Atmospheric Thermodynamics

Lecture 2: Humidity

2.1 What is humidity?

- Humidity is the amount of water vapor, an invisible gas, in the air.
- Warm air can 'hold' more water vapor than cold air; in fact, air at 35°C can hold six times more water vapor than air at 5°C. Humidity can affect comfort levels – a hot day will feel oppressive and muggy if the air is very humid, and it may lead to heat stress because sweat cannot evaporate from your body to cool you down.

2.2 How is Humidity Measured?

- Instruments for measuring relative humidity are called hygrometers.
- A traditional weatherman's tool is the 'wet and dry-bulb thermometer' (or Mason's hygrometer). Two identical thermometers are mounted side by side, but the bulb of one of them is kept wet by a wick dipping into a water reservoir. Evaporation from the wet bulb causes it to cool – you can demonstrate this by licking your finger and blowing on it - and the amount it cools depends on the humidity. So to measure the RH, we take the air temperature and the difference between the wet bulb and the dry bulb and use a table (supplied with the thermometer) to read off the RH.
- Electronic sensors with digital displays now are also available.

2.3 Water Vapor

Water vapor is important for moving energy around the Earth. Energy from the Sun evaporates water into water vapor, which can easily be moved around the atmosphere. The water vapor eventually condenses back into water releasing energy as heat. This heat is a key component in the formation of storms.

2.4 Moisture Variables

There are a number of ways to specify the amount of moisture (referred to as humidity) in the air such as:

(1) Absolute humidity (2) specific humidity (3) vapor pressure
(4) saturation vapor pressure (5) relative humidity (6) mixing ratio
(6) saturation mixing ratio (7) wet-bulb temperature (8) dew-point temperature

2.4.1 Absolut humidity

The Absolute Humidity of a parcel of air is expressed as:

$$Specific\ Humidity = \frac{mass\ of\ water\ vapor}{volume\ of\ air}$$

So, Absolute Humidity is like a water vapor density (ρ_v), commonly express in $\frac{grams}{m^3}$

Question: Is Absolute Humidity a good variable to use for measuring moisture in air?

Answer: No, because it is sensitive to changes in both the air temperature and atmospheric pressure.

How? Consider a parcel of air that is rising and expanding and recall that absolute humidity = mass of H2O / volume. The amount of moisture in the parcel does not change as it rises. However, the absolute humidity is not constant as the parcel rises even though the amount of moisture is not changing because the volume is changing.

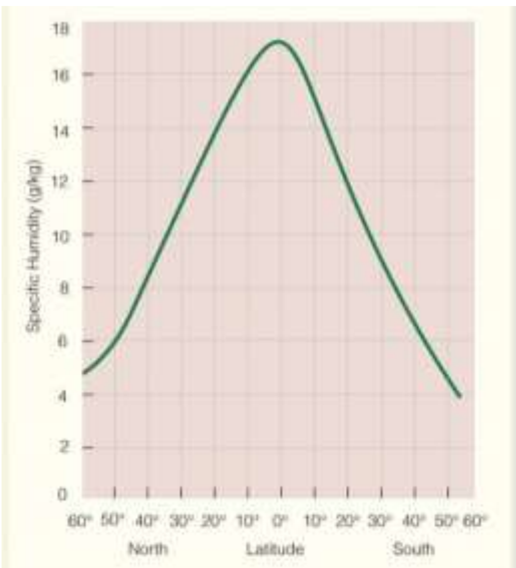
2.4.2 Specific humidity

Specific humidity is defined as:

$$Specific\ Humidity = \frac{mass\ of\ water\ vapor}{total\ mass\ of\ air}$$

example: In a given parcel, the mass of water vapor is 1 gm, the total mass of the parcel (N2, O2, AR, H2O, other trace gasses) is 1 kg. Then the specific humidity is: 1 gm/kg

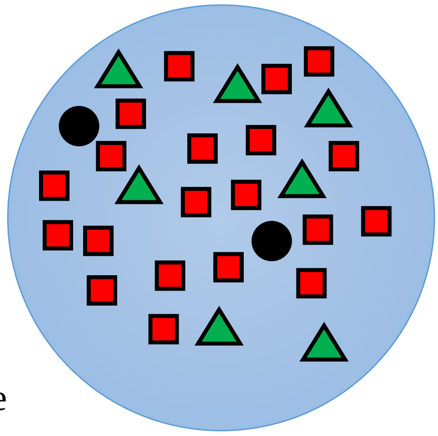
Q: what will the latitudinal distribution of specific humidity look like?



