4-4 Optimal Strategy in Other Two-Person Zero-Sum Games

In this section we shall give techniques for finding Optimal strategies when the matrix not 2×2.

Dominant/Recessive Row: If a matrix A contains a row r^* with entries that are all less than or equal to the Corresponding entries in some other row r, then row r is said to dominate row r^* and r^* is said to be recessive.

Ex 1: In the matrix

$$A = \begin{array}{c} \begin{bmatrix} -6 & -3 & 2 & 2 \\ -2 & 0 & 3 & 2 \\ 5 & -2 & 4 & 0 \end{bmatrix} \\ \begin{matrix} 5 & 0 & 4 & 2 \end{matrix} \end{array} \begin{matrix} -6 \\ -2 \\ -2 \end{matrix}$$

row1 is dominated by row2, since each entry in row1 is less than or equal to its Corresponding entry in row2. We could represent it by the reduced matrix.

$$\begin{bmatrix} -2 & 0 & 3 & 2 \\ 5 & -2 & 4 & 0 \end{bmatrix}$$

in which row1 of matrix A is eliminated since it would never be chosen.

Dominant/Recessive Column: If a matrix A contains a column c^* with entries that are all greater than or equal to the Corresponding entries in some other col. c, then col. c is said to dominate col. c^* and c^* is said to be recessive.

Ex 2: In the matrix $A = \begin{bmatrix} -6 & 2 & 4 \\ 4 & 4 & 2 \\ 1 & 3 & -1 \end{bmatrix}$

Col.1 dominates Col.2, since each entry in Col.2 is greater than or equal to its Corresponding entry in Col.1. We could represent it by the reduced matrix

$$\begin{bmatrix} -6 & 2 \\ 4 & 4 \\ 1 & -1 \end{bmatrix}$$

tegies

1. Find the reduced form of the ollowing matrix

$$A = \begin{bmatrix} -6 & -4 & 2 \\ 2 & -1 & 2 \\ -3 & 4 & 4 \end{bmatrix}$$

Sol: First we look at the rows of the matrix A, we find that row1 is recessive, then the reduced matrix is:

$$\begin{bmatrix} 2 & -1 & 2 \\ -3 & 4 & 4 \end{bmatrix}$$

Now we looking at the col's, then col3 is recessive col., then the reduced matrix is:

$$\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$$

Ex14: Find the optimal strategy for each player, $A = \begin{bmatrix} -6 & -4 & 2 \\ 2 & -1 & 2 \\ -3 & 4 & 4 \end{bmatrix}$

Find the value of the two-person, zero sum game.

Sol: By eliminating recessive rows and cols, this matrix reduces to $\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$.

using ratio technique, we find that:

$$P_1 = \frac{7}{10},\ P_2 = \frac{3}{10} \text{ and } q_1 = \frac{5}{10},\ q_2 = \frac{5}{10}$$

$$P = \left[\frac{7}{10} \ \ \frac{3}{10}\right],\ Q = \begin{bmatrix} \frac{5}{10} \\ \frac{5}{10} \end{bmatrix}, \text{ then}$$

$v = \frac{5}{10}$., Thus the game is favorable to player A

H.W: Find the optimal strategy for each player and what is the value of the following games.

a. $\begin{bmatrix} 8 & 3 & 8 \\ 6 & 5 & 4 \\ -2 & 4 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 2 & 1 & 0 & 6 \\ 3 & -2 & 1 & 2 \end{bmatrix}$ c. $\begin{bmatrix} 4 & -5 & 5 \\ -6 & 3 & 3 \\ 2 & -6 & 3 \end{bmatrix}$

d. $\begin{bmatrix} 6 & -4 & 2 & -3 \\ -4 & 6 & -5 & 7 \end{bmatrix}$ e. $\begin{bmatrix} 4 & 3 & -1 \\ 1 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ f. $\begin{bmatrix} 1 & 3 & 0 \\ 0 & -3 & 1 \\ 0 & 4 & 1 \\ -2 & 1 & 1 \end{bmatrix}$

