tami that number of dimes.

Sol:  

		Lura	
		1	2
Tami	1	-20	30
	2	+30	-40

H.W: Write the matrix game that corresponds to each two-person conflict situation.

1. Tami and Lura simultaneously each show one or two fingers. If they show the same number of fingers, Tami pays Lura one dime, else, Lura pays Tami one dime.
2. Tami and Lura simultaneously and independently, each write down one of the numbers 1, 4 or 7 (3, 6 or 8). If the sum of the numbers is even, Tami pays Lura that number of dimes, if the sum is odd, Lura pays Tami that number of dimes.

#### 4-2 Solution of Two-Person Zero-Sum Games

Strictly Determined Game: A game defined by a matrix is said to be strictly determined iff there is an entry of the matrix that is the smallest element in its row and is also the largest element in its column. This entry is then called a saddle point and is the value of the game.

If a game has a positive value, the game favors player A. If a game has a negative value, the game favors player B. Any game with a value of 0 is termed a fair game. Such games are also called games of pure strategy. The row containing the saddle point is the best strategy for A and the column containing it is the best strategy for B.

8.3 Determine whether the game defined by the matrix below is strictly determined, if it is so, find the value of the game

(a)

$$\begin{bmatrix} 3 & 0 & -2 & -1 \\ 2 & -3 & 0 & -1 \\ 4 & 2 & 1 & 0 \end{bmatrix} \begin{matrix} -2 \\ -3 \\ \textcircled{0} \end{matrix} \text{ Maxmin}$$

$$4 \quad 2 \quad 1 \quad \textcircled{0}$$

Minmax

Since  $\text{Maxmin} = \text{Minmax} = 0$

The game is strictly determined. Its value is 0, so the game is fair.

(b)

$$\begin{bmatrix} 1 & -3 \\ 4 & -2 \end{bmatrix} \begin{matrix} -3 \\ \textcircled{-2} \end{matrix} \text{ Maxmin}$$

$$4 \quad \textcircled{-2}$$

Minmax

$\text{Maxmin} = \text{Minmax} = -2$

The game is strictly determined and the saddle point is -2. This game is favorable to B.

4.10 Determine which of the two-person, zero-sum games are strictly determined. For those that are, find the value of the game.

1.  $\begin{bmatrix} -1 & 2 \\ -3 & 6 \end{bmatrix}$   $\begin{matrix} \textcircled{1} \\ -3 \end{matrix}$   $\begin{matrix} -1 \\ \textcircled{-1} \end{matrix}$

2.  $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$   $\begin{matrix} \textcircled{2} \\ 1 \end{matrix}$   $\begin{matrix} 2 \\ \textcircled{2} \end{matrix}$

3.  $\begin{bmatrix} 2 & 0 & -1 \\ 3 & 6 & 0 \\ 1 & 3 & 7 \end{bmatrix}$

4.  $\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

5.  $\begin{bmatrix} 6 & 4 & -2 & 0 \\ -1 & 7 & 5 & 2 \\ 1 & 0 & 4 & 4 \end{bmatrix}$