

Application of Derivatives

- a. Increasing or decreasing function : the sign of dy/dx .
- b. Significance of the sign of the second derivative.
- c. Curve plotting.
- d. Maxima and minima , problems .
- e. Concavity and point of inflection .

Application of derivative

1. The slop of curve

Secant to the curve is a line through two points on a curve.

Slope and tangent lines

1. We start with what we can calculate, namely the slope of the secant through P and a point Q nearby on the curve.
2. We find the limiting value of the secant slope (if it exists) as Q approaches P along the curve.
3. We take this number to be the slope of the curve at P and define the tangent to the curve at P to be the line through P with this slope.

$$\text{The slop } m = f'(x) = \frac{dy}{dx}$$

EXAMPLE 1: Write an equation for the tangent line at $x = -1$ of the curve $f(x) = y = 4 - x^2$

Sol:

$$\frac{dy}{dx} = -2x$$

The slope at $x = -1$

$$y'|_{x=-1} = (-2x)_{-1}$$

$$= -2 \cdot (-1) = 2$$

$$m = 2$$

$$y = 4 - (-1)^2 = 3$$

The line through $(-1, 3)$ with slope $m = 2$

$$y - 3 = (2)(x - (-1))$$

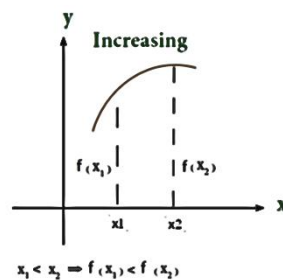
$$y = 2x + 2 + 3$$

$$y = -x + 5$$

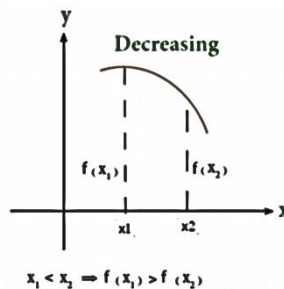
Increasing and decreasing function

Let f be a function defined on an interval I and let x_1 and x_2 be any two points in I .

1. If $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ then f is said to be increasing on I .



2. If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$ then f is said to be decreasing on I .



First Derivative Test

1. If $f'(x) > 0$ at each point $x \in (a, b)$, then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ at each point $x \in (a, b)$, then f is decreasing on $[a, b]$.

Definition Concave Up, Concave Down

The graph of a differentiable function $y = f(x)$ is

- (a) **Concave up** on an open interval I if f' is increasing on I
- (b) **Concave down** on an open interval I if f' is decreasing on I.

Second Derivative Test

- 1- If $f''(x) > 0$ on I, the graph of f over I Concave up
- 2- If $f''(x) < 0$ on I, the graph of f over I Concave down

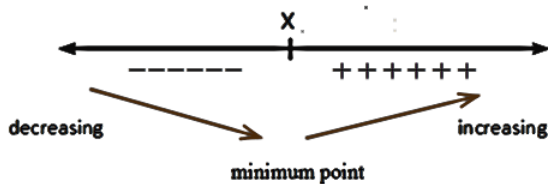


To find critical point (local maximum point and local minimum), concavity (Concave up and Concave down) and point of inflection point.

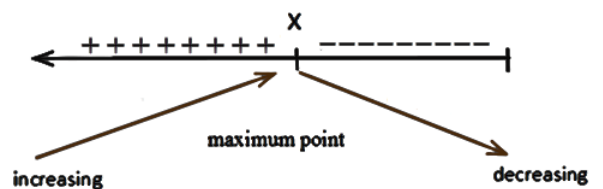
لايجاد النقاط الحرجة النهاية العظمى والصغرى والتحدب بنوعيه ونقطة الانقلاب.

1. First derivative test for local extrema f' or y'
2. $y' = 0$ the First derivative is zero at $x = ?$, find the value of x

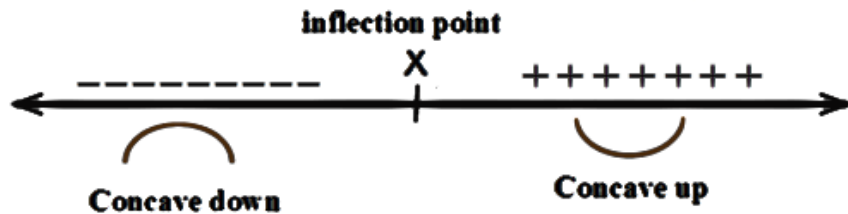
either



or



3. Second derivative test for concavity
4. Also $y'' = 0$ the second derivatives is zero that mean find value of x



EXAMPLE 1: Find all critical points, local minimum and maximum, concavity and inflection point. $y = 2x^3 - 3x^2 - 12x + 3$

Sol:

Test 1

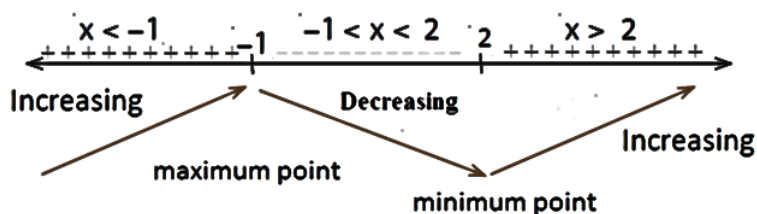
$$y' = 0 \Rightarrow y' = 6x^2 - 6x - 12 = 0$$

$$\div 6 \Rightarrow x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x - 2 = 0 \Rightarrow x = 2$$

$$x + 1 = 0 \Rightarrow x = -1$$



لكي نجد مناطق التزايد والتناقص نختبر المشتقة الاولى في خط الاعداد باخذ عدد اكبر من 2 ونلاحظ اشارة المشتقة وكذلك عدد في الفترة $(-1, 2)$ وكذلك عدد عدد اقل من -1 كما في الرسم اعلاه

1. $\{x: x \in R, x > 2\}$ الدالة تكون متزايدة
 $\{x: x \in R, x < -1\}$
2. $(-1, 2)$ ومنتناقص في الفترة

Sub -1 in $y = 2x^3 - 3x^2 - 12x + 3$

$$y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 3 = 10$$

$P = (-1, 10)$ **Maximum point** نقطة عظمى

Sub 2 in $y = 2x^3 - 3x^2 - 12x + 3$

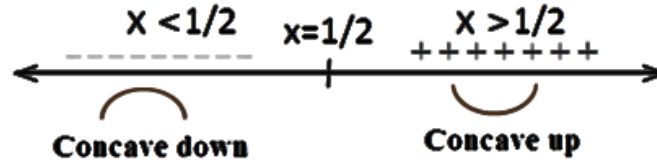
$$y = 2(2)^3 - 3(2)^2 - 12(2) + 3 = -17$$

$P = (2, -17)$ **Minimum point** نقطة صغرى

Test 2

$$y'' = 12x - 6 \quad y'' = 0$$

$$12x - 6 = 0 \Rightarrow x = 1/2$$



لكي نجد مناطق التفرع والتحدب نختبر المشتقة الثانية في خط الاعداد باخذ عدد اكبر من $1/2$ ونلاحظ اشارة المشتقة وكذلك عدد اقل من $1/2$ ونلاحظ اشارة المشتقة كما في الرسم اعلاه

1. $\{x : x \in \mathbb{R}, x > 1/2\}$ **Concave up** منطقة التفرع
2. $\{x : x \in \mathbb{R}, x < 1/2\}$ **Concave down** منطقة النحدب

Sub $1/2$ in $y = 2x^3 - 3x^2 - 12x + 3$

$$y = 2(1/2)^3 - 3(1/2)^2 - 12(1/2) + 3 = -3.5 \quad \text{at } x = 1/2 \quad y = -3.5$$

$P = (1/2, -3.5)$ **Inflection point** نقطة الانقلاب

EXAMPLE 2: Find all critical points, local minimum and maximum, concavity and inflection point. $y = (x - 4)^2$

Sol:

$$y' = 2(x - 4)(1)$$

$$y' = 2x - 8 \Rightarrow y' = 0$$

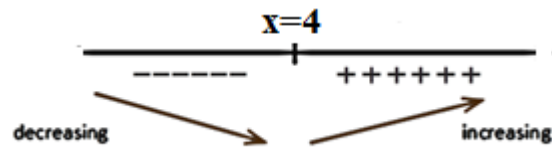
$$2x - 8 = 0 \Rightarrow 2x = 8$$

$$x = 4$$

sub 3 in $y' \Rightarrow y' < 0$ decreasing

sub 5 in $y' \Rightarrow y' > 0$ increasing

$x = 4$ Sub in $y = (x - 4)^2$



$$y = (4 - 4)^2$$

$$y = 0$$

$= (4, 0)$ **Minimum point**

$$y'' = 2$$

$y'' > 0$ **Increasing Concave up**

Inflection point dose not exit

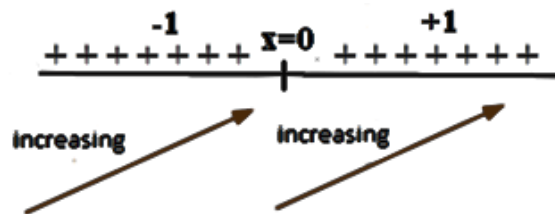
EXAMPLE 3: Sketch and Find all critical points, local minimum and maximum, concavity, concave up, concave down and inflection point. $y = x^3$

Sol: **Test 1**

$$y' = 3x^2 \Rightarrow y' = 0$$

$$3x^2 = 0 \Rightarrow x = 0$$

sub $\begin{matrix} 1 \\ -1 \end{matrix}$ $\Rightarrow y' > 0$ increasing



Test 2

$$y'' = 6x \quad y'' = 0$$

$$6x = 0 \Rightarrow x = 0$$

also $\begin{matrix} 1 \\ -1 \end{matrix}$ in $y'' \Rightarrow$ at $x = 1 \Rightarrow y'' > 0$ **Concave up**

at $x = -1 \Rightarrow y'' < 0$ **Concave down**

$$x = 0 \quad \text{subin } y = x^3$$

$y = 0$ p $(0,0)$ **Inflection point**

x	$y = x^3$
0	0
-1	-1
1	1
-2	-8
2	8