4-4-2 2xm or mx2 Matrix Games

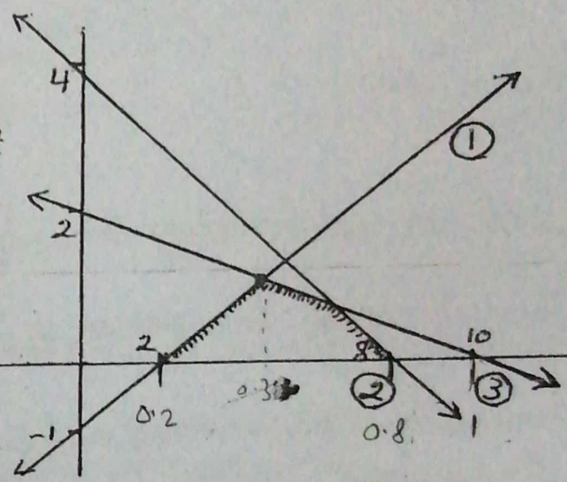
Let A be a two-person Zero-sum game matrix with 2xm or mx2 (m>2) degree which is not strictly determine, and that Contains no recessive rows or Cols.

I. 2xm Matrix Game

For a 2xm matrix game, player A has two strategies and B has m strategies.

<u>Ex15</u>: Find the optimal strategy foreach play in $A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix}$ what is the value of this game.

<u>Sol</u>: Suppose p is the prob. that player A plays row1, Then 1-p is the prob. that row2 is played. If B elects to play Col1, then E_A of A ① $E_A = 5p-1$. and ② $E_A = 4-5p$. ③ $2-2p$.



player A will choose the value of P that yields the most earning for him.

This occurs at the intersection of the lines ① $E_A = 5p-1$ and ③ $E_A = 2-2p$

then: $p = \frac{3}{7}$ and $E_A = \frac{8}{7} \Rightarrow P = [\frac{3}{7} \;\; \frac{4}{7}]$

To find the optimal Strategy for player B, Since Col2. is eliminated from the matrix, then the reduced matrix is: $\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

by using ratio technique we obtain:

$5p-1 = 4-5p$

$10p = 3 \to p = \frac{3}{10}$

$$q_1 = \frac{2}{7},\; q_2 = 0,\; q_3 = \frac{5}{7} \Rightarrow Q = \begin{bmatrix} \frac{2}{7} \\ 0 \\ \frac{5}{7} \end{bmatrix}.$$

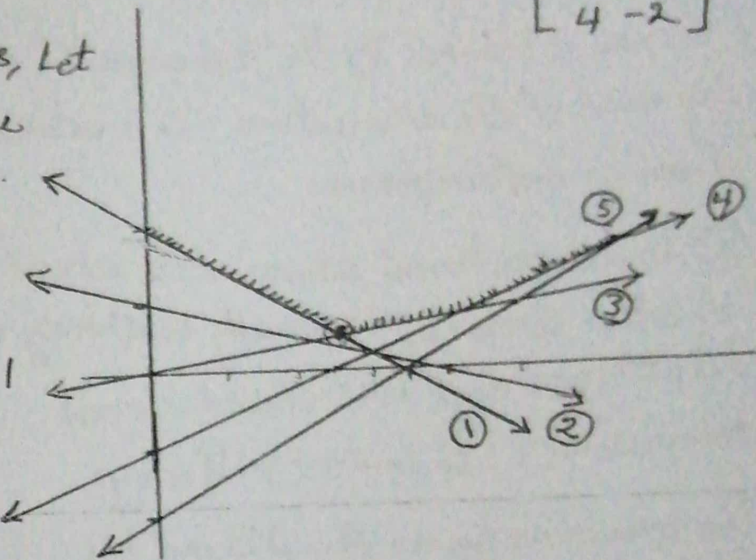
thats mean Col2 never played. by Player B.

a m×2 matrix game, play A has m strategies and B has 2 strategies only.

