<u>The Inverse hyperbolic function</u> If is u any differentiable function of x then:

1. $\frac{d}{x}\sinh^{-1}u = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$

2. $\frac{d}{x}\cosh^{-1}u = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$

3. $\frac{d}{x}\tanh^{-1}u = \frac{1}{1-u^2}\frac{du}{dx} \qquad u<1$

4. $\frac{d}{x}\coth^{-1}u = \frac{1}{1-u^2}\frac{du}{dx} \qquad u>1$

5. $\frac{d}{x}\operatorname{sech}^{-1}u = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx}$

6. $\frac{d}{x}\operatorname{csch}^{-1}u = -\frac{-1}{u\sqrt{1+u^2}}\frac{du}{dx}$

EXAMPLE 1: Find $\frac{dy}{dx}$ for the following function.

1. $y = \cosh^{-1}(\sec x)$

Sol:

$$y' = \frac{\sec x \tan x}{\sqrt{(\sec x)^2 - 1}} = \frac{\sec x \tan x}{\sqrt{\tan^2 x}}$$

$$y' == \frac{\sec x \tan x}{\tan x} = \sec x \qquad \text{where} \quad \tan x > 0$$

2. $y = \tanh^{-1}(\cos x)$

Sol:

$$y' = \frac{-\sin x}{1-\cos^2 x} = \frac{-\sin x}{\sin^2 x} = -\csc x$$

3. $y = \coth^{-1}(\sec x)$

Sol:

$$y' = \frac{\sec x \tan x}{1 - \sec^2 x} = \frac{\sec x \tan x}{-\tan^2 x} = -\csc x$$

4. $y = \text{sech}^{-1}(\sin 2x)$

Sol:

$$y' = -\frac{2\cos 2x}{\sin 2x \sqrt{1 - \sin^2 2x}} = \frac{-2\cos 2x}{\sin 2x \cos 2x} = -2\csc 2x \qquad \text{where } 2x > 0$$

EXAMPLE 2: Verify the following formulas:

1. $\dfrac{d}{x}\cosh^{-1} u = \dfrac{1}{\sqrt{u^2 - 1}}\dfrac{du}{dx}$

Sol:

Let $y = \cosh^{-1} u$

$$u = \cosh y$$

$$\frac{du}{dx} = \sinh\ y \frac{dy}{dx}$$

$$y' = \frac{dy}{dx} = \frac{1}{\sinh y}\frac{du}{dx}$$

$$\cosh^2 y - \sinh^2 y = 1 \qquad \Rightarrow u^2 - \sinh^2 y = 1$$

$$\Rightarrow \ \sinh\ y = \sqrt{u^2 - 1}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{u^2 - 1}} \cdot \frac{du}{dx}$$

2. $\dfrac{d}{x}\tanh^{-1} u = \dfrac{1}{1 - u^2}\dfrac{du}{dx}$

Sol:

Let $y = \tanh^{-1} u$

$u = \tanh\, y$

$$\frac{du}{dx} = \operatorname{sech}^2 y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\operatorname{sech}^2 y} \frac{du}{dx}$$

$$\operatorname{sech}^2 y + \tanh^2 y = 1 \quad \Rightarrow \operatorname{sech}^2 y = 1 - \tanh^2 y = 1 - u^2$$

$$\frac{dy}{dx} = \frac{1}{1-u^2} \frac{du}{dx}$$

$$\Rightarrow \quad \frac{d}{x} \tanh^{-1} u = \frac{1}{1-u^2} \frac{du}{dx}$$

مراجعة

((اسئلة اضافية))

EXAMPLE 1: Find $\frac{dy}{dx}$ for the following function.

1. $y = mx + b$

Sol:

$$y' = m$$

2. $y = \frac{1}{x}$

Sol:

$$y' = \frac{-1}{x^2}$$

3. $y = \frac{x}{x-1}$

Sol:

$$y' = \frac{(x-1)(1) - x(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

4. $y = \sqrt{x}$

Sol:

$$y = x^{1/2} \Rightarrow y' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

EXAMPLE 2: Find the value of the derivative.