2.4.3 Vapor pressure

Given a parcel of air comprised of only nitrogen, oxygen, and water vapor:

[■ is N2 78% , ▲ is O2 21% , ● is H2O less than 1%]



The total pressure of the air parcel is due to the sum of "partial pressures" of each of the gasses comprising the parcel. The partial pressure due to water vapor is called the "vapor pressure".

- We usually denote vapor pressure as e.
- vapor pressure is related to absolute humidity via the ideal gas law: $e = \rho_v \, R_v \, T$
  Where R_v is the specific gas constant for water vapor (461 J kg^{-1} K^{-1}).
- Vapor pressure similar to absolute humidity is not very convenient expressions for humidity (at least for meteorologists).

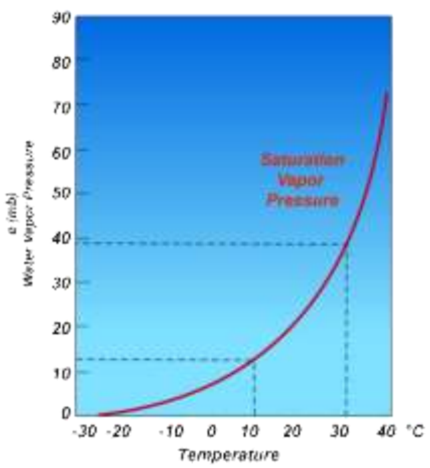
2.4.4 Saturation vapor pressure

When an air parcel is saturated, the vapor pressure is then the saturation vapor pressure (e_s).

- The saturation vapor pressure is a function of temperature, and is given by the Clausius-Clapeyron equation

$$e_s = e_0 \, exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$

where e_0 is the vapor pressure at some known temperature T_0 , and L_v is the latent heat of vaporization. We typically use $T_0 = 273\,K, e_0 = 611\,Pa\,, and\; L_v = 2.5 \times 10^6\,J\,kg^{-1}$.



Q: For an unsaturated parcel of air at a given temperature, there are two ways to saturate it where the vapor pressure will then equal the saturation vapor pressure. What are they?

Ans.: (1) cool the parcel until it is saturated. (2) add more moisture to it.

2.4.5 Relative humidity

It is the most common variable used to describe atmospheric moisture and is defined as the ratio of the vapor pressure to the saturation vapor pressure,

$$RH = \frac{water\ vapor\ content}{water\ vapor\ capacity}$$

$$RH = \frac{e}{e_s} \times 100\ \%$$

$$RH = \frac{mixing\ ratio}{saturation\ mixing\ ratio}$$

- Relative humidity tells us how close an air parcel is to saturation but it does not directly tell us how much water vapor is in the parcel!

2.4.6 Mixing ratio

Mixing ratio is defined as:

$$mixing\ ratio = \frac{mass\ of\ water\ vapor}{mass\ of\ dry\ air}$$

Example: in a given parcel, the mass of water vapor is 1 gm, the mass of dry air in the parcel (N2, O2, AR, other trace gasses) is 1.0 kg

Then the mixing ratio is: 1 gm/kg

- Mixing ratio can be related to vapor pressure via:

$$mixing\ ratio\ (r) = \frac{\rho_v}{\rho_d} = \frac{\frac{e}{R_v T}}{\frac{p_d}{R_d T}} = \frac{R_d}{R_v}\frac{e}{p_d} = \varepsilon \frac{e}{p-e}$$

  where $\varepsilon = \frac{R_d}{R_v}$

- Specific humidity is very close to mixing ratio, as shown:

$$Specific\ humidity\ (q) = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_d + \rho_v} = \frac{\frac{\rho_v}{\rho_d}}{1 + \frac{\rho_v}{\rho_d}} = \frac{r}{1+r} \approx r$$

  since $r \ll 1$

- In meteorology, mixing ratio is used far more than specific humidity, and for most purposes the two can be considered as equivalent.
- The saturation mixing ratio, r_s , is found by using e_s in the formula.

