the flux density of diffuse solar radiation reaching the surface. The absorption and back-scattering by the aerosol layer reduced the total (direct plus diffuse) solar radiation reaching the surface by ~3%. In response to the reduction in total incident solar radiation, global-mean surface air temperature tends to be significantly lower in the wake of major volcanic eruptions than in the long-term average, as shown in Fig. 10.35.

The absorption of incident solar radiation by sulfate aerosols results in a pronounced warming of the lower stratosphere in response to volcanic eruptions, as shown in Fig. 10.36. The widths of the spikes in the temperature time series suggests that the residence time of the aerosols is on the order of 1–2 years, consistent with Fig. 10.34 and the discussion in Section 5.7.3.

The duration of the cooling at the Earth's surface in the wake of volcanic eruptions, as indicated by Fig. 10.35, appears to be somewhat longer than the 1- to 2-year residence time of the stratospheric aerosols. The difference is attributable to the involvement of the ocean mixed layer, with its large heat capacity. Throughout the time interval that the aerosols remain in the stratosphere, the Earth's surface and the air immediately above it tends to be anomalously cool. Latent and sensible heat fluxes through the ocean surface tend to be enhanced in response to the enhanced air–sea temperature difference. The enhanced fluxes reduce the amplitude of the atmospheric cooling by extracting heat from the ocean mixed layer and depositing it in the atmosphere. As the ocean mixed layer loses heat it becomes anomalously cool. With the removal of the aerosols the insolation incident upon the troposphere returns to normal and the anomalous fluxes at the air–sea interface reverse direction. An additional year or two is required for the ocean mixed layer to regain the heat that it lost while the aerosols were present in the atmosphere. Not until the ocean comes back into equilibrium does global-mean surface air temperature return to its preeruption level. In a similar manner, the exchange of heat between the atmosphere and the ocean mixed layer damps and delays the response of local sea-surface temperature to the annual cycle and to random month-to-month and year-to-year atmospheric variability.

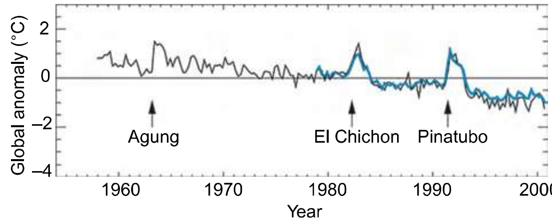

Fig. 10.36 Time series of the globally-averaged temperature in the lower stratosphere. The blue curve is based on the lower stratospheric channel of the microwave sounding unit, which is representative of the layer 15–20 km above the Earth's surface and the black curve is based on radiosonde data for the same layer. [Adapted from Intergovernmental panel on climate change, *Climate Change 2001: The Scientific Basis*, Cambridge University Press, p. 121 (2001).]

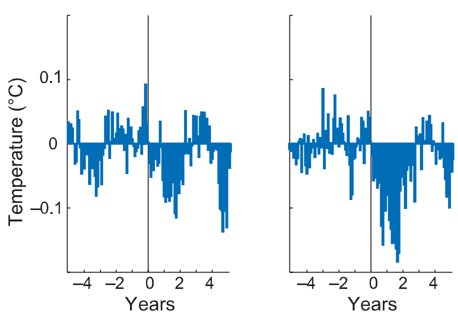

Fig. 10.35 (Left) Composite of global-mean surface air temperature around the time of seven major volcanic eruptions.[18] The time of the eruption corresponds to Year 0 on the *x* axis and positive values on the *x* axis denote time after the eruption. The 2- to 3-year-long cool interval beginning at the time of the eruption shows that the eruptions affected global-mean temperature. (Right) The same temperature data, adjusted to remove the temperature variability that is attributable to El Niño, the annular mode and the PNA pattern, so as to more clearly reveal the radiatively induced temperature change following these eruptions. The temperature scales for the two panels are the same. [Courtesy of David W. J. Thompson.]

10.3 Climate Equilibria, Sensitivity, and Feedbacks

To gain a more quantitative understanding of the climate response to a prescribed forcing, it will be useful to place the concepts of climate feedbacks and climate sensitivity into a mathematical framework.

