Trigonometric function integral $\sin u$, $\cos u$

1. $\sin u$ $\qquad \frac{d}{x}\sin u = \cos u \frac{du}{dx}$

$$\boxed{\int \cos u \, du = \sin u + c}$$

2. $\cos u$ $\qquad \frac{d}{x}\cos u = -\sin u \frac{du}{dx}$

$$\boxed{\int \sin u \, du = -\cos u + c}$$

EXAMPLES:

1. $\int \cos 3x \, dx = \frac{\sin 3x}{3} + c$

2. $\int \sin 7x \, dx = \frac{-\cos 7x}{7} + c$

3. $\int \sin(3x-1) \, dx = \frac{-\cos(3x-1)}{3} + c$

اذا كانت الدالة اسية ومشتقة داخل القوس متوفرة عندها نستخدم القوانين التالية

1. $\boxed{\int \sin^n au \cos au \, du = \frac{\sin^{n+1} au}{(n+1)a} + c}$

2. $\boxed{\int \cos^n au \sin au \, du = \frac{-\cos^{n+1} au}{(n+1)a} + c}$

EXAMPLE: **1.** $\int \sin^7 3x \cos 3x \, dx = \frac{\sin^8 3x}{(8)(3)} + c$

2. $\int \sin 2x \cos^3 2x\, dx = -\dfrac{\cos^4 2x}{(4)\,(2)} + c$

3. $\int \sin x \cos x\, dx = \int (\sin^1 x) \cos x\, dx = \dfrac{\sin^2 x}{2a} + c$

اذا كانت الدالة اسية والمشتقة غير متوفرة نتبع مايلي

1. اذا كانت الدالة اسية والمشتقة غير متوفرة وكان الاس عدد زوجي

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

EXAMPLES: **1.** $\int \sin^2 x\, dx = \int (\frac{1}{2} - \frac{1}{2}\cos 2x)\, dx$

$$= \int \frac{1}{2} dx - \frac{1}{2}\int \cos 2x\, dx$$

$$= \frac{1}{2}x - \frac{1}{2}\frac{\sin 2x}{2} + c$$

2. $\int \cos^2 3x\, dx = \int (\frac{1}{2} + \frac{1}{2}\cos 2(3x))\, dx$

$$= \int \frac{1}{2} dx + \frac{1}{2}\int \cos 6x\, dx$$

$$= \frac{1}{2}x + \frac{1}{2}\frac{\sin 6x}{6} + c$$

2. اذا كانت الاس عدد فردي نستخدم القانون

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$\cos^2 x = 1 - \sin^2 x$

EXAMPLE 1 $\int \sin^3 x\,dx = \int \sin^2 x \sin x\,dx$

$$= \int (1 - \cos^2 x) \sin x\,dx$$

$$= \int (\sin x - \cos^2 x \sin x)\ dx$$

$$= \int \sin x\,dx - \int \cos^2 x \sin x\,dx$$

$$= -\cos x + \frac{\cos^3 x}{3} + c$$

EXAMPLE 2 $\int \cos^3 2x\,dx = \int \cos^2 2x \cos 2x\,dx$

$$= \int (1 - \sin^2 2x) \cos 2x\,dx$$

$$= \int (\cos 2x - \sin^2 2x \cos 2x)\ dx$$

$$= \int \cos 2x\,dx - \int \sin^2 2x \cos 2x\,dx$$

$$= \frac{\sin 2x}{2} - \frac{\sin^3 2x}{3(2)} + c$$

EXAMPLE 3: $\int \sin^4 x \quad dx = \int (\sin^2 x)^2\ dx$

$$= \int (\frac{1}{2} - \frac{1}{2}\cos 2x)^2\ dx$$

$$= \int (\frac{1}{4} - \frac{1}{4} 2\cos 2x + \frac{1}{4}\cos^2 2x)\,dx$$

$$= \int \frac{1}{4}dx - \frac{1}{2}\int \cos 2x\,dx + \frac{1}{4}\int \cos^2 2x\ dx$$

$$=\frac{1}{4}x-\frac{1}{2}\frac{\sin 2x}{2}+\frac{1}{4}\int(\frac{1}{2}+\frac{1}{2}\cos 4x)\,dx$$

$$=\frac{1}{4}x-\frac{1}{2}\frac{\sin 2x}{2}+\frac{1}{4}\frac{1}{2}x+\frac{1}{4}\frac{1}{2}\frac{\sin 4x}{4}+c$$

$$=\frac{1}{4}x-\frac{1}{4}\sin 2x+\frac{1}{8}x+\frac{\sin 4x}{32}+c$$

EXAMPLE 4: $\int\cos^4 3x\quad dx=\int(\cos^2 3x)^2\,dx$

$$=\int(\frac{1}{2}+\frac{1}{2}\cos 6x)^2\,dx$$

$$=\int(\frac{1}{4}+\frac{1}{2}\cos 6x+\frac{1}{4}\cos^2 6x)\,dx$$

$$=\int\frac{1}{4}dx+\frac{1}{2}\int\cos 6x\,dx+\frac{1}{4}\int\cos^2 6x\;dx$$

$$=\frac{1}{4}x+\frac{1}{2}\frac{\sin 6x}{6}+\frac{1}{4}\int(\frac{1}{2}+\frac{1}{2}\cos 12x)\,dx$$

$$=\frac{1}{4}x+\frac{1}{12}\sin 6x+\frac{1}{4}(\frac{1}{2}x+\frac{1}{2}\frac{\sin 12x}{12})+c$$

EXAMPLE 5: $\int\sin^5 x\,dx=\int(\sin^2 x)^2\sin x\,dx$

$$=\int(1-\cos^2 x)^2\;\sin x\,dx$$

$$=\int(1-2\cos^2 x+\cos^4 x)\;\sin x\;dx$$

$$=\int(\sin x-2\cos^2 x\sin x+\cos^4 x\sin x)\;dx$$

$$= -\cos x + \frac{2\cos^3 x}{3} - \frac{\cos^5 x}{5} + c$$