1. $\left.\frac{ds}{dt}\right|_{t=1}$ if $s = 1 - 3t^2$

Sol:

$$\frac{ds}{dt} = 0 - 6t \qquad \left.\frac{ds}{dt}\right|_{t=1} = -6(-1) = 6$$

2. $\left.\frac{dy}{dx}\right|_{x=\sqrt{3}}$ $\quad y=1-\frac{1}{x} \quad \Rightarrow \quad y=1-x^{-1}$

Sol:

$$\frac{dy}{dx}=0-(-1)(x^{-2})$$

$$\frac{dy}{dx}=\frac{1}{x^2} \qquad \Rightarrow \quad \left.\frac{dy}{dx}\right|_{x=\sqrt{3}}=\frac{1}{(\sqrt{3})^2}=\frac{1}{3}$$

3. $\left.\frac{dr}{d\theta}\right|_{\theta=0}$ $\quad$ if $r=\frac{2}{\sqrt{4-\theta}} \quad \Rightarrow \quad r=2(4-\theta)^{-1/2}$

Sol:

$$\frac{dr}{d\theta}=-\frac{1}{2}(2)(4-\theta)^{-3/2}(-1)$$

$$\left.\frac{dr}{d\theta}\right|_{\theta=0}=\frac{1}{4^{3/2}}=8$$

EXAMPLE 3: Find the derivative of $y=\frac{t^2-1}{t^2+1}$

Sol:

$$\frac{dy}{dt}=\frac{(t^2+1)(2t)-(t^2-1)(2t)}{(t^2+1)^2}$$

$$\frac{dy}{dt}=\frac{4t}{(t^2+1)^2}$$

EXAMPLE 4: Find an equation for the tangent to the curve $y=x+\frac{2}{x}$ at the point $(1,3)$

Sol:

$$\frac{dy}{dt}=1+\frac{-2}{x^2}$$

The slope at $x=1$

$$y'|_{x=1}=[1-\frac{2}{x^2}]_{x=1}$$

$$=1-2=-1$$

$$m=-1$$

The line through $(1,3)$ with slope $m=-1$

$$y-3=(-1)(x-1)$$

$$y=-x+1+3$$

$$y=-x+4$$

EXAMPLE 5: Find higher derivatives $y=x^3-3x^2+2$

Sol:

First	$y'=3x^2-6x$
Second	$y''=6x-6$
Third	$y'''=6$
Fourth	$y''''=0$

EXAMPLE 6: Find $\dfrac{dy}{dx}$ for the following Trigonometric function.

1. $y=\dfrac{\sin x}{x}$

Sol:

$$y'=\frac{x\cos x-\sin x(1)}{x^2}=\frac{x\cos x-\sin x}{x^2}$$

2. $y=x^2\sin x$

Sol:

$$y'=x^2\cos x+2x\sin x$$

3. $y=\sin x\cos x$

Sol:

$$y'=\sin x(-\sin x)+\cos x\cos x$$

$$y'=\cos^2 x-\sin^2 x$$

4. $y=\dfrac{\cos x}{1-\sin x}$

Sol:

$$y'=\frac{(1-\sin x)(-\sin x)-\cos x(-\cos x)}{(1-\sin x)^2}$$

$$y'=\frac{-\sin x+\sin^2 x+\cos^2 x}{(1-\sin x)^2}=\frac{1-\sin x}{(1-\sin x)^2}$$

$$=\frac{1}{1-\sin x}$$

5. If $y=\sec^2 x$ find y''

Sol:

$$y'=\sec x\tan x$$

$$y''=\sec x\sec^2 x+\sec x\tan x\tan x$$

$$y''=\sec^3 x+\sec x\tan^2 x$$

6. Find the slope of the line tangent to the curve $y=\sin^5 x$ at point where $x=\pi/3$

Sol:

$$\frac{dy}{dt}=5\sin^4 x\cos x$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/3}=5(\frac{\sqrt{3}}{2})^4\cdot\frac{1}{2}=\frac{45}{32}$$