[18] The seven largest volcanic eruptions in the modern record: Krakatoa (August 1883), Tarawera (June 1886), Pelee/Soufriere/Santa Maria (May–October 1902), Katmai (June 1912), Agung (March 1963), El Chichon (April 1982), and Pinatubo (June 1991).

This same framework will provide a basis for understanding the nature of "climate surprises"—instances in which a modest (or gradual) climate forcing could, at least in principle, give rise to a large and abrupt climate change that would be irreversible.

In the interest of brevity we focus on variations in only one climate variable, namely global-mean surface air temperature T_s, which we treat as being governed by three factors: the incident solar radiation, the planetary albedo, and the strength of the greenhouse effect averaged over the Earth's surface. A number of auxiliary variables (e.g., the concentrations of atmospheric water vapor and ozone and the fractions of the surface area of the Earth covered by clouds with tops in various altitude ranges and by ice and snow) enter into the determination of the albedo and the strength of the greenhouse effect. We will treat the forcing and the response as spatially uniform and consider only globally averaged quantities.

The *radiative forcing* F is defined as the net downward flux density or irradiance at the top of the atmosphere[19] that would result if it were applied instantaneously (i.e., without giving the surface air temperature or the vertical temperature profile any time to adjust to it). For example, if the flux density of solar radiation were to increase by the amount dS, the net radiation at the top of the atmosphere would initially be downward and equal to dS. Surface air temperature T_s would gradually rise in response to this imbalance until the outgoing flux of the atmosphere increased by the amount dS, at which time the Earth system can be said to have equilibrated with the forcing. When the system reached its new equilibrium, T_s would have increased by the amount dT_s in response to the solar forcing dS.

In a similar manner, an increase in the greenhouse effect can be expressed as a net downward irradiance dG at the top of the atmosphere, equal to the initial decrease in the upward irradiance of longwave radiation due to the enhanced blanketing effect of the atmosphere. As in the previous example, T_s would rise in response to the imbalance until the outgoing irradiance at the top of the atmosphere became equal to the incoming solar radiation. When the system had fully adjusted to the change, T_s would have increased by the amount dT_s in response to the enhanced greenhouse forcing dG.

The sensitivity of T_s to the radiative forcing F, namely $\lambda \equiv dT_s/dF$, is called the *climate sensitivity*. Consistent with its definition in terms of the total derivative, the climate sensitivity takes into account the changes in the auxiliary variables y_i (the concentration of atmospheric water vapor, the fraction of the Earth's surface covered by snow and ice, low clouds, etc.) that influence T_s. The various processes that play a role in determining the climate sensitivity can be evaluated by expanding the total derivative using the chain rule, which yields

$$\lambda = \frac{dT_s}{dF} = \frac{\partial T_s}{\partial F} + \sum_i \frac{\partial T_s}{\partial y_i}\frac{dy_i}{dF} \tag{10.5}$$

The partial derivative term $\partial T_s/\partial F$ is the climate sensitivity λ_0 that would prevail in the absence of feedbacks involving the auxiliary variables:

$$\lambda_0 \equiv \frac{\partial T_s}{\partial F} \approx \frac{dT_E}{dF}$$

where T_E is the equivalent blackbody temperature, as estimated in Exercise 4.6.

Exercise 10.3 Estimate the sensitivity of the Earth's equivalent blackbody temperature to a change in the solar radiation F_s incident upon the the top of the atmosphere.

[19] The current operational definition of *radiative forcing* uses the tropopause rather than the top of the atmosphere as a reference level with the proviso *"after the stratosphere has come into a new thermal equilibrium state."* This definition is motivated by the following considerations:

1. The tropospheric temperature profile, surface temperature, and the temperature of the ocean mixed layer are strongly coupled because the tropical lapse rate is constrained to be close to moist adiabatic. In contrast, the stratosphere, with its stable stratification, is decoupled from the underlying media. Hence, variations in surface temperature are related more strongly to the radiative forcing at the tropopause than at the top of the atmosphere.
2. The stratosphere equilibrates with an abrupt change in radiative forcing within a matter of months, whereas the troposphere requires decades to fully adjust because of the large thermal inertia of the oceans.

