The domains D_f and range R_f

Root function:

General form $y = \sqrt{f(x)} + k$

$D_f \longrightarrow f(x) \geq 0$

R_f we have two cases

1. If the value of D_f unbounded

   $R_f = \{y: \ y \geq k\}$

Find domain D_f and range R_f of

EXAMPLE 1: $y = \sqrt{x}$

Sol

$x \geq 0$

$D_f = \{x : x \geq 0\}$

$R_f = \{y : y \geq 0\}$

EXAMPLE 2: $y = 1 + \sqrt{7 - 10x}$

Sol

$7 - 10x \geq 0$

$-10x \geq -7$

$10x \leq 7$

$x \leq 7/10$

$D_f = \{x : x \leq 7/10\}$

$R_f = \{y : y \geq 1\}$

H.W

Ex 3: $y = \sqrt{4 - x}$

Ex 4: $y = \sqrt{1 - 3x}$

Ex 5: $f(x) = \sqrt{2x - 3} + 7$

EXAMPLE 6: $y = \sqrt{x^2 - 1}$

Sol

$x^2 - 1 \geq 0$

$x^2 \geq 1$

$x \geq \pm\sqrt{1}$

$x \geq \pm 1$

$x \geq 1$ or $x \leq -1$

$D_f = \{x : x \geq 1 \text{ or } x \leq -1\}$

$R_f = \{y : y \geq 0\}$

H.W **Ex 7**: $y = \sqrt{x^2 - 9}$

2. If the value of D_f bounded $(-a \leq x \leq a)$

$R_f = (0 \leq y \leq a)$

EXAMPLE 8: $y = \sqrt{4 - x^2}$

Sol

$4 - x^2 \geq 0$

$-x^2 \geq -4$

$x^2 \leq 4$

$x \leq \pm 2$

$D_f = \{x : -2 \leq x \leq 2\}$

$R_f = \{y : 0 \leq y \leq 2\}$ just positive

H.W **EX 9**: $y = \sqrt{1 - x^2}$

EXAMPLE 10: $y = \sqrt{6 - x^2}$

Sol

$$6-x^2 \geq 0$$
$$-x^2 \geq -6$$
$$x^2 \leq 6$$
$$x \leq \pm\sqrt{6}$$
$$D_f = \{x: -\sqrt{6} \leq x \leq \sqrt{6}\}$$
$$R_f = \{y: 0 \leq y \leq \sqrt{6}\}$$

EXAMPLE 11: $y = \sqrt{2-\sqrt{x}}$

Sol

1) $\sqrt{x}$
   $x \geq 0$

2) $2-\sqrt{x} \geq 0$
   $2 \geq \sqrt{x}$
   $\sqrt{x} \leq 2$
   $x \leq 4$

$$D_f = \{x: 0 \leq x \leq 4\}$$
$$R_f = \{y: 0 \leq y \leq 4\}$$

H.W **Ex 12**: $y = \sqrt{1-\sqrt{x}}$

Ex 13: $y = \sqrt{7x-3}$

Ex 14: $y = 1-\sqrt{x}$

Relative function

General form $y = \dfrac{f(x)}{g(x)}$

$$D_f = R/\{g(x) = 0\}$$

To find R_f rewrite function ($x \to f(y)$)

Find domain D_f and range R_f

EXAMPLE 1: $y = \dfrac{1}{x}$

$$D_f = R/\{x = 0\}$$

$R_f \rightarrow \quad y = \frac{1}{x}$

$x = \frac{1}{y}$

$R_f = R/\{y = 0\}$

EXAMPLE 2: $y = \frac{3}{2x - 7}$

Sol

$2x - 7 = 0$

$2x = 7$

$x = 2/7$

$D_f = R/\{x = 2/7\}$

To find R_f

$y = \frac{3}{2x - 7}$

$2xy - 7y = 3$

$2xy = 3 + 7y$

$x = \frac{3 + 7y}{2y}$

$R_f = R/\{y = 0\}$

H.W **Ex 3**: $y = \frac{x}{x - 1}$

Ex 4: $y = \frac{3x - 3}{1 - 7x}$

Ex 5: $y = \frac{2x - 4}{x + 3}$

EXAMPLE 6: $y = \frac{1}{\sqrt{7x - 1}}$

Sol

$$7x-1>0$$
$$7x>1$$
$$x>1/7$$
$$D_f=R/\{x:\ x>1/7\}$$
$$R_f=R/\{y=0\}$$

EXAMPLE 7: $y=\sqrt{\frac{1}{x}-1}$

Sol

$$y=\sqrt{\frac{1-x}{x}}$$
$$y=\frac{\sqrt{1-x}}{\sqrt{x}}$$

1) $\sqrt{1-x}$

$$1-x\geq 0$$
$$-x\geq -1$$
$$x\leq 1$$

2) $\sqrt{x}>0$

$$D_f=x:\quad 0<x\leq 1$$
$$R_f=y:\quad 0<y\leq 1$$

H.W **Ex 8**: $y=\sqrt{\frac{1+x}{1-x}}$

Ex 9: $f(x)=1+x^2$

Ex 10: $f(t)=\frac{1}{\sqrt{t}}$