2.4.7 Wet bulb temperature

The wet bulb temperature is the lowest temperature that can be reached by evaporating water into the air.

Note: the wet bulb temperature will always be less than or equal to the air temperature.

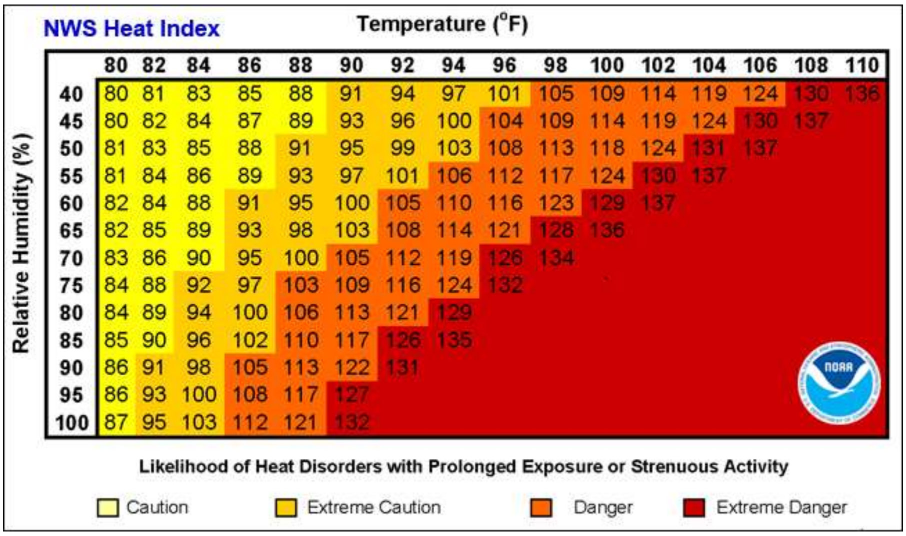
Example: If you are a runner:

T = 90 F, RH = 90% => high wet-bulb temp.

T = 90 F, RH = 10% => low wet-bulb temp.

feel more comfortable when wet-bulb temperature is low

Wet-bulb temperature is related to the heat index (see the figure below).



NWS Heat Index

Temperature (°F)

Relative Humidity (%)	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110
40	80	81	83	85	88	91	94	97	101	105	109	114	119	124	130	136
45	80	82	84	87	89	93	96	100	104	109	114	119	124	130	137	
50	81	83	85	88	91	95	99	103	108	113	118	124	131	137		
55	81	84	86	89	93	97	101	106	112	117	124	130	137			
60	82	84	88	91	95	100	105	110	116	123	129	137				
65	82	85	89	93	98	103	108	114	121	128	136					
70	83	86	90	95	100	105	112	119	126	134						
75	84	88	92	97	103	109	116	124	132							
80	84	89	94	100	106	113	121	129								
85	85	90	96	102	110	117	126	135								
90	86	91	98	105	113	122	131									
95	86	93	100	108	117	127										
100	87	95	103	112	121	132										

Likelihood of Heat Disorders with Prolonged Exposure or Strenuous Activity

Caution   Extreme Caution   Danger   Extreme Danger

QUESTIONS FOR THOUGHT:

1. On a warm, muggy day, the air is described as "close". What are several plausible explanations for this expression?
[“The air is close”, is generally referring to a hot, muggy (humid) room, with no ventilation. It's a feeling of not having enough air to breathe.]
2. Why are evaporative coolers used in Arizona, Nevada, and California, but not in Florida, Georgia, or Indiana?

2.4.8 Dew point temperature (T_d)

Temperature to which one must cool air for it to reach saturation. It's a pretty good approximation to amount of vapor in air.

The difference between T, T_d is proportional to the relative humidity.

- The dew point temperature can be found from the Clausius-Clapeyron equation by using the actual vapor pressure instead of the saturation vapor pressure, and solving for T. This gives:

$$T_d = \left[\frac{1}{T_0} - \frac{R_v}{L} ln\left(\frac{e}{e_0}\right)\right]^{-1}$$

2.5 Key difference between the measures of humidity

- There are two ways to change the relative humidity, or absolute humidity of an air parcel:
    - Add or subtract water vapor.
    - Change the temperature.
- There is only one way to change mixing ratio, specific humidity, vapor pressure, or dew point (assuming pressure is constant):
    - Add or subtract water vapor.