Once the stratosphere has come into thermal equilibrium with the new radiative forcing, the net irradiance through the tropopause and the top of the atmosphere are the same.

Solution: From the Stefan–Boltzmann law (Eq. 4.12)

$$T_E = \left(\frac{F_s}{\sigma}\right)^{1/4}$$

or, taking the natural log of both sides,

$$\ln T_E = \frac{1}{4}\ln F_s - \frac{1}{4}\ln\sigma$$

Taking the differential yields

$$\frac{dT_E}{T_E} = \frac{1}{4}\frac{dF_s}{F_s}$$

Hence,

$$\frac{dT_E}{dF_s} = \frac{1}{4}\frac{T_E}{F_s}$$

Exercise 4.6 shows that for the Earth $F_s = 239.4$ W m^{-2} and $T_E = 255$ K. Hence,

$$\frac{\partial T_s}{\partial F_s} = 0.266\ \mathrm{K(W\ m^{-2})^{-1}}$$

Or, taking the reciprocal, it can be inferred that the Earth's equivalent black body temperature rises 1 K for each 3.76 W m^{-2} of (downward) radiative forcing at the top of the atmosphere. ■

The changes in the auxiliary variables in the last term in (10.5) are a consequence of their temperature dependence. Hence,

$$\frac{dy_i}{dF} = \frac{dy_i}{dT_s}\frac{dT_s}{dF}$$

Substituting into (10.5) we obtain

$$\frac{dT_s}{dF} = \frac{\partial T_s}{\partial F} + \frac{dT_s}{dF}\sum_i f_i \qquad (10.6)$$

where

$$f_i = \frac{\partial T_s}{\partial y_i}\frac{dy_i}{dT_s} \qquad (10.7)$$

are dimensionless *feedback factors* associated with the various feedback processes to be discussed in the following subsections. The feedback factor f_i will be positive if the two derivatives of which it is comprised in (10.7) are of like sign, and it will be negative if they are of opposing sign. [For example, if a rise in T_s results in a decrease in y_i (as is the case for planetary albedo) and if a decrease in y_i favors a further rise in T_s, then the feedback is positive.] The feedback factors are additive, i.e.,

$$f = \sum_i f_i \qquad (10.8)$$

where the algebraic sign of the feedback is taken into account in the summation.

Solving (10.6) for the sensitivity of T_s to the forcing F, we obtain

$$\frac{dT_s}{dF} = \frac{\partial T_s/\partial F}{1-f} \qquad (10.9)$$

Hence, the gain $g \equiv \lambda/\lambda_0$ due to the presence of climate feedbacks is

$$g = \frac{1}{1-f} \qquad (10.10)$$

provided that $f < 1$. A value of $f \geq 1$ corresponds to the case of infinite sensitivity, in which even an infinitesimal forcing may cause the climate system to diverge from its present equilibrium state and seek a new one, as in the mechanical analogs pictured in Fig. 3.17.

Exercise 10.4 Estimate the apparent climate sensitivity $\delta T_s/\delta F$ based on estimates of the differences in T_s and F between current climate and the climate at the time of the last glacial maximum (LGM) around 20,000 years ago, using the relation

$$\frac{\delta T_s}{\delta F} = \frac{T_s\,(\mathrm{current}) - T_s\,(\mathrm{LGM})}{F\,(\mathrm{current}) - F\,(\mathrm{LGM})}$$

Solution: Global-mean surface air temperature is estimated to have been ~5 °C lower at the time of the LGM than it is today. On the basis of ice core data it is known that atmospheric CO_2 concentrations were 180 ppmv, slightly less than half their current values. On the basis of radiative transfer models it is estimated that the climate forcing due to a doubling of the CO_2 concentration is 3.7 W m^{-2}. On the basis of what is known about the coverage of the continental ice sheets and the extent of sea ice at the time of the LGM, it is estimated that the planetary albedo was ~0.01 higher at the time of the LGM than it is today. Assuming that the flux density of solar radiation at the time of the LGM is the same as the current value (342 W m^{-2}), the corresponding increase in the flux of reflected solar radiation at the top of the atmosphere is $342 \times 0.01 = 3.4$ W m^{-2}. Substituting these values into the above expression yields

$$\frac{\delta T_s}{\delta F} = \frac{5\text{ K}}{(3.7 + 3.4)\text{ W m}^{-2}} = 0.70\text{ K per W m}^{-2}$$