$\cos\pi/3=1/2$

$\sin\pi/3=\sqrt{3}/2$

7. If $x=2t+3$ and $y=t^2-1$ Find the value of $\dfrac{dy}{dx}$ at $t=6$

Sol:

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{2t}{2}=t$$

$$\frac{dy}{dx}=6 \qquad t=6$$

Hint: Note that we are also able to find $\dfrac{dy}{dx}$ as a function of x

$x=2t+3$ $\qquad$ $y=t^2-1$

$x-3=2t$

$t=\dfrac{x-3}{2}$

8. Find $\frac{dy}{dx}$ if $y^2 = x$

Sol:

$$2y\,y' = 1$$

$$y' = \frac{1}{2y}$$

9. Find the slope of circle $x^2 + y^2 = 25$at the point $(3, -4)$

Sol:

$$2x + 2y\,y' = 0$$

$$y' = \frac{-2x}{2y} \quad \Rightarrow \; y' = -\frac{x}{y}$$

$$y'\big|_{(3,-4)} = -\frac{3}{-4} = \frac{3}{4}$$

10. Find $\frac{dy}{dx}$ if $y^2 = x^2 + \sin xy$

Sol:

$$2y\,y' = 2x + \cos xy(x\,y' + y)$$

$$2y\,y' = 2x + x\,y' \cos xy + y \cos xy$$

$$2y\,y' - x\,y' \cos xy = 2x + y \cos xy$$

$$y'(2y - x \cos xy) = 2x + y \cos xy$$

$$y' = \frac{2x + y \cos xy}{2y - x \cos xy}$$

11. Find $\frac{d}{dx}(\cos x)^{-1/5}$

Sol:

$$-\frac{1}{5}(\cos x)^{-6/5}(-\sin x)$$

$$\frac{1}{5}\sin x(\cos x)^{-6/5}$$

12. Find $\frac{d}{dx}\ln 2x$

Sol:

$$y' = \frac{1}{2x}\frac{d}{dx}(2x) = \frac{1}{2x}\cdot 2 = \frac{1}{x}$$

13.Find $\frac{d}{dx}\ln(x^2+3)$

Sol:

$$y' = \frac{1}{x^2+1}\cdot(2x) = \frac{2x}{x^2+1}$$

14.Find $\frac{d}{dx}\ln x^r$

Sol:

$$y' = \frac{1}{x^r}\cdot rx^{r-1}$$

$$y' = \frac{1}{x^r} r\, x^r\, x^{-1} = \frac{r}{x}$$

15.Find $\frac{dy}{dx}$ if $y = \frac{(x^2+1)(x+3)^{1/2}}{x-1}$ $\quad x > 1$

Sol: we take natural logarithm of the both side

$$\ln y = \ln\frac{(x^2+1)(x+3)^{1/2}}{x-1}$$

$$\ln y = \ln(x^2+1) + \ln(x+3)^{1/2} - \ln(x-1)$$

$$\ln y = \ln(x^2+1) + \tfrac{1}{2}\ln(x+3) - \ln(x-1)$$

$$\frac{y'}{y} = \frac{2x}{(x^2+1)} + \frac{1}{2}\frac{1}{(x+3)} - \frac{1}{(x-1)}$$

$$y' = y\left(\frac{2x}{x^2+1} + \frac{1}{2x+6} - \frac{1}{x-1}\right)$$

16. Find $\dfrac{dy}{dx}$ if 1) $y = 5e^{x}$ 2) $y = e^{-x}$ 3) $y = e^{\sin x}$

Sol:

1) $y' = 5e^{x}$

2) $y' = -e^{-x}$

3) $y' = e^{\sin x}\cos x$

17.Find $\dfrac{dy}{dx}$ if 1) $y = x^{\sqrt{2}}$ 2) $y = (2+\sin 3x)^{\pi}$

Sol:

1) $y' = \sqrt{2}\, x^{\sqrt{2}-1}$

2) $y' = \pi(2+\sin 3x)^{\pi-1}(\cos 3x)\cdot 3$

$y' = 3\pi\,(2+\sin 3x)^{\pi-1}(\cos 3x)$

18. Find $\dfrac{dy}{dx}$ if 1) $y = 3^{x}$ 2) $y = 3^{-x}$ 3) $y = 3^{\sin x}$

Sol:

1) $y' = 3^{x}\ln 3$

2) $y' = -3^{x}\ln 3$

3) $y' = 3^{\sin x}(\ln 3)\cos x$

19. Find $\dfrac{dy}{dx}$ if $y = x^{x}$ $x > 0$

Sol:

$\ln y = \ln x^{x}$

$\ln y = x \ln x$

$\dfrac{y'}{y} = (x\dfrac{1}{x} + \ln x)$

$y' = y\ (1+\ln x)$

$y' = x^{x}\ (1+\ln x)$

Or write x^x as a power of e

$$y = x^x = e^{x\ln x}$$

$$\frac{dy}{dx} = \frac{d}{dx} e^{x\ln x}$$

$$\frac{dy}{dx} = e^{x\ln x}(\frac{d}{dx} x\ln x)$$

$$\frac{dy}{dx} = x^x \ (x\frac{1}{x} + \ln x)$$

$$= x^x \ (1 + \ln x)$$

20. Find $\frac{d}{dx}\log_{10}(3x+1)$

Sol:

$$\frac{d}{dx}\log_{10}(3x+1)$$

$$\frac{d}{dx}\frac{\ln(3x+1)}{\ln 10}$$

$$\frac{1}{\ln 10}\frac{3}{3x+1}$$

21. If $\sinh u = \frac{e^u - e^{-u}}{2}$ find $\frac{d}{dx}\sinh u$

Sol:

$$\frac{d}{dx}\sinh u = \frac{e^u(1) - e^{-u}(-1)}{2}$$

$$= \frac{e^u + e^{-u}}{2}$$

$$= \cosh u$$

22. Find $\frac{d}{dt}\tanh\sqrt{1+t^2}$

Sol:

$$= \operatorname{sech}^2 \sqrt{1+t^2}\ \frac{d}{dt}(1+t^2)^{1/2}$$

$$= \operatorname{sech}^2 \sqrt{1+t^2}\ \frac{1}{2}\frac{1}{\sqrt{1+t^2}}\cdot 2t$$

$$= \operatorname{sech}^2 \sqrt{1+t^2}\ \ \frac{t}{\sqrt{1+t^2}}$$

Hint:

1) $\operatorname{sech}^{-1}x = \cosh^{-1}(1/x)$
2) $\operatorname{csch}^{-1}x = \sinh^{-1}(1/x)$
3) $\coth^{-1}x = \tanh^{-1}(1/x)$

23. Find y' or $\dfrac{dy}{dx}$ $\sin^{-1}(x\,y) = \cos^{-1}(x-y)$

Sol:

$$\frac{xy'+y}{\sqrt{1-(xy)^2}} = \frac{-(1-y')}{\sqrt{1-(x-y)^2}}$$

$$\frac{xy'}{\sqrt{1-(xy)^2}} + \frac{y}{\sqrt{1-(xy)^2}} = \frac{-1}{\sqrt{1-(x-y)^2}} + \frac{y'}{\sqrt{1-(x-y)^2}}$$

$$y'[\frac{x}{\sqrt{1-(xy)^2}} - \frac{1}{\sqrt{1-(x-y)^2}}] = -\frac{1}{\sqrt{1-(x-y)^2}} - \frac{y}{\sqrt{1-(xy)^2}}$$

