MEASURES OF CENTRAL TENDENCY

The fundamental measures of tendencies are:

1/ mean
2/ Median
3/ Mode
4/ Geometric mean
5/ Harmonic mean
6/ Weighted averages.

1/ Mean (average)

The mean also called the arithmetic mean, is the most frequently used measure of central tendency.

* Mean for ungrouped data.

The mean for ungrouped data is obtained by dividing the sum of all values by the numbers of values in the data set.

$$(\bar{X})\ \text{Mean} = \frac{X_1 + X_2 \cdots X_n}{n} = \frac{\sum X_i}{n}$$

This method is known as the "Direct Method".

Example:

calculate the mean of the following data.

110 , 117 , 129 , 195 , 95 , 100 , 100 , 175 , 250 , 750

Solution

$$\sum x_i = 110 + 117 + 129 + 195 + 95 + 100 + 100 + 175 + 250 + 750$$

$$\sum x_i = 2021$$

$$n = 10 \qquad \text{So, } \bar{x} = \frac{2021}{10} = 202.1$$

* Mean for grouped data.

In the grouped data, every value (x_i) is multiplied by its corresponding frequency ($f_i x_i$), and then their total (sum) is found ($\sum f_i x_i$). ~~The mean~~ mean is obtained by dividing the sum ($\sum f_i x_i$) by the total frequency ($\sum f_i$).

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Example:

Find the mean (grouped data) of the following 50 observations.

19, 19, 20, 20, 20, 19, 20, 18, 21, 19, 20, 20, 19, 19
20, 19, 21, 19, 19, 21, 18, 20, 18, 18, 17, 20, 20, 22, 20
20, 20, 20, 20, 21, 20, 17, 23, 18, 17, 21, 20, 21
20, 20, 20, 18, 21, 19, 20, 19.

Solution:

observation x_i	frequency f_i	$f_i x_i$
17	3	$17 \times 3 = 51$
18	6	$18 \times 6 = 108$
19	11	209
20	20	400
21	8	168
22	1	22
23	1	23
Total	$\sum f_i = 50$	$\sum f_i x_i = 981$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{981}{50} = 19.62$$

Example

from the folloing data, Find the mean.

149, 406, 183, 107, 426, 97

$$\hat{x} = \frac{1368}{6} = 228.$$

* indirect method.

The mean for ungrouped data (indirect method) can be given by:

$$\bar{u} = \frac{\sum u_i}{n} \quad \text{where } u_i = x_i - A$$

A is any value in the data set.

$$\bar{X} = A + \bar{u}$$

Example.

calculate the mean (indirect method) of the folloing data.

110, 117, 129, 195, 95, 100, 100, 175, 250, 750.

let $A = 175$ then

$\sum u_i = x_i - A \Rightarrow$

$\sum u_i = (-65) + (-58) + (-46) + (20) + (-80) + (-75) + (-75) + (0) + (575)$

$\sum u_i = 670 - 399 = 271$

$$\bar{u} = \frac{271}{10} = 27.1$$

$\bar{X} = A + \bar{u} \Rightarrow 175 + 27.1 = 202.1$

properties of the mean.

1. The algebnic sum of the deviations of a set of numbers from their mean is zero, $\Sigma(x_i - \bar{x}) = 0$

If $\bar{x}$ is the mean of n observation $x_1, x_2, x_3 \cdots x_4$ then $(x_1 \mp \bar{x}) + (x_2 \mp x) + \cdots (x_n - \bar{x}) = 0$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{x_1 + x_2 + \cdots x_n}{n}$$

$$x_1 + x_2 + \cdots x_n = n\bar{x} \longrightarrow (1)$$

$$\vdots$$

$$\Sigma(x_i - \bar{x}) = 0$$

2. The mean of n observation $x_1, x_2 \cdots x_n$ is $\bar{x}$ if each observation is increased by p, the mean of new observation is $(\bar{x} + p)$.

3. The mean of observation $x_1, x_2 \cdots x_n$ is $\bar{x}$, if each observation is decreased by p, the mean of new observation is $(\bar{x} - p)$.

4. The mean of n observation $x_1, x_2 \cdots x_n$ is $\bar{x}$, if each observation is multiplied (divided) by a non zero number p, then mean of new observation is $p\bar{x}$ $\left(\frac{\bar{x}}{p}\right)$.

Example :

From the following observation

50 , 60 , 70 , 80

if number (let it be 5) is added to each observation , then new mean will be equal to original mean plus the number adde (5) to each observation.

Solution.

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{260}{4} = 65$$

x_i	$x_i + 5$
50	55
60	65
70	75
80	85
~~68~~	~~700~~
260	280

~~$\bar{x} = 70 = 65 + 5$~~

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{280}{4} = 70$$

$$70 = 65 + 5$$

if number (5) is subtracted from each observation then new will be equal to orginal mean less the number subtracted from observation.

x_i	$x_i - 5$
50	45
60	55
70	65
80	75
260	240

$$x_i = \frac{\Sigma x_i}{n} = \frac{260}{4} = 65$$

$$x_i = \frac{\Sigma x_i}{n} = \frac{240}{4} = 60$$

Example.

Find the sum of the deviations of the values 3, 4, 6, 8, 14 from their mean.

Solution

$$\bar{x} = \frac{\sum x_i}{n} = \frac{3+4+6+8+14}{5}$$

$$\bar{x} = \frac{35}{5} = 7$$

x_i	$x_i - \bar{x}$
3	$3-7$
4	$4-7$
6	$6-7$
8	$8-7$
14	$14-7$
	$\sum(x_i-\bar{x})=0$

$\therefore \sum(x_i - \bar{x}) = 0$