Comparing this result with the climate sensitivity in the absence of feedbacks $\lambda_0 = 0.266$ (W m^{-2})$^{-1}$, as computed in Exercise 10.3, and the apparent sensitivity of the climate system is enhanced by a factor of 0.70/0.266 = 2.7 due to the presence of feedbacks. ■

10.3.1 Transient versus Equilibrium Response

Because of the large heat capacity of the Earth system (in particular, the oceans and the cryosphere), global-mean surface air temperature exhibits a delayed response to climate forcing. An equilibrium response to an abrupt change in the forcing would be achieved only after all components of the system have had adequate time to adjust to the change. The adjustment time is different for different components of the Earth system: the atmosphere adjusts to changes in climate forcing within a matter of a few months, the ocean mixed layer requires a few years, the full depth of the ocean requires centuries, and the continental ice sheets perhaps even longer. The respective adjustment times depend on both the heat capacity of the various components of the Earth system and on the climate sensitivity.

We can gain some insight into the nature of this adjustment process by considering the effect of including a hypothetical ocean mixed layer whose temperature adjusts instantaneously to changes in surface air temperature. With this assumption we can treat the ocean mixed layer as a slab with mean temperature $T = T_0 + T'$, where T_0 is the equilibrium temperature before the forcing F is applied and T' is the time varying (i.e., transient) response to a change Q' in the climate forcing. Based on the energy balance at the top of the atmosphere, we can write

$$c\frac{dT'}{dt} = -\frac{T'}{\lambda} + Q' \qquad (10.11)$$

where c is the heat capacity of the slab in units of J m^{-2} K^{-1} averaged over the surface area of the Earth and λ is the climate sensitivity dT_s/dF. The left-hand side of (10.11) is the rate at which energy is stored in the slab, and the right hand side as the imbalance between the forcing Q' and the increase in outgoing longwave radiation at the top of the atmosphere due to the warming of the slab.

Now let us assume that the perturbation in the forcing Q' is "turned on" at time $t = 0$ and maintained at a constant value after that time. In this case (10.11) can be rewritten as

$$\frac{dT'}{dt} + \frac{T'}{\tau} = \frac{Q'\lambda}{\tau}$$

where $\tau = c\lambda$, and solved to obtain

$$T' = \lambda Q' \left(1 - e^{-t/\tau}\right) \qquad (10.12)$$

Hence, after the forcing is turned on, T' exponentially approaches the equilibrium solution $\lambda Q'$, increasing rapidly at first and then more gradually as the solution is approached. The *e*-folding timescale for the approach to the equilibrium solution is proportional, not only to the heat capacity of the mixed layer, but also to the climate sensitivity. Hence the existence of positive climate feedbacks lengthens the time it takes the climate system to equilibrate with a change in the forcing. It can be shown that for a linear rate of increase in climate forcing, the response of T' is also linear and delayed by the same response time $\tau = c\lambda$.[20]

The time delay in the response to climate forcing due to the heat capacity of the ocean mixed layer is on the order of a decade or less (see Exercise 10.21). The heat capacity of the oceans as a whole is about 50 times larger than that of the ocean mixed layer; the continental ice sheets also possess a high effective heat capacity because of the large magnitude of the latent heat of fusion of water. If the atmosphere exchanged heat freely with these large reservoirs, the response time, as defined in the context of (10.11), would be on the order of centuries or longer. Short-term perturbations of the climate system such as volcanic eruptions would be almost entirely damped out and the impacts of longer term changes in forcing, such as those associated with greenhouse warming, would become apparent only over the course of centuries.

The atmosphere does, in fact, exchange significant quantities of heat with these large reservoirs, but the rates of exchange are much slower than with the ocean mixed layer. The thermohaline circulation ventilates the deep layers of the ocean on timescales of centuries,

[20] See D. L. Hartmann, *Global Physical Climatology*, Academic Press, Section 12.6 (1994).