$$y' = \frac{-\dfrac{1}{\sqrt{1-(x-y)^2}} - \dfrac{y}{\sqrt{1-(xy)^2}}}{\dfrac{x}{\sqrt{1-(xy)^2}} - \dfrac{1}{\sqrt{1-(x-y)^2}}}$$

$$y' = \frac{\dfrac{-\sqrt{1-(xy)^2} - y\sqrt{1-(x-y)^2}}{\sqrt{1-(x-y)^2}\ \sqrt{1-(xy)^2}}}{\dfrac{x\sqrt{1-(x-y)^2} - \sqrt{1-(xy)^2}}{\sqrt{1-(xy)^2}\ \sqrt{1-(x-y)^2}}}$$

$$y' = \frac{-\sqrt{1-(xy)^2} - y\sqrt{1-(x-y)^2}}{x\sqrt{1-(x-y)^2} - \sqrt{1-(xy)^2}}$$

24. Show for $y = \frac{U}{V}$ that $y'' = \frac{V(VU'' - UV'') - 2V'(VU' - UV')}{V^3}$

Sol:

$$y = \frac{U}{V}$$

$$y'' = \frac{VU' - UV'}{V^2}$$

$$y'' = \frac{V^2(VU'' + V'U' - UV'' - U'V') - (VU' - UV')2VV'}{V^4}$$

$$= \frac{V^2(VU'' + V'U' - UV'' - U'V') - 2V'(V^2U' - UVV')}{V^4}$$

$$= \frac{V[V(VU'' - UV'') - 2V'(VU' - UV')]}{V^4}$$

$$= \frac{V(VU'' - UV'') - 2V'(VU' - UVV')}{V^3}$$

25.Show that $y = 35x^4 - 30x^2 + 3$ satisfies $(1-x^2)y'' - 2xy' + 20y = 0$

Sol:

$$y' = 140x^3 - 60x \qquad y'' = 420x^2 - 60$$

$$(1-x^2)(420x^2 - 60) - 2x(140x^3 - 60x) + 20(35x^4 - 30x^2 + 3)$$

$$420x^2 - 60 - 420x^4 + 60x^2 - 280x^4 + 120x^2 + 700x^4 - 600x^2 + 60$$

$$420x^2 + 60x^2 + 120x^2 - 600x^2 - 420x^4 - 280x^4 + 700x^4 - 60 + 60 = 0$$

<u>H.W Derivative</u>

1) Find $\frac{dy}{dx}$ for the following function

1. $y = \csc^{-2/3}\sqrt{5x}$ **ans:** $\frac{5}{3\sqrt{5x}} \cdot \frac{\cot\sqrt{5x}}{\csc^{2/3}\sqrt{5x}}$

2. $y = (x-3)(1-x)$ **ans:** $4-2x$

3. $y = \frac{ax+b}{x}$ **ans:** $-\frac{b}{x^2}$

4. $y = \ln(\cos x)$ **ans:** $-\tan x$

5. $y = \tan x \sin x$ **ans:** $\sin x + \tan x \sec x$

6. $y = \frac{3x-4}{2x-3}$ **ans:** $\frac{1}{(2x+3)^2}$

7. $y = (\sqrt{x^3} - \frac{1}{\sqrt{x}})^2$ **ans:** $\frac{3(x^5-1)}{x^4}$

8. $y = \frac{\cos x}{x}$ **ans:** $\frac{-x \cdot \sin x + \cos x}{x^2}$

9. $y = \tan(\sec x)$ **ans:** $\sec^2(\sec x)\sec x \cdot \tan x$

10. $y = x^2 \sin x$ **ans:** $x^2\cos x + 2x \cdot \sin x$

11. $y = \sin^{-1}(5x^2)$ **ans:** $\frac{10x}{\sqrt{1-25x^4}}$

12. $y = \cot^3(\frac{x+1}{x-1})$ **ans:** $\frac{6}{(x-1)^2}\cot^2(\frac{x+1}{x-1})\csc^2(\frac{x+1}{x-1})$