Ex16: Find the Optimal strategy for each player in 5×2 matrix game, what is the value of this game

$$A = \begin{bmatrix} -2 & 2 \\ -1 & 1 \\ 2 & 0 \\ 3 & -1 \\ 4 & -2 \end{bmatrix}$$

Sol: Here, player B has two strategies, Let q be the prob. of Col1 and $1-q$ is the prob. of choosing Col2. player A earnings E_A are then



① $E_A = -4q+2$ ② $E_A = -2q+1$

③ $E_A = 2q$ ④ $E_A = 4q-1$

⑤ $E_A = 6q-2$

player B wants the earning of $A(E_A)$ to be small as possible. This occurs at the intersection of ① and ③, then:

$q_1 = \frac{1}{3}$ and $E_A = \frac{2}{3} \Rightarrow$ of Course $q_2 = \frac{2}{3}$, then $Q = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$

we eliminate rows 2, 4 and 5, the reduced matrix is $\begin{bmatrix} -2 & 2 \\ 2 & 0 \end{bmatrix}$

then $P_1 = \frac{-2}{-6} = \frac{1}{3}$, $P_3 = \frac{-4}{-6} = \frac{2}{3} \Rightarrow P = [\frac{1}{3}\ 0\ \frac{2}{3}\ 0\ 0]$

H.W.: ① An investor plans to invest \$10,000 during a period of international uncertainty as to whether there will be peace, a Continuation of the Cold war, or an actual war. Investment can be made in government bounds, armament stocks, or industrial stocks. The game is struggle between the investor and nature. The matrix gives the rate of interset for each player

	nature		
Investor	hot war	cold war	Peace
Government bonds	2	3	3.2
Armament stocks	18	6	-2
Idustrial stocks	2	7	12

② (Behavior of Jamaican fisherman)

Each fishing Crew is confronted with a three-choice decision of fishing in the inside banks, the outside banks, or a combination of inside-outside banks. The fish caught estimates under the two conditions that current is present, or not present. If the crew is player A and the environment as another player, An estimation of income claimed by the fishermen using each of the alternatives. This estimate is given in matrix form.

$$\begin{array}{r} \text{Crew} \\ \text{Inside} \\ \text{Inside - outside} \\ \text{outside} \end{array} \begin{array}{c} \text{Environment} \\ \begin{array}{cc} \text{Current} & \text{no Current} \end{array} \\ \begin{bmatrix} 17.3 & 11.5 \\ 5.2 & 17.0 \\ -4.4 & 20.6 \end{bmatrix} \end{array}$$

③ In a department store, one area (A) is usually very crowded and the other area (B) is usually relatively empty. The store employs two detectives and has closed-circuit television (T) to control pilferage. The television covers A and B and the detectives can be in either area A or area B or watching the television (T). The matrix gives an estimate of the probability of the detectives finding and arresting a thief. Here, TT means both detectives are at the television. TA means the first detective is at the television and the second is in the area A, and so on. Find the optimal strategy for the thief and the detectives. What is the value of the game?

$$\begin{array}{r} \text{Detectives} \\ TT \\ AA \\ BB \\ TA \\ TB \\ AB \end{array} \begin{array}{c} \text{Thief} \\ \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.51 & 0.75 \\ 0.64 & 0.36 \\ 0.19 & 0.91 \\ 0.58 & 0.6 \\ 0.37 & 0.85 \\ 0.56 & 0.76 \end{bmatrix} \end{array}$$

④ (Effectiveness of Antibiotics) 3 types of Antibiotics A_1, A_2 and A_3, and five types of bacilli; M_1, M_2, M_3, M_4, and M_5, are involved in a study of the effectiveness of antibiotics on bacilli, with A_1 having a prob. 0.3 of destroying M_1, and so on, as given in the matrix.

$$\begin{array}{r} \text{Antibiotics} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \text{Bacilli} \\ \begin{array}{ccccc} M_1 & M_2 & M_3 & M_4 & M_5 \end{array} \\ \begin{bmatrix} 0.3 & 0.4 & 0.5 & 1 & 0 \\ 0.2 & 0.3 & 0.6 & 0 & 1 \\ 0.1 & 0.5 & 0.3 & 0.1 & 0 \end{bmatrix} \end{array}$$

In what ratio should the antibiotics be mixed to have the greatest prob. of being effectiveness.