**Lecture 1**

**CH1: Functions**

**S1.1 : Functions and Their Graphs**

**Definition : A function *f* ( or a mapping *f* ) from a set  *A* to a set *B* is a rule that assigns to each element a of *A* exactly one element b of *B* . The set *A* is called the domain of *f* and the set *B* is called the codomain of *f* . If *f* assigns b to a , then b is called the image of a under *f* . The subset of *B* comprised of all the images of elements of *A* under *f* ( which is denoted by  ) is called the image of *A* under *f* ( or the range of *f* ) .**

**We use** ** to mean that *f*  is a function from *A* to *B* . We will write *f* ( a ) = b to indicate that b is the image of a under *f .***

**Example 1.1.1:**

**Let *A* = { 2, 4, 5 } ,  *B* = { 1, 2, 3, 6 } , and  be the function defined by  *f* ( 2 ) = 1 , *f* ( 4 ) = 3 , *f* ( 5 ) = 6 . Then the domain of *f* is**

 ***A* = { 2, 4, 5 } , the codomain of *f*  is *B* = { 1, 2, 3, 6 }, and the range**

**of  *f* = { 1, 3, 6 } .**

**Counter example :**

**Let *C* = { 1, 2, 3, 4 } and *D* = { 2, 3, 4, 5 } , and let  *h* be the rule defined by  *h* ( 1 ) = 2 , *h* ( 1 ) = 4 , *h* ( 2 ) = 3 , *h* ( 3 ) = 5 , *h* ( 4 ) = 4 , then *h* is not a function from  *C* to *D* since there are two different elements ( 2 and 4 ) belong to *D* are assigned to the same element 1 of  *C* .**

**Example 1.1.2: Find the domain and the range of the function *f* defined by  .**

**Solution : For  to be real , *x* + 10 must be greater than or equal to 0 . That is ,  which means that  .**

**Thus the domain is  and the range is  .**

**Exercises:**

**1) Let *A* = { 2, 4, 5,7 } ,  *B* = { 1, 2, 3, 6,9 } , and  be the**

 **function defined by  *f* ( 2 ) = 9 , *f* ( 4 ) = 3 , *f* ( 5 ) = 6 , *f* ( 7 ) = 2. Find**

 **the domain of *f*  , the codomain of *f*  , and the range of *f* .**

**2) Find the domain and the range of the function *f*  defined by**

 **.**

**3) Find the domain and the range of the function *f*  defined by**

 ** .**

**Definition: The graph of a function *f* is the line passing through all**

**the points ( *x* , ) on the  *x y* - plane .**

**Definition: The *y* - coordinate of the point where a graph of a function**

**intersect the *y* - axis is called the *y* - intercept of the function .**

**Definition: The *x* - coordinate of a point where a graph of a function**

**intersects the *x* - axis is called an *x* - intercept of the function .**

**Remarks :**

**1) The graph of any function *f* has at most one *y* - intercept . The**

 **graph of the function *f* has exactly one *y* - intercept if 0 is in the**

 **domain of the function *f* and the *y* - intercept is *f* (** 0 **) .**

**2) The graph of any function *f* has no *x* - intercept if there is no *x***

 **in the domain of the function *f* such that *f* ( *x* ) =** 0 **.**

 **The graph of a function *f* has one or more than one *x* - intercepts**

 **if *f* ( *x* ) =** 0 **for some *x* in the domain of *f* , and the number of**

 ***x* - intercepts is the number of the distinct solutions of the**

 **equation *f* ( *x* ) =** 0 **.**

**Properties of Functions :**

1. **A function is called an even function of if**

 **.**

1. **A function is called an odd function of if**

 **.**

**Lecture 2**

**S1.2 : Linear Functions and their Graphs**

**Definition: A function  is called a linearfunction if *f* is defined by  , **

**where  *a* and *b* are real numbers .**

**Example 1.2.1: The function  defined by **

**is a linear function .**

**Example 1.2.2: The function  defined by **

**is a linear function .**

**Example 1.2.3: The function  defined by **

**is a linear function .**

**Example 1.2.4: Let  be the linear function defined by  . Find the *x* - intercept and the *y* - intercept of *f* .**

**Solution:   **

 ** **

 ** **

**Therefore the  *x* - intercept is **

**  the *y* - intercept is 10 .**

**Example 1.2.5: Let  be the linear function defined by  . Find the *x* - intercept and the *y* - intercept of *g* .**

**Solution:   **

 **   **

**Therefore the  *x* - intercept is 30**

**  the *y* - intercept is  .**

**Lecture 3**

**Graph of a linear function :**

**The graph of a linear function  *f* is the straight line passing through the two points (** a **,** 0 **) and (** 0 **,** b **) where a is the *x* - intercept of the function *f* and b is the *y* - intercept of the function *f* .**