13. $y = \sin(\ln\ x) + \cos(\ln\ x)$ **ans:** $2\cos(\ln\ x)$

14. $y = \cot^{-1}(\frac{1+x}{1-x})$ **ans:** $-\frac{1}{1+x^2}$

15. $y = \tan^{-1}\sqrt{4x^3 - 2}$ **ans:** $-\frac{6x^2}{(4x^3-1)\sqrt{4x^3-2}}$

16. $y = \sec^{-1}(3x^2+1)^3$ **ans:** $\frac{18x}{(3x^2+1)\sqrt{(3x^2+1)^6-1}}$

17. $y = \sin^{-1}2x\cos^{-1}2x$ **ans:** $\frac{2(\cos^{-1}2x - \sin^{-1}2x)}{\sqrt{1-4x^2}}$

18. $y = \tan^{-1}\ln x$ **ans:** $\frac{1}{x(1+(\ln x)^2)}$

19. $y = (\cos x)^{\sqrt{x}}$ **ans:** $\frac{y}{2\sqrt{x}}(\ln\cos x - 2x\tan x)$

20. $y = (\sin x)^{\tan x}$ **ans:** $y(1+\sec^2 x\ln\sin x)$

21. $y = \sqrt{2x^2 + \cosh^2(5x)}$ **ans:** $\frac{2x + 5\cosh(5x)\cdot\sinh(5x)}{\sqrt{2x^2+\cosh^2(5x)}}$

22. $y = \sinh(\cos\ 2x)$ **ans:** $-2\sin\ 2x\cosh(\cos\ 2x)$

23. $y = \csc(1/x)$ **ans:** $\frac{1}{x^2}\cdot\csc(1/x)\cdot\cot(1/x)$

24. $y = x^2\cdot\tanh^2\sqrt{x}$ **ans:** $x\cdot\tanh\sqrt{x}(\sqrt{x}\,\mathrm{sech}^2\sqrt{x} + 2\tanh\sqrt{x})$

25. $y = \ln\frac{\sin x\cos x + \tan^3 x}{\sqrt{x}}$ **ans:** $\frac{\cos^2 x - \sin^2 x + 3\tan^2 x}{\sin\ x\cos x + \tan^3 x} - \frac{1}{2x}$

26. $y = \log_4 \sin\ x$ **ans:** $\frac{\cot x}{\ln 4}$

27. $y = e^{(x^2 - e^{5x})}$ **ans:** $(2x - 5e^{5x})e^{(x^2 - e^{5x})}$

28. $y = e^{x^2 \tan x}$ **ans:** $(x^2 \sec^2 x + 2x \tan x)e^{x^2 \tan x}$

29. $y = 7^{\csc\sqrt{2x+3}}$ **ans:** $\dfrac{-7^{\csc\sqrt{2x+3}} \ln 7 \csc\sqrt{2x+3}\ \cot\sqrt{2x+3}}{\sqrt{2x+3}}$

30. $y = [\ln(x^2+2)^2]\cos x$ **ans:** $\dfrac{4x\cos x}{x^2+2} - 2\ln(x^2+2)\sin x$

31. $y = \sinh^{-1}(\tan x)$ **ans:** $\sec x$

32. $y = \sqrt{1+(\ln x)^2}$ **ans:** $\dfrac{\ln x}{x\sqrt{1+(\ln x)^2}}$

33. $\dfrac{e^x}{\ln x}$ **ans:** $\dfrac{e^x(\ln x - 1)}{x(\ln x)^2}$

34. $x^3 \log_2(3-2x)$ **ans:** $2x^2 \log_2(3-2x) - \dfrac{2x^3}{(3-2x)\ln 2}$

35. $y = 2\cosh^{-1}\dfrac{x}{2} + \dfrac{x}{2}\sqrt{x^2-4}$ **ans:** $\dfrac{x^2}{\sqrt{x^2-4}}$