2.6 Humidity Measurement

- Meteorologists commonly measure humidity by measuring the wet-bulb temperature.
- The wet-bulb temperature is the lowest temperature that can be achieved by evaporating water into the air parcel at constant pressure (the evaporation requires heat, which is supplied by the air parcel).
- Wet-bulb temperature is measured using a psychrometer.
- The difference between the air temperature (dry-bulb temperature) and the wet-bulb temperature is called the wet-bulb depression.
    - The wet-bulb depression is a relative measure of the moisture content of the air. Dry air can be cooled much further by evaporation than moist air, so a larger wet-bulb depression means less humidity (for the same dry-bulb temperature).
    - The dew-point temperature and relative humidity are found by using psychrometric tables, with dry-bulb temperature and wet-bulb depression as the independent variables.

2.7 Virtual Measurement

- Moist air is a mixture of two ideal gases: dry air and water vapor.
- The ideal gas law for moist air is:

$$p = p_d + e = (\rho_d R_d + \rho_v R_v)T$$

- We can manipulate this in the following manner:

$$p = \rho R_d T(\frac{\rho_d}{\rho} + \frac{\rho_v}{\rho}\frac{R_v}{R_d} = \rho R_d T(\frac{\rho - \rho_v}{\rho} + \frac{\rho_v}{\rho}\frac{R_v}{R_d})$$

$$p = \rho R_d T(1 - \frac{\rho_v}{\rho} + \frac{\rho_v}{\rho}\frac{R_v}{R_d}) = \rho R_d T(1 - q + q/\varepsilon)$$

- If we define a new temperature, T_v such that

$$T_v = T(1 - q + q/\varepsilon)$$

then we can write the ideal gas law for moist air as

$$p = \rho R_d T_v$$

- T_v is called the virtual temperature.
- For moist air we can use the ideal gas law for dry air, only using the virtual temperature in place of the actual temperature.
- Virtual temperature is always greater than or equal to the actual temperature.
  - The addition of water vapor causes the air to behave as though it is warmer. This makes sense, because moist air is lighter than dry air.
- Since $\varepsilon = 0.622$, we have

$$T_v = T(1 + 0.61\,q) \qquad Virtual\,Temperature$$

- Virtual temperature can also be written in terms of mixing ratio as

$$T_v = T\left(\frac{1 + \frac{r}{\varepsilon}}{1 + r}\right)$$

- Since mixing ratio and specific humidity are so close, we often write virtual temperature using mixing ratio as

$$T_v \cong T(1 + 0.61\,r)$$

- In all of the preceding equations for virtual temperature we must use the absolute (Kelvin) temperature and the dimensionless form of mixing ratio or specific humidity.
  - However, there is an approximate formula for virtual temperature in Celsius that uses the dimensional (g/kg) form of mixing ratio or specific humidity.

    This formula is

$$T_{v[^\circ C]} = T_{[^\circ C]} + \frac{r_{[g/kg]}}{6}$$

EXERCISES

1. Show that the specific gas constant for water vapor, $R_v , is\ 462\ J\ kg^{-1}\ K^{-1}$.

   ($R =\ 8.3145\ J\ mol^{-}K^{-1}$, and the molecular weight of water is 18.001 g/mol)

2. An air sample at standard sea level pressure, and with a volume of 1 m^3 at 20^o C, contains 7 grams of water vapor.

a. What is the vapor pressure?

b. What is the saturation vapor pressure?

c. What is the relative humidity?

d. What is the absolute humidity?

e. What is the mixing ratio?

f. What is the specific humidity?

g. What is the dew-point temperature?

3. The air parcel in question 2 is cooled (at constant pressure) to 10 ^{o}C.

a. What is the vapor pressure?

b. What is the saturation vapor pressure?

c. What is the relative humidity?

d. What is the absolute humidity?

e. What is the mixing ratio?

f. What is the specific humidity?

g. What is the dew-point temperature?

h. Which humidity variables remained constant as a result of the cooling?