**Remark : The graph of any linear function *f* has exactly one**

***x* - intercept and has exactly one *y* - intercept .**

**Example 1.2.6: Let  be the linear function defined by  . Find the *x* - intercept and the *y* - intercept of *f* ,**

**then graph the function *f* .**

**Solution:   **

 ** **

 ** **

**Therefore the  *x* - intercept is**  **.**

**  the *y* - intercept is** 7 **.**

**Thus the graph of the function *f* is the straight line passing through**

 **the two points (** 3.5 **,** 0 **) and (** 0 **,** 7 **) .**

**Thus the graph of the function *f* is** **the following graph**

 ***y-axis***

 (3.5,0)

 (0,7)

 ●

 ●

 ***x-axis***

****

 (0,0)

**Example 1.2.7: Let  be the linear function defined by  . Find the *x* - intercept and the *y* - intercept of *g* , then**

**graph the function *g* .**

**Solution:   **

 ** **

 ** **

**Therefore the  *x* - intercept is **

**  the *y* - intercept is  .**

**Thus the graph of the function *g* is the straight line passing through**

**the two points (,** 0 **) and (** 0 **, ) .**

**Thus the graph of the function *g* is** **the following graph**

 ***x-axis***

 (-3,0)

 (0,12)

 ●

 ●

***y-axis***

 ****

 (0,0)

**Exercises:**

1. **Let**  **be the linear function defined by**  **.**

 **Find the *x* - intercept and the *y* - intercept of *f* .**

 **2) Let  be the linear function defined by  .**

 **Find the *x* - intercept and the *y* - intercept of *g* .**

 **3) Let  be the linear function defined by  .**

 **Find the *x* - intercept and the *y* - intercept of *f* , then graph the**

 **function *f* .**

 **4) Let  be the linear function defined by  .**

 **Find the *x* - intercept and the *y* - intercept of *g* , then graph the**

 **function *g* .**

**Lecture 4**

**S1.3 : Some well-known Functions and their Graphs**

1. **A function where c is a fixed number is called a**

 **constant function .**

**Example 1.3.1 : The function is a constant function**

**and its graph is**

***y-axis***

 ****

***y =* 1**

***x-axis***

1. **The absolute value function is defined by the**

 **formula**

**and its graph is**

***y-axis***

 ****

***x-axis***

***y = x***

***y = - x***

**Remember that .**

1. **A function where is a real number is called a power function .**

**Example 1.3.2 :**

**The function is a power function ( which is also a quadratic function ) and its graph is**

 

***x-axis***

***y-axis***

**Example 1.3.3 :**  **The function is a power function and its graph is**

***y-axis***

 

***x-axis***

**Example 1.3.4 : The function is a power function and its graph is**

***y-axis***

 

***x-axis***

**Example 1.3.5 : The function is a power function and its graph is**

 ****

***x-axis***

***y-axis***

1. **Let a be a positive real number other than 1 . The function**

 **is called the exponential function with base a .**

**Example 1.3.6 : Graph the exponential function**

**Answer : To draw the graph of , we can make use of a table**

**give values for  *x* and find the corresponding values for *y***

***x* = 0 gives *y* =  = 1 ,**

***x* = 1 gives *y* =  = 2 ,**

***x* = –1 gives *y* =  =  .**

**Following the process we make the table**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ***x*** | **– 4** | **– 3** | **– 2** | **– 1** | **0** | **1** | **2** | **3** | **4** |
| **2*x*** | **0.0625** | **0.125** | **0.25** | **0.5** | **1** | **2** | **4** | **8** | **16** |



**Example 1.3.7 : The function is an exponential function and its graph is**

**Answer :**

***x* = 0 gives *y* =  = 1 ,**

***x* = 1 gives *y* =  = 5 ,**

***x* = –1 gives *y* =  = 0.2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***x*** | **– 2** | **– 1** | **0** | **1** | **2** |
| **5*x*** | **0.04** | **0.2** | **1** | **5** | **25** |

***y-axis***

 

***x-axis***

**Exercise 1.3.8 :**  **Graph the exponential function  .**

**The properties of exponential function and their graph**

* **The domain is R (set of real numbers) .**
* **The range is R+ (set of positive real numbers) .**
* **The graph is always continuous (no break in the graph) .**

**Rules of Exponents : If and , the following rules of exponent should be hold for all real numbers and :**















1. **The function is called the natural exponential function whose base is , and its graph is**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***x*** | **– 2** | **– 1** | **0** | **1** | **2** |
| **e *x*** | **0.1353** | **0.3679** | **1** | **2.718** | **7.389** |



***y-axis***

***x-axis***

**Remark : Graph of e *x* and e –*x* are reflections of each other .**

1. **The function is called the logarithm function with base b where b is a positive number ; and , and the graph**

**of where b is greater than is the following graph**

****

**Remark :**  **means that** .

**Example 1.3.9 : The function is a logarithm function with base 2 and its graph is**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  ***x*** |  **0.25** |  **0.5** |  **1** |  **2** |  **4** |
| ***y* = log2 *x*** | **2** |  **1** |  **0** |  **1** |  **2** |

***y-axis***

 

***x-axis***

**Example 1.3.10 : Draw the graph of log 10 *x* .**

**Answer :**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ***x*** |  **0.5** |  **1** |  **5** |  **10** |  **15** |  **20** |  **50** |  **100** |
| ***y* = log10 *x*** |  **0.301** |  **0** |  **0.699** |  **1** | **1.176** | **1.301** | **1.699** |  **2** |

***y-axis***



***x-axis***

**Rules of logarithm : For and , and b is a positive number**

**we have the following rules :**

1.
2.
3. ** , where c can be any base .**

**Remarks :**

* **The logarithm of any number to the base of the same number**

**will be 1 (** ,  **etc …** **) .**

* **Logarithm of 1 to any base is 0 (**  **,**  **etc … ) .**
* **The logarithm function is defined only for positive numbers .**
* **The domain of the logarithm function is R+ .**
* **The range of the logarithm function is R .**
1. **The logarithm function with base e is called the natural logarithm function and will be denoted by ( i.e. ) and its graph is**



**Remarks :**

* **ln e = 1 ( since ln e = loge e )**
* **ln 1 = 0**

**Exercises 1.3.11 :** **Draw the graph for the following logarithmic**

**functions:**

1. **log 5 *x***
2. **log 8 *x***
3. **log 3 *x***
4. **A polynomial function is defined as**

 **where**

 **are constants .**

**Example 1.3.12 : The function is a polynomial function .**

***y-axis***

 ****

***x-axis***

**Lecture 5**

**Algebra of Functions**

**Definition: The sum , difference , product , and quotient of the functions *f* and *g* are the functions defined by**

**( *f* + *g* ) ( *x* ) = *f* ( *x* ) + *g* ( *x* ) sum function**

**( *f* – *g* ) ( *x* ) = *f* ( *x* ) – *g* ( *x* ) difference function**

**( *f* . *g* ) ( *x* ) = *f* ( *x* ) . *g* ( *x* ) product function**

** quotient function**

**The domain of each function is the intersection of the domains of *f* and *g* , with the exception that the values of *x* where *g*( *x* ) = 0 must be excluded from the domain of the quotient function .**

**Definition: Let *f* and *g*  be functions , then *f***  o ***g*  is called the composite of *g* and *f* and is defined by the equation**

**( *f***  o ***g* )( *x* ) = *f* ( *g* ( *x* ) ) .**

**The domain of *f***  o ***g* is the set  .**

**Example 1.3.13 : Let *f* and *g* be the functions defined by**

** and  . Find the functions *f* + *g* , *f* – *g***

 **, *f* . *g* , , *f***  o ***g*  , *g*** o ***f* and find their domains .**

**Solution :**

**( *f* + *g* ) ( *x* ) = *f* ( *x* ) + *g* ( *x* ) =  +  **

**( *f* – *g* ) ( *x* ) = *f* ( *x* ) –  *g* ( *x* ) =   **

**( *f* . *g* ) ( *x* ) = *f* ( *x* ) . *g* ( *x* ) =  .  **

****

**( *f***  o ***g* )( *x* ) = *f* ( *g* ( *x* ) ) = *f*  =  =**

**( *g***  o ***f* )( *x* ) = *g* ( *f* ( *x* ) ) = *g*  = **

 **= =**

**The domain of *f* = R**

**The domain of *g* = R**

**The intersection of the domains of *f* and *g* is R**

**Thus the domain of each of the functions  *f* + *g* , *f* – *g* , *f* . *g* , *f***  o ***g***

**, and *g*** o ***f* is R .**

**The domain of** **** = **** .

**Remark : The domain of any polynomial function is R .**

**Example 1.3.14 : Let *f* and *g* be the functions defined by**

***f* ( *x* ) = *x* + 5 and *g* ( *x* ) , Find *f* o *g* ( *x* ) , *g* o *f* ( *x* ) ,**

***f* o *g* ( 3) and *g* o *f* ( 3) .**

**Solution: *f* o *g* ( *x* ) = *f* ( *g* ( *x* ) )**

 ***g* o *f* ( *x* ) *g* ( *f* ( *x* ) ) *g* ( *x* + 5 )**

***f* o *g* ( 3)**

***g* o *f* ( 3)**

**Exersice 1.3.15 : Let *f* and *g* be the functions defined by**

 ***f* ( *x* ) = *x* – 4 and  . Find the functions *f* + *g* , *f* – *g***

 **, *f* . *g* ,  and find their domains .**

**Lecture 6**

**S 1.4 : Unit Circle and Basic Trigonometric Functions**

**Definition 1:** Let ***x*** be any real number and let *U* be the unit circle

with equation  ( the centre of the circle *U* is the point O (0,0) ,

and the radius of the circle *U* equals 1 ) . Start from the point A(1,0) on

*U* and proceed counterclockwise if  is positive and clockwise if

is negative around the unit circle  *U* until an arc length of has

been covered . Let P(***a* , *b***) be the point at the terminal end of the arc .

The measurement of the angle AOP is radians .

 If radians = ( degrees ) ,

 then the following six

 trigonometric functions of

 are defined in terms of the

 coordinates of the circular

 point P(***a , b***) :

 ●

  *b*

 **P**(*a*, *b*)

 *units*

 **A**(1,0)

 **O**(0,0)

 ●

 ●

 *radians*

  *a*

**1)  = *b* =  ( *x* radians) **

**2) =  *a* =  ( *x* radians) **

**3) =  **

 **=  ( *x* radians) **

**4) =  **

 **=  ( *x* radians) **

**5)  =  **

 **= ( *x* radians) **

**6)  =  **

 **=  ( *x* radians) **

**Remark 1:** Definition 1 uses the standard function notation , , with *f* replaced by the name of a particular trigonometric function . For example ,  actually means  and  actually means  .

**Remark 2:** Remember that  and

 .

**Theorem 1:**

For any real number *x* we have the following trigonometric identities :

**1)  .**

**2)  .**

**3)  .**

**4)  .**

**5)  .**

**6)  .**

**7)  .**

**8)  .**

**9)  .**

**10)  .**

**11) .**

**12) .**

**S 1.5: Graphs of Sine and Cosine Functions**

**1.5.1: Table for values of sin *x* , cos *x* , and tan *x* for selected values**

**of *x***

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Values of*****x*** | **Degrees** | **0** | **30** | **45** | **60** | **90** |
| **Radians** | **0** |  |  |  |  |
| **sin *x*** | **0** |  |  |  | **1** |
| **cos *x*** | **1** |  |  |  | **0** |
| **tan *x*** | **0** |  | **1** |  |  **Undefined** |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Values of*****x*** | **Degrees** | **120** | **135** |  **150** | **180** | **270** |
| **Radians** |  |  |  |  |  |
| **sin *x*** |  |  |  |  **0** |  |
| **cos *x*** |  |  |  |  | **0** |
| **tan *x*** |  |  |  |  **0** |  **Undefined** |

**Definition:** A function *f* is periodic if there exists a positive real number

 *p* such that  for all *x* in the domain of *f* .

The smallest such positive number *p*  is the period of *f .*

**Remarks :**

1. The functions sin *x* , cos *x* , sec *x* , and csc *x* are periodic functions

 with period 2 .

2) The functions tan *x* and cot *x* are periodic functions with period  .

**1.5.2: The Graph of sin *x***

The graph of the function *y* = sin *x* is the line passing through all the points ( *x* ,sin *x*) on the  *x y* - plane .

 The graph of the function *y* = sin *x* for the interval  is the line

passing through the points ( 0 , 0 ), ,  ,  , ,

 ,  ,  , and  which is shown in the

 **●**

**● ● ●**

 **● ●**

 **●**

following figure

 **ו**

 **ו**

 



 





The graph of the function *y* = sin *x* is shown in the following figure

 

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The period of the function  *y* = sin *x* is 2 . The domain of the function

 *y* = sin *x* is the set of all real numbers R .

The range of the function *y* = sin *x* is the interval ** .**

**1.5.3: The Graph of cos *x***

The graph of the function ***y*** = **cos *x*** is the line passing through all

the points **( *x* ,cos *x*)** on the  *x y* - plane .

The graph of the function *y* = cos *x* for the interval  is the line

Passing through the points **(** 0 **,** 1 **), ,  ,  , ,  ,  , ** , and  which is shown in the

following figure

 **● ●**

**● ● ●**

 **●**  **●**

 **● ●**

 **●**

 ****

 0



 

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 **ו**

 **ו**

The graph of the function *y* = cos *x* is shown in the following figure



 -

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The period of the function *y* = cos *x* is 2 .

The domain of the function *y* = cos *x* is the set of all real numbers R .

The range of the function *y* = cos *x* is the interval  .

**1.5.4: The Graphs of tan *x* and sec *x***

The graph of the function **** is the line passing through all

the points **( *x* ,tan *x*)** on the  *x y* - plane .

The graph of  is shown in the following figure

 ****

**0**

***x-axis***

 ***y-axis***

**The graph of  is shown in the following figure**

***y-axis***

 ****

**0**

***x-axis***

**Exercise: Draw the graph of** **the following** **trigonometric functions :**