2) Verify the following derivative

1. $\dfrac{d}{dx}[5x + (\sqrt{x} + \dfrac{1}{\sqrt{x}})^2 = 6 - \dfrac{1}{x^2}$

2. $\dfrac{d}{dx}[\sqrt{x} + (ax^2 + bx + c)] = \dfrac{1}{2\sqrt{x}}(5ax^2 + 3bx + c)$

3) Find the derivative of y with respect to x in the following function:

1. $y = \dfrac{u^2}{u^2+1}$ and $u = 3x^2 - 2$ **ans:** $\dfrac{18x^2y^2}{(3x^3-2)^3}$

2. $y = \sqrt{u} + 2u$ and $u = x^2 - 3$ **ans:** $\dfrac{x}{\sqrt{x^2-3}} + 4x$

4) Find the second derivative for the following function

1. $y=(x+\frac{1}{x})^3$ **ans:** $6x+\frac{6}{x^3}+\frac{12}{x^5}$

2. $y=\sqrt{2x}+\frac{2\sqrt{2}}{\sqrt{x}}$ at $x=2$ **ans:** $\frac{1}{4}$

3. $y=x^2-2xy+y^2-16x=0$ **ans:** $\pm x^{-3/2}$

5) Find the third derivative of the function $y=\sqrt{x^3}$ **ans:** $-\frac{3}{8y}$

6) Show that $y=\frac{U}{V}$ that $y''=\frac{V(VU''-UV'')-2V'(VU'-UV')}{V^3}$

7) Show that $y=35x^4-30x^2+3$ satisfies $(1-x^2)y''-2xy'+20y=0$

8) Find $\frac{dy}{dx}$ for the following implicit function:

1. $\sqrt{xy}+1=y$ **ans:** $\frac{y}{2\sqrt{xy}-x}$

2. $\sinh y=\tan^2 x$ **ans:** $\frac{2\tan x\cdot\sec^2 x}{\cosh y}$

3. $x^3+4x\sqrt{y}-\frac{5y^2}{x}=3$ **ans:** $\frac{3x^2+5y^2x^{-2}+4\sqrt{y}}{10x^{-1}y-(2x/\sqrt{y})}$

4. $3xy=(x^3+y^3)^{3/2}$ **ans:** $\frac{3x^2\sqrt{x^3+y^3}-2y}{2x-3y^2\sqrt{x^3+y^3}}$

5. $\sin^{-1}(xy)=\cos^{-1}(x-y)$ **ans:** $\frac{y\sqrt{1-(x-y)^2}+\sqrt{1-(xy)^2}}{\sqrt{1-(xy)^2}-x\sqrt{1-(x-y)^2}}$

6. $y^2\sin(xy)=\tan x$ **ans:** $\frac{\sec^2 x-y^3\cos(xy)}{2y\sin(xy)+xy^2\cos(xy)}$

7. $x^3+x\tan^{-1}y=y$ **ans:** $\frac{(1+y^2)(3x^2+\tan^{-1}y)}{1+y^2-x}$

9) Prove the following function

1. $\frac{d}{x}\cot u = -\csc^2 u \frac{du}{dx}$

2. $\frac{d}{x}\csc u = -\csc u \cot u \frac{du}{dx}$

3. $\frac{d}{x}\cosh^{-1} u = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

4. $\frac{d}{x}\mathrm{sec}h^{-1} u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

5. $\frac{d}{x}\sin\mathrm{h}u = \cosh u \frac{du}{dx}$

6. $\frac{d}{x}\csc\mathrm{h}u = -\csc\mathrm{h}u \coth u \frac{du}{dx}$

7. $\frac{d}{x}\sinh^{-1} u = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$

8. $\frac{d}{x}\sec\mathrm{h}^{-1} u